

Regulated Rate of Return: Beta and Its Drivers

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Abstract: Using the CAPM algebra that appeared first in Lintner's original derivation of CAPM, I explain how the firm's beta is affected by the mean-variance parameters of its cash flow or payoff distribution. By looking at CAPM in payoffs form, rather than its equivalent but more common returns form, we can understand beta not only in terms of asset returns and returns covariances but in terms of the firm's substantive risks and underlying cash flow characteristics. The insights gained are of deep importance for utilities regulation because they reveal how rulings that affect the mean and market covariance of the utility's cash flow must also affect its CAPM beta and cost of capital. The results of this analysis are not well known and contradict aspects of common CAPM discourse. They are nonetheless easily proved using nothing but existing CAPM algebra and simple numerical examples.

Two identical factories produce the same outputs at the same cost, but one sits on an earthquake fault. Both businesses are affected equally by the usual ups and downs in the economy. We would agree that the business with the shaky geological foundations has lower market value, but why? The CAPM answer is twofold. First, the threat to its cash flows tells us that its future cash flow has lower mean. Second, its lower mean combined with the same cash flow covariance with the market implies that it must have a higher discount rate. The first of these explanations is well known. The second is widely unknown.

1 Introduction

Logically correct interpretation of CAPM reveals that "unsystematic" cash flow risks, such as the risk of asset stranding, must affect the utility's cost of capital and should affect its regulated WACC.¹ This finding goes against standard textbooks and conventional CAPM wisdom but stands firmly on CAPM foundations. To discourage readers from dismissing such an "obviously false" conclusion, let me say immediately that the logic of this paper was explained by Lintner (1965) in his original CAPM exposition, and has appeared in the works of very well known finance theorists including Fama (1977). It was explained more recently in detail by Lambert, Leuz and Verrecchia (2007) and Johnstone and Grant (2025).

Each of these papers starts the same way by defining the assets or firms in a market by the joint probability distribution of their future cash flows. Each asset is boiled down under CAPM to its cash flow mean and covariance with the market (the market is understood as the sum of all assets in the reference set; e.g., all stocks on the ASX). The CAPM is written in its "certainty equivalent" form, in which we find the price P of a future cash flow C by the discounted sum

$$P = \frac{1}{R_f} [E[C] - \lambda \text{cov}(C, M)] \quad (1)$$

of (i) its mean cash flow $E[C]$, and (ii) a multiple $\lambda > 0$ of its cash flow covariance

¹The "regulated WACC" is the number (say 7.5%) that the regulator deems to be the NSP's cost of capital.

$\text{cov}(C, M)$ with the market. The market constant λ captures the inherent human risk aversion (or, more specifically, its aversion to payoff variance).²

When written in certainty equivalent form, the CAPM is an equation (2) *explicitly* for the asset price. The much more familiar way of writing the CAPM is by an equation describing not the price of the asset but its required expected market return. In that equation, the asset price is implied in a roundabout elliptical way by implication rather than by an explicit function.

If we take the asset's cash flow as C , its return factor is the ratio C/P and hence its expected return $E[C]/P$ depends on P and hence on every variable that affects its CAPM price P . Since P in CAPM is a function of the asset's expected (mean) cash flow and its cash flow covariance with the market, so must be the asset's return and expected return. Lintner (1966) showed that this relationship is delightfully simple. He proved that the CAPM expected return on an asset depends on its "Lintner ratio" of cash flow covariance to CAPM mean.

That simple ratio is the underlying mathematical driver of the asset's beta and expected return. It is more fundamental than beta because it involves just two parameters, whereas beta is a function of those parameters and three others as well (see below). In Johnstone (2017, 2020), I referred to the same ratio as "Fama's ratio" because the earliest explanation I had found was in Fama (1977). Later I read Lintner's famous paper more carefully and came to the original source.³

1.1 A Quick Shortcut Explanation

Before going into a full explanation and illustration, let me explain a simple example that proves the point. Suppose that there are two assets A and B with prices P_A and P_B . Asset A has positive beta β_A and asset B is a purely random future cash flow that is completely independent of the market. Hence, asset B 's cash flow risk is "unsystematic", in the sense that it is not dependent on market conditions, so its market correlation and beta are zero.

Now, if we combine the two assets into one, we are adding a zero beta cash flow to asset A . According to well known textbooks, asset A 's cost of capital stays unchanged and we should discount the combined expected cash flow of the newly merged asset

²In a full derivation of CAPM equation (1), the constant λ is traced to the investors' utility functions and their aggregate risk aversions. See e.g., Lambert et al (2007) and Johnstone (2025).

³Other authoritative sources with the same explanation are Stapleton (1971) and Hull (1986).

at the same discount rate as previously applied to asset A.

That conclusion is badly wrong. The merged asset has CAPM price $P_A + P_B$ and its beta is the price-weighted average of the individual asset's betas

$$\beta_{A+B} = \frac{P_A \beta_A + P_B \beta_B}{P_A + P_B}.$$

Or put another way, the appropriate discount rate for the new asset is the price weighted average of the existing discount rate on asset A and the risk-free rate applicable to asset B. This simple proof shows how a purely unsystematic or idiosyncratic cash flow affects the cost of capital. The proof requires just CAPM and rests on the mathematical fact that asset prices under CAPM are additive (Luenberger, 1988).

It is widely known that the beta of the sum of two assets, or two assets viewed as one, is the price-weighted average beta of the separate assets. Similarly, the expected return on the merged asset is the price-weighted average of the expected returns on the individual assets. So how did finance come to a different view?

It seems that the mistake and all surrounding confusion comes from assuming that what is true of returns must also be true of cash flows. Specifically, we know from the usual exposition of CAPM that returns that are uncorrelated with the market are unsystematic and unpriced. That correct conclusion has been taken wrongly as if it applies also to cash flows. Specifically, the false connection is that cash flows that are uncorrelated with the market are unpriced, so risks of loss due to purely random events like storms and industrial accidents must also be unpriced. That ignores the CAPM fact that cash flows C are exogenous ("from nature") but returns are endogenous, simply because you need the CAPM price P before you can calculate a return C/P on that price.

To avoid this mistake, we need to understand and reconcile the conclusions drawn from CAPM in both its returns form and its payoffs form. These are merely different mathematical forms of the very same model, one representing the asset by its price P and the other by its expected return, $E[R] \equiv E[C]/P$.

By viewing CAPM through both formats, it becomes clear that the expected return on an asset - that is the ratio of its expected payoff $E[C]$ to its price P - is not linear in $E[C]$, simply because that ratio $E[C]/P$ when expanded shows that the

expected return is the non-linear function

$$\frac{E[C]}{E[C] - \lambda \text{cov}(C, M)}.$$

The common mistake is to interpret the expected return $E[C]/P$ as if P is an exogenous constant, thus concluding falsely that expected returns are linear in expected cash flows $E[C]$.

CAPM algebra indeed recognizes P as a constant but moreover as an *unknown endogenous constant* formed under market equilibrium. Lintner's great advance was to reveal that unknown constant as a function (1) of the cash flow's parameters, thus proving not only how to find the equilibrium value of P but also what drives it in the most fundamental terms.

1.2 Key to CAPM Statistical Algebra

To understand Lintner's CAPM algebra, the "trick" is to recognize that the asset price P is a constant for mathematical purposes, albeit an unknown constant. Thus, when it comes to writing the asset's returns variance in terms of its cash flow variance, we can use the usual statistical laws. Specifically, defining returns as either $R = C/P$ or equivalently lower case $r = C/P - 1$, the returns variance $\text{var}(R) = \text{var}(1 + r)$ can be written as a function of the payoff variance and payoff covariance with the market return r_M , as respectively

$$\text{var}(1 + r) \equiv \text{var}(R) \equiv \text{var}\left(\frac{C}{P}\right) = \frac{1}{P^2} \text{var}(C)$$

and

$$\text{cov}(R, r_M) = \text{cov}\left(\frac{C}{P}, r_M\right) = \frac{1}{P} \text{cov}(C, r_M).$$

These mathematical rules of variance and covariance are essential knowledge when moving back and forth between the payoffs and returns expressions of CAPM. They do not appear in most undergrad finance textbooks and, for that reason alone, little is revealed about the advantages and insights available from viewing CAPM simultaneously through both of its mathematical forms.

2 Correct Interpretation of CAPM

Fundamental implications of CAPM are best exposed by expressing endogenous market outcomes (asset prices, expected returns and betas) as functions of exogenous inputs.

The exogenous inputs are the firm's cash payoff parameters (means and covariances) along with the market's risk aversion and the prevailing risk-free interest rate. The payoffs approach, advocated by Cochrane (2001) and common in financial economics theory, contrasts with models favored by empiricists that take returns rather than payoffs as primitive⁴

... thinking in terms of returns takes us away from the central task of finding asset prices. (Cochrane 2001, p.11)

The advantage of focusing on payoffs is evident even in empirical finance where observed returns and related observations are often explained by reference to firms' cash flow characteristics. For instance, much empirical research (e.g., Chen, Da and Zhao, 2013) links stock returns to various fundamental risks affecting cash flows, such as for example the variability in cash flows brought by higher operating leverage. Similarly, an important strand of accounting research (e.g., Lambert, Leuz and Verrecchia, 2007) links the firm's cost of capital to investors' perceptions about the risk of its cash flows (see also Johnstone, 2016). In the following analysis, I apply both the returns and payoffs expressions of CAPM, which are mathematical analogies of each other and merely alternative expressions of the same equilibrium model. Ultimately, they have exactly the same implications.

To see the advantage of this approach, consider now the well trodden *returns* form of the CAPM

$$E[r] = r_f + \beta (E[r_M] - r_f). \quad (2)$$

In this familiar equation, the firm's endogenous expected return $E[r]$ is written as a function of two endogenous variables, the firm's beta β and the market risk premium $E[r_M] - r_f$. There is no explicit role in this CAPM format for the firm's underlying exogenous payoff parameters, yet ideally we seek to understand their effects on the firm's and market's required return. I therefore call on the payoffs form of the model

⁴Empirical finance treats the market returns process as primitive whereas economists' asset pricing models treat the firm's raw cash flows as primitive.

to introduce assets' payoff parameters and reveal their effects on observable CAPM market outcomes - including specifically asset prices, returns and betas. The results uncover a key equation for the firm's cost of capital lying "hidden" in Lintner's original CAPM exposition, together with an elegant new equation for the firm's beta.

2.1 Simple Numerical Example

I start by considering a textbook style numerical example where an idiosyncratic risk is proved to affect the firm's cost of capital.⁵ I then explain and clarify Lintner's CAPM algebra to understand in general CAPM terms how that happens.

A firm is faced with a new idiosyncratic zero-beta risk threatening its reputation and bank account. Using textbook CAPM mechanics matching those applied by Grinblatt and Titman (1998, pp.385-7), I show that the firm's cost of capital jumps, and explain why. The existing firm is a business that pays net stochastic cash amount X at period end. At period start, the business has an expected terminal cash payoff of $E[C] = 100$ and a payoff variance of $\text{var}(C) = 1500$. The payoff correlation with the market return r_M is $\rho = 0.7$, which implies that the payoff covariance with the market return is

$$\text{cov}(C, r_M) = \rho \sqrt{\text{var}(C) \text{var}(r_M)}.$$

Letting $\text{var}(r_M)$ take the realistic amount of about 0.03 based for reality on the last 50 years of S&P 500 data

$$\begin{aligned} \text{cov}(C, r_M) &= \rho \sqrt{\text{var}(C) \text{var}(r_M)} \\ &= 0.7 \sqrt{1500 \times 0.03} \\ &= 4.70. \end{aligned}$$

By the usual "certainty equivalent" CAPM formula, as shown in Lintner (1965, 1970), Brealey et al. (2023), Fama and Miller (1972) and Luenberger (1998), the firm's CAPM price at period start is

$$P = \frac{E[C] - \lambda \text{cov}(C, r_M)}{R_f}, \quad (R_f \equiv 1 + r_f) \quad (3)$$

⁵Grinblatt and Titman (1998, p.385-7) describe similar CAPM calculations as "scenario analysis". See their example using scenarios or different payoff possibilities to calculate betas from the probability distribution of payoffs.

where λ is an exogenous market constant driven by investors' aggregate risk aversion⁶ and R_f is the return factor on the risk-free asset (the validity of the calculations and their meaning are unaffected by the assumed constant $\lambda > 0$). The investors' risk aversions are not directly observable but I show in Appendix A using CAPM algebra that the market risk aversion parameter has theoretical value

$$\lambda = \frac{E[r_M] - r_f}{\text{var}(r_M)}.$$

which is an observable quantity called the "market price of risk".⁷

The specific value of $\lambda > 0$ is not important for my point, however I make the calculations as realistic as possible by assuming exogenous constants $\lambda = 1.59$ and $R_f \equiv (1 + r_f) = 1.0667$. These constants are calibrated like $\text{var}(r_M)$ against the last 50 years of US financial market history (6.7% is the 50 year average 10 year annual bond return). The market return over that period is 11.45%, implying a market risk premium of $(E[r_M] - r_f) = 0.0478$ and a CAPM "price of risk"

$$\begin{aligned} \lambda &= \frac{E[r_M] - r_f}{\text{var}(r_M)} \\ &= \frac{0.1145 - 0.0667}{0.03} = 1.59. \end{aligned}$$

The firm's asset price is therefore

$$\begin{aligned} P &= \frac{E[C] - \lambda \text{cov}(C, r_M)}{R_f} \\ &= \frac{100 - 1.59(4.70)}{1.0667} \\ &= 86.73. \end{aligned}$$

and the firm's CAPM price-implied cost of capital is by definition

⁶Lintner (1965; 1970) derived a theoretical value of the CAPM market parameter λ as a function of the investors' personal risk aversions. See the more recent derivation in Lambert et al. (2007) and Johnstone (2025). Both assume a market where assets have joint normal payoffs and investors have exponential utility.

⁷This theoretical value for λ is similarly defined in Grinblatt and Titman (1998, p.389).

$$E[r] = \frac{E[C]}{P} - 1 = \frac{100}{86.73} - 1 = 15.3\%. \quad (4)$$

Now, also at period start, the firm recognizes a new firm-specific risk. To its great concern, the firm receives legal instructions that it is subject to a claim by parties affected by its historical, albeit long ceased, activities (the James Hardie asbestos related mesothelioma case in Australia is a large and realistic example). No other firm is affected, making this threat a textbook case of zero-beta "idiosyncratic" or "diversifiable risk".⁸

The legal action has for simplicity just two possible outcomes, neither positive. If the case is lost, which expert legal counsel assesses as having 50% probability, the firm loses all of the year's expected payoff of \$100 because it pays out damages and costs of that given amount. If the firm wins, its originally estimated expected payoff of \$100 will be unaffected because its costs will be paid by the litigants. The firm's payoff is now its pre-existing payoff (which has covariance of 4.70 with the market) plus the new payoff (actually payout) stemming from the court case, which has zero covariance with the market, so the firm's total covariance stays unchanged at 4.70 (I have assumed that the covariances of the firm's other activities with the market are independent of the outcome of its court appearance). That is

$$\begin{aligned} \text{new payoff covariance} &= \text{old covariance} + \text{covariance of damages} \\ &= 4.70 + 0 = 4.70. \end{aligned}$$

Although the payoff covariance is unchanged, the firm's new mean payoff is now much lower at $E[C] = 50$ because it now holds 50% probability of its previously assumed mean of 100 and 50% probability of zero payoff after compensating the litigants and legal costs. This reduced mean changes the market's view of the company. Once informed of the firm's newly "acquired" firm-specific risk, the market applies CAPM and discounts its stock price to

$$P = \frac{E[C] - \lambda \text{cov}(C, r_M)}{R_f} = \frac{50 - 1.59(4.70)}{1.0667} = 39.85.^9 \quad (5)$$

⁸In reality, there is likely no risk that is purely or exactly zero-beta. If you win at the casino in blackjack, you still have to get your money and that might not happen if the economy and casino are broke.

⁹This values the payout as just another uncertain cash flow, in the eyes of the market (i.e., from

Thus by doing just what CAPM says, the market imposes a new, much higher, cost of capital

$$E[r] = \frac{E[C]}{P} - 1 = \frac{50}{39.85} - 1 = 25.5\%.$$

It is clear therefore that the usual CAPM doctrine on idiosyncratic risk is misleading. The firm pays for its new firm-specific risk in two ways - its mean payoff drops from 100 to 50 and its *endogenous* price-implied cost of capital increases from 15.3% to 25.4%. The combined effect of these simultaneous changes is a price decrease from 86.73 to 39.85. Put simply, the market has effectively changed the numerator (expected cash flow) from 100 to 50 and implicitly changed the discount rate from 15.3% to 25.5%.

This is unquestionably what CAPM equilibrium requires. It is hard to overstate the pedagogical importance of such elementary but insightful CAPM calculations. Had we done what is suggested in Brealey, Myers, Allen and Edmans (2023, p.260-1)¹⁰ we would have discounted the new mean at the old discount rate and obtained a new price of $50/1.153 = 43.36$, which compares with the logically correct CAPM price of 39.85. That would have brought a $(43.36 - 39.85)/39.85 = 9\%$ overstatement error in the revised price.¹¹

It is tempting to say that the change in mean without change in discount rate reduced the asset price from 86.73 to 43.36 and the change in discount rate caused the last bit of the price reduction to a final price of 39.85. However, the conceptually correct view according to Lintner, and explained later in the paper, is that there is a *single endogenous reset* under CAPM where the new asset price, beta and discount rate are functions of the new mean and can't be separated from it.

Lastly, I note of course that CAPM produces additive asset prices so the firm's new price can be written as the sum

$$\begin{aligned} P_{\text{new}} &= P_{\text{old}} + (-P_{\text{damages}}) \\ &= 86.73 - 46.87 = 39.85, \end{aligned}$$

the perspective of someone who might buy the right to it). That is why I have it here as a positive with positive price.

¹⁰Similarly, Edmans (2023b, p.5) writes incorrectly: "Finance 101 tells us that the discount rate is affected only by market risk and not by company-specific risk, ... The discount rate only increases if the risk of the disaster is correlated with market conditions, that is, is greater in bad times."

¹¹The 9% error represents the difference between the correct price and the misconceived price, and offers an arbitrage opportunity presuming that the market will at some point adjust to the correct price.

because the market value P_{damages} of the possible payout from the court case is given by its expectation of 50 discounted at the risk-free rate, that is

$$P_{\text{damages}} = \frac{50}{R_f} = \frac{50}{1.0667} = 46.87.$$

3 Lintner's CAPM Algebra

So far I have written in (3) the asset price as a function of strictly exogenous inputs. I now do the same for the firm's endogenous cost of capital and beta. This analysis stands on no more than CAPM mathematics and leads to results that can only be seen by looking at CAPM in its payoffs expression. The general CAPM law, applying to typical assets that have positive covariance with the market, is that a lower payoff mean $E[C]$, of itself, brings a higher CAPM cost of capital. The firm's CAPM cost of capital is embedded in its equilibrium price which changes with any *cet. par.* change in *either* its expected payoff $E[C]$ or its payoff covariance $\text{cov}(C, r_m)$. Substituting for P in (4) gives the fundamental CAPM corollary

$$E[R] = \frac{E[C]}{P} = R_f \left[\frac{E[C]}{E[C] - \lambda \text{cov}(C, r_M)} \right] = R_f \left[1 - \lambda \frac{\text{cov}(C, r_M)}{E[C]} \right]^{-1}. \quad (6)$$

This equation reveals how the firm's payoff parameters drive its CAPM cost of capital in equilibrium. It is obvious from this equation that the CAPM cost of capital $E[R]$ approaches the risk-free rate as the expected cash flow $E[C]$ increases. This should be in mind whenever a change in regulation alters the NSP's expected cash flow.

Conveniently, the singular characteristic of the firm's cash flow that drives its discount rate is the ratio Λ of the firm's payoff risk (covariance) to its payoff mean

$$\Lambda \equiv \frac{\text{cov}(C, r_M)}{E[C]}. \quad (7)$$

Provided that the firm has positive covariance with the market, its cost of capital $E[R] \equiv E[C]/P$ is monotonically increasing in Λ . This simply proved CAPM corollary shows why a revised payoff expectation with no change in payoff covariance is sufficient of itself to shift the firm's cost of capital or discount rate.

The implication for regulators is that risk is best understood in Lintner's way by its effect on the firm's payoff covariance *per unit of mean*. It follows directly from (6)

that Λ is the fundamental CAPM risk factor, and is more primitive than beta.¹² In what follows, I call Λ "Lintner's ratio" because it was first written and interpreted by Lintner (1965, p.27). Later in this discussion I detail Lintner's CAPM prescription and some related citations from classical early CAPM literature.

3.1 General Implications

Lintner's conclusion can be generalized as follows. The firm is the sum of its discrete cash flows $C = c_A + c_B$, where c_A is the "payoff" from its "normal" activities and c_B is an independent cash flow that may or may not arise depending on some idiosyncratic event or shock (e.g., the firm's product is made illegal, as occurred with some gambling services in the US).¹³ While the normal cash flow c_A is typically correlated (positively) with the market, the idiosyncratic cash flow c_B is by definition a random variable with zero market covariance. The effect of idiosyncratic payoff c_B on the firm's Lintner ratio Λ and cost of capital is therefore:

$$\left\{ \begin{array}{ll} \text{positive} & \text{when } E[c_B] < 0 \\ \text{zero} & \text{when } E[c_B] = 0 \\ \text{negative} & \text{when } E[c_B] > 0. \end{array} \right.$$

In short, if c_B is the incremental cash outflow from some independent random event (like a lightning strike), the potential loss ($E[c_B] < 0$) is idiosyncratic but is nonetheless "priced" by CAPM. Thus, I question Brealey et al. (2023) when they say that the risk of the firm's satellite being hit by space junk is unpriced. If in reality the risk is "practically" unpriced, that is simply because it has such low ex ante probability that the expected negative payoff c_B is virtually zero (i.e, $E[c_B]$ is negative but too small to have any effect on the firm's overall mean payoff and Lintner ratio Λ).

The pathological case of $E[c_B] = 0$ is raised by Ingersoll (1987, pp.207-8) and described as a "fair game". In his highly general asset pricing model, subsuming CAPM, the only truly "unpriced" random payoff c_B is one with both (i) $E[c_B] = 0$ and (ii) zero covariance with the market. We find that Ingersoll's explanation fits

¹²Both beta and Lintner's ratio Λ are "sufficient statistics" for the cost of capital, but Λ is the "minimal sufficient statistic".

¹³CAPM asset values are additive and the value of the firm is the sum of the CAPM values of its separate cash flows.(Luenberger 1998, pp.188-9).

perfectly with Lintner's argument within CAPM because a cash flow with zero mean and zero covariance obviously leaves the firm's Lintner ratio unchanged.

Note also that idiosyncratic incremental cash flows with positive expectations $E[c_B] > 0$ should per Lintner's argument *reduce* the firm's CAPM cost of capital. Consistent with that conclusion, it has been suggested under CAPM that a business that is actively producing positive cash flows using ways quite divorced from market influences will have a lower beta.¹⁴

3.2 Beta in Terms of Lintner's Ratio

The relationship between Lintner's ratio (7) and the conventional definition of CAPM beta is easily revealed. We can start by writing the CAPM in its usual "returns" form, as shown in every introductory finance textbook

$$E[r] = r_f + \beta (E[r_M] - r_f), \quad (8)$$

where $r \equiv R - 1 = C/P - 1$ is the stock return based on payoff C , and where the firm's beta is

$$\beta = \frac{\text{cov}(r, r_M)}{\text{var}(r_M)}. \quad (9)$$

The common CAPM expression shown in (8) is merely the alternative and much more familiar way to express (3). One expression implies the other (they are mathematically equivalent representations of the same pricing model).

We now show how the firm's CAPM beta is driven fundamentally by its ratio (7) of payoff risk to mean. We then simply plug in the new parameter values from our numerical example to show how an idiosyncratic risk can greatly increase the firm's beta. Using straightforward statistical algebra, the usual definition of beta (9) is

¹⁴Albuquerque, Koskinen and Zhang (2019) found empirical evidence of firms having lower betas when they have a greater involvement in CSR activities that do not depend as much as other "normal" businesses on a strong economy.

rewritten as

$$\begin{aligned}
\beta &= \frac{\text{cov}\left(\frac{C}{P} - 1, r_M\right)}{\text{var}(r_M)} \\
&= \frac{1}{P} \frac{\text{cov}(C, r_M)}{\text{var}(r_M)} \\
&= \frac{R_f}{E[C] - \lambda \text{cov}(C, r_M)} \frac{\text{cov}(C, r_M)}{\text{var}(r_M)} \\
&= \frac{R_f}{\text{var}(r_M)} \left[\frac{E[C]}{\text{cov}(C, r_M)} - \lambda \right]^{-1} \\
&= \frac{R_f}{\text{var}(r_M)} \frac{\Lambda}{(1 - \lambda\Lambda)}. \tag{10}
\end{aligned}$$

We can see therefore that beta is driven by a firm characteristic that is economically more fundamental than beta itself, namely the Lintner ratio of risk to mean, $\Lambda = \frac{\text{cov}(C, r_M)}{E[C]}$, defined above. Noting that I have defined the premium required by the market for risk as $\lambda \equiv (E[r_M] - r_f) / \text{var}(r_M)$, Levy (2025) comes to the very same revealing payoffs-based expression for beta as mine; see his equation 24. In that equation, he shows how the asset's payoff mean helps determine the discount rate applicable to that asset.¹⁵ Levy (2025) goes further by revealing that the CAPM discount rate "explodes" asymptotically towards infinity when, in our terms, the asset's Lintner ratio $\text{cov}(C, r_M) / E[C]$ approaches $\text{var}(r_M) / (r_M - r_f)$ from below.

An interesting immediate implication of (10) is that any two assets with the same Lintner ratio Λ must always have the same CAPM cost of capital, regardless of: (i) the market's overall returns risk, $\text{var}(r_M)$, (ii) the market's risk aversion λ , and (iii) R_f . Both assets will have the same CAPM cost of capital but that amount will differ across different markets (e.g., markets with different risk aversions λ). Thus, if we list the two firms in another market, they will still have the same beta as each other, but it will be different because of that market's different parameters R_f , $\text{var}(r_M)$ and λ .

To confirm our analysis and example calculations, I go back to the previous numerical example and calculate the firm's beta both before and after the bad news of the pending class action (i.e., before and after the new firm-specific risk). The firm's

¹⁵ An expansion that includes both the asset's Lintner ratio and the overall market Lintner ratio is given by Johnstone (2017, p.131).

beta before being faced with that risk is

$$\begin{aligned}
\beta &= \frac{R_f}{\text{var}(r_M)} \left[\frac{E[C]}{\text{cov}(C, r_M)} - \lambda \right]^{-1} \\
&= \frac{1.0667}{0.03} \left[\frac{100}{4.70} - 1.59 \right]^{-1} \\
&= 1.81
\end{aligned} \tag{11}$$

Similarly, after the market is informed of the new risk, the only change is a drop in expected payoff to 50, and the post-news beta is much higher

$$\begin{aligned}
\beta &= \frac{R_f}{\text{var}(r_M)} \left[\frac{E[C]}{\text{cov}(C, r_M)} - \lambda \right]^{-1} \\
&= \frac{1.067}{0.03} \left[\frac{50}{4.70} - 1.59 \right]^{-1} \\
&= 3.93.
\end{aligned}$$

Now, to complete the circle, I plug these betas into the usual CAPM returns equation (8), revealing that the "before" and "after" discount rates are respectively

$$\begin{aligned}
E[r] &= r_f + \beta [E[r_M] - r_f] \\
&= 0.0667 + 1.81 [0.1145 - 0.0667] \\
&= 15.3\%,
\end{aligned}$$

and

$$\begin{aligned}
E[r] &= r_f + \beta (E[r_M] - r_f) \\
&= 0.0667 + 3.93 (0.1145 - 0.0667) \\
&= 25.5\%,
\end{aligned}$$

which are the same rates as obtained above using the payoffs CAPM, thus completing the circle.

I suggest that no CAPM exposition is truly complete without the equation for beta shown in (11). The end result is that we have demonstrated coherently that a "diversifiable" cash flow risk can add markedly to the firm's beta and cost of capital,

contrary to conventional wisdom and accepted finance doctrine. If the effect of that risk is to increase the firm's Lintner's ratio, the firm's cost of capital must increase.

It follows directly that the risk of an idiosyncratic loss will be more heavily priced when either it has a larger potential dollar amount or a greater probability of occurring. That conclusion upsets most textbook statements on idiosyncratic risk. For example, textbooks hold that events like spacejunk crashing into the company's billion dollar satellite are unpriced, but CAPM reveals that they are priced, and priced more heavily when they are potentially larger or more likely to occur.

My belief is that CAPM misconceptions have arisen when the asset's *returns* covariance with the market is treated as exogenous rather than endogenous. Although returns observed in the market look like exogenous outputs from a "black box", the return on a stock depends on the stock price, which under CAPM is an endogenous outcome and depends in equilibrium on the asset's expected payoff.¹⁶ Returns and their market covariance or beta are therefore driven latently by the asset's market price, which is a function of its exogenous payoff parameters (mean and covariance). Empirical finance assumes that this endogeneity works its magic beneath the surface and then observes the resulting stochastic market asset returns and their covariances. The payoffs form of CAPM looks deeper because it shows explicitly how a change in the firm's expected payoff (cash flow) will affect its price and hence its returns and their covariance with the market. Another way to say this is that we cannot look at an asset return $C/P - 1$ as having the same covariance with the market as C , because P is not a fixed exogenous constant. Instead, P depends on the statistical parameters of cash flow C .

3.3 "But Diversifiable Risks Have Zero Beta"

A likely reaction to the Lintner style analysis set out above is to think "that's weird, a risk that has zero correlation with the market has zero beta, so how can the firm's beta increase when it becomes subject to such a risk?" This apparent paradox is resolved with just conventional CAPM algebra.

¹⁶Market equilibrium in CAPM is defined as demand equal to supply. That is the equilibrium criterion by which the CAPM price is derived (e.g., see the explicit derivations in Lambert et al. (2007) and Johnstone (2025). Supply is taken as fixed and demand is determined under the economic rationality principle that all investors maximize their expected utility. Unfortunately this derivation of CAPM in terms of cash flows and the investors' utility of wealth does not appear in mainstream textbooks.

Imagine the firm as the sum of its old self and a new zero-beta business activity called "class action". The firm's new beta is the price-weighted average of the old beta of 1.806 and the class action's beta of zero. The pre-risk business had price $P_{old} = 86.74$. The class action - from which no good can come - has a CAPM market price $P_{damages}$ given by its expected cash flow of 50 discounted correctly by CAPM at the risk-free rate (since it has zero beta).¹⁷ Thus,

$$P_{damages} = \frac{50}{1.0667} = 46.87,$$

which is of course the firm's reduction in market value given as above by $86.73 - 39.85$, or the difference in its value before and after the threat of litigation.

So the firm's new beta based on its new risk is the price-weighted average beta of the two separate sources of cash flow

$$\begin{aligned}\beta &= \frac{P_{old}(1.81) - P_{damages}(0)}{P_{old} - P_{damages}} \\ &= \frac{86.73(1.81) - 46.87(0)}{86.73 - 46.87} = 3.93,\end{aligned}$$

which is as again just as already found (this cross-checking circularity is part and parcel of the internal coherence of CAPM).

This CAPM re-analysis has the important implication that any firm-specific zero-beta business activity that has negative expected payoff, and which is uncorrelated with the market, will increase the firm's cost of capital. All firms have such risks. For example, the firm-specific possibility of the company's products or activities causing environmental or social harm, or workers being hurt (as in the Union Carbide Bhopal explosion) are negative expected payoff zero-beta events.

Examples of idiosyncratic cash flow risks in utility regulation are well known. Lately, gas networks have become more liable to asset standing as gas consumption falls. Recognizing that risk is no different to recognizing the risk of a bad outcome in litigation. It affects the utility's expected cash flow badly, driving its Lintner ratio of payoff covariance to mean upwards, leaving it with not only a lower CAPM price but implicitly also a higher cost of capital.

¹⁷Its negative mean payoff does not imply that the firm will lose the case, it merely allows that losing is a strong possibility while winning in the sense of generating a positive net cash flow from the case is impossible.

On this basis, utilities such as gas networks can argue correctly that risks to their cash flow that have previously been regarded as unpriced (e.g., the risk of losses from asset stranding) are not be seen that way by rational investors in a free market. Instead, a CAPM market will discount the suitably reduced expected cash flow of the firm at a higher rate for any such risk (remembering of course that risks of cash windfalls rather than cash losses would have had the opposite effect).

This conclusion seems to be supported instinctively by company managers, who see any large risks as threats to the company's prospects and survival. Documenting the pragmatic view, Jagannathan, Matsa, Meier, and Tarhan (2016) find that firm managers regard firm-specific risks as adding to the firm's cost of capital:

Almost two-thirds of respondents report that idiosyncratic risk is important in determining their firm's discount rate (65.4%), about the same number as for market risk (63.4%). (Jagannathan, et al. 2016, p.458)

Idiosyncratic risks exist in all business activities (e.g., developing a new business that requires but may not get regulatory approval). While relatively small and diversifiable at the market portfolio level, they are material threats *at firm level* and can greatly impact the firm's own cost of capital. The firm managers cannot take the portfolio theory viewpoint of a well diversified stock market investor, for whom one firm's fortunes are not material.

4 Multiplicative Idiosyncratic Risk

Most idiosyncratic risks are additive in the sense that they add another increment of uncertain cash flow to the firm's existing uncertain cash flow. For instance, the risk of losing at court described in our numerical example is additive because it adds an uncertain cash flow of either exactly -100 or 0 to the firm's pre-existing cash flow. It is possible however that a purely random shock to the firm's cash flow can be multiplicative rather than additive. Imagine for example a risk where the firm's cash payoff is sometimes, quite randomly, penalized by a fixed multiple $0 < t < 1$, effectively like a tax. Being idiosyncratic, the penalty occurs, or not, independently of the market. For example, a regulator might choose to "tax" the firm's payoff for reasons completely unrelated to the economy. Ignoring the potential tax, the anticipated payoff is say V but if the tax at rate t is applied the firm's share of

the payoff is just gV where constant $g = (1 - t)$. Thus, the firm's ultimate payoff, $C \in (V, gV)$, is drawn randomly from one of two distributions depending on whether the tax is applied or not. It is either V (untaxed) or $(1 - t)V$ (taxed), where V is the random variable.

Taking Lintner's approach, the first point to make is that the risk of the payoff being taxed leaves its Lintner ratio unchanged, because its covariance and mean are reduced by the same proportion. See the proof in Appendix B. The possibility of being taxed reduces the covariance of the firm's payoff with the market and reduces the firm's mean payoff, both by the same proportion. The firm's value is then its revised mean payoff discounted at an unchanged discount rate, and is therefore fraction g of its original value.

Many cash flow risks, idiosyncratic or not, have both additive and multiplicative elements. For example, a newly imposed sales tax can have both fixed and variable components. However, firms usually have "constants" in their payoffs, including most obviously fixed costs and payments to debt holders. Their effect means that a change in the covariance of the firm's net payoff is rarely matched by an exactly proportionate change in the mean.

Returning to related literature, Lambert et al. (2007) caused a deep rethink in accounting theory and empirical research by addressing how information affects the firm's Lintner ratio of payoff covariance to mean. They suggested that while it is common in finance to assume that the payoff mean and covariance move in exactly the same proportions,¹⁸ there is no evidence supporting that assumption:

While it is common in some corporate finance and valuation models to assume that the level of cash flow and the covariances move in exact proportion to each other (i.e., all cash flow is from the same risk class), we are unaware of any theoretical results or empirical evidence to suggest this should be the case. On the contrary, the existence of fixed costs in the production function, economies of scale, et cetera, generally make the expected values and covariances of firms' cash flows change in ways that are not exactly proportional to each other. Moreover, there is ample

¹⁸While apparently so, that assumption is rarely made explicitly. More commonly, Lintner's point is simply overlooked. The issue for both accounting and finance research shown up by Lambert et al. (2007) is that researchers trace changes in the firm's cost of capital to changes in its payoff risk (covariance) rather than risk per unit of mean.

empirical evidence that betas vary over time , which implies the ratio of expected cash flow to overall covariance varies... (Lambert, Leuz and Verrecchia 2007, pp.393-4)

Note how the authors attribute changes in the firm's beta to changes in its Lintner ratio. The embedded CAPM relationship between beta and the Lintner ratio is shown explicitly in our equation (11).

5 Classical CAPM Literature

In textbooks, it is suggested that we can find the present value of an asset with expected payoff $E[C]$ by the simple shortcut formula $E[C]/(1 + r)$ where r is "the" assumed discount rate. However, in his original CAPM derivation, Lintner (1965, p.27) warned explicitly against any hope that we can "know" and plug in an "off the shelf" discount rate (e.g., an industry norm or firm hurdle rate). Lintner specifically emphasized that the theoretically correct discount rate under CAPM is a "derived variable" (i.e., endogenous) and depends logically on each of the exogenous variables mentioned in equation (6) above. Remarkably, although hidden in a lengthy and difficult paper, Lintner (1965, p. 27) wrote this very equation (see Figure 1 below).

Figure 1

The Easily Overlooked Lintner (1965, p.27) Equations

$$\begin{aligned}
 (29'') \quad V_{0i} &= \frac{\bar{R}_i}{k_{ri}} = \frac{\bar{R}_i}{r^* (\mathbf{I} - W_i / \check{R}_i)^{-1}} \\
 &= \frac{\bar{R}_i}{r^* (\mathbf{I} - \gamma \Sigma_j \check{R}_{ij} / \bar{R}_i)^{-1}} \\
 \text{so that} \\
 (30) \quad k_{ri} &= r^* (\mathbf{I} - \gamma \Sigma_j \check{R}_{ij} / \bar{R}_i)^{-1}.
 \end{aligned}$$

While not absorbed into mainstream finance textbooks, some of which apply the payoffs-based CAPM to capital budgeting exercises, Section IV of Lintner's original CAPM exposition was based explicitly on asset cash payoffs as the underlying driver

of returns.¹⁹ To reveal in his terms the driver of the CAPM cost of capital, I provide an excerpt from Lintner (1965) reproduced as a picture in Figure 1. The figure shows Lintner's notation, which is cumbersome and often hard to follow. Our own equation (6) appears in Lintner's paper as his equation 30.

To see the equivalence of Lintner's equation 30 and our (6) I summarize his notation as follows:

V_{0i} is "the aggregate market value of the i^{th} stock at time zero" (period start); Lintner (1965, p.26)

R_i is "the aggregate return on the i th stock (the sum of aggregate cash dividends paid and appreciation in aggregate market value over the transaction period)"; Lintner (1965, p.26)

r^* is the risk free interest rate (NB. r^* is our $R_f = (1 + r_f)$)

$\widetilde{R}_{ij}$ is the covariance of the aggregate dollar returns of the i^{th} and j^{th} stocks (Lintner 1965, p.26), implying that the i^{th} asset has market payoff variance $\sum_j \widetilde{R}_{ij}$

k_{ri} is the risk adjusted discount rate for asset i .

Allowing for the differences in notation, Lintner's equation 30 corresponds to my (6). Specifically, with asset cash flow C and subscript M representing the market aggregate asset, which has total cash payoff C_M and payoff variance $\text{var}(C_M)$, we have

$$\begin{aligned} E[R] &= R_f \left[1 - \lambda \frac{\text{cov}(C, r_M)}{E[V]} \right]^{-1} \\ &= R_f \left[1 - \kappa \frac{\text{cov}(C, C_M)}{E[C]} \right]^{-1}, \end{aligned} \tag{12}$$

where the market's exogenous risk (variance) aversion parameter is λ in (12) or alternatively $\kappa \equiv \lambda/P_M$ in Lintner's equation 30 (since $\text{cov}(C, r_M) = \text{cov}(C, C_M/P_M) =$

¹⁹ "... I shall assume that investors' probability distribution pertain to dollar returns rather than rates of return - the dollar return in the period being the sum of the cash dividend and the increase of market price during the period." (Lintner 1965, p.25)

$\text{cov}(C, C_M)/P_M$).²⁰

It follows immediately that the CAPM theoretical discount rate is a function of all of the parameters appearing in this equation (12), and cannot therefore be plucked off the shelf, and certainly cannot be held unchanged when any of these parameters change (including the asset's expected payoff).

In particular, the two parameters of the assets payoff that matter are its mean and market covariance, and these can be further summarized by just their Lintner ratio (7), $\Lambda \equiv \frac{\text{cov}(C, r_M)}{E[X]}$. It is clear in Lintner's paper that he anticipated that CAPM users would not follow or correctly understand this CAPM corollary. Lintner admonished practitioners in advance as follows:

Those who like (or hope) to find a "risk" discount rate k_r , with which to discount expected values under uncertainty will find from [his equation] (29) thatthe risk discount rate k_{r_i} is unique to each individual company in a competitive equilibrium; (ii) that efforts to derive it complicate rather than simplify the analysis, since (iii) it is a derived rather than a primary variable; and that (iv) it explicitly involves all the elements required for the determination of V_{0_i} itself, and, (v) does so in a more complex and non-linear fashion. (Lintner 1965, pp.27-8)

To this day we can see that Lintner's proof is mostly ignored or was just never made known. Over-simplified arguments including that put by Edmans (2023a,b) suggest that when a new firm-specific risk is recognized, the correct CAPM analysis is to revise downwards the firm's expected payoff but to leave its risk-adjusted discount rate unchanged. This widely practiced CAPM interpretation (e.g., Brealey et al. 2023, p.261) is proved by Lintner's argument to be logically wrong on its own CAPM terms. It leads to wrong CAPM asset prices that can be arbitrated against, which of all theoretical vices according to efficient markets is the least acceptable.

While similarly overlooked, Stapleton (1971, pp.109-110, 115) wrote the same equation as Lintner's in his equation 58. Interpreting that equation, he stated directly that the CAPM risk of any venture or potential cash flow is measured by the ratio which I have called Lintner's ratio, namely the asset's payoff covariance divided by it

²⁰Lintner wrote our P_M as his T , defined as " $T = \sum_i V_{0_i}$, the aggregate market value of *all* stocks in the market at time zero" (Lintner 1965, p.26).

payoff mean.²¹ A very similar version of the same equation is provided by Hull (1986, p.446) who concluded (p.449) that the risk of a cash flow C "can be measured as the covariance of $C/E[C]$ with the return on the market". The most straightforward statement I have found is in Fama (1977). Using letter V to represent the asset's cash payoff and subscripts i for the specific firm and t for time, Fama wrote that the firm's beta β_i and expected return $E[R_{i,t}]$ are affected as follows, where M is the market aggregate:

...differences in the values of β_i [firm beta] are the source of differences in the values of $E[\tilde{R}_{i,t}]$ for different firms. ... only the ratio of $\text{cov}(\tilde{V}_{i,t}, \tilde{V}_{M,t})/E[\tilde{V}_{i,t}]$ changes from one firm to another, and this ratio of covariance to expected value is the source of differences in the values of $E[\tilde{R}_{i,t}]$ for different firms. It follows that the relative risk measure β_i is directly related to $\text{cov}(\tilde{V}_{i,t}, \tilde{V}_{M,t})/E[\tilde{V}_{i,t}]$, the ratio in which the firm produces contributions to the aggregate risk and to the expected value of invested market wealth. (Fama 1977, p.7)

Before Stapleton (1971) and Fama (1977), the role of the ratio specified in this quote was documented but somewhat hidden by an overly complicated notation in Lintner's famous yet no longer widely read 1965 classic. Lintner exposed this ratio as the fundamental risk measure that determines the firm's CAPM cost of capital. I have shown in (11) its role as the sole asset parameter driving the asset's returns beta (see above). Put succinctly, the minimal sufficient statistic describing the risk of a random cash flow is its Lintner ratio. According to CAPM, no other attribute of the anticipated cash flow has any role. We can say that (i) beta drives the cost of capital but, more fully, that (ii) beta is a function of Lintner's ratio and Lintner's ratio is more primitive.

It shows in (??) that beta involves parameters beyond Λ , namely $\text{var}(r_M)$, R_f and λ , so requires more input and context than Lintner's ratio. The very same business will have different betas in different markets. In the extreme, although obviously unlikely, a stock can have high cash flow covariance with the market return in one market and lower or even negative covariance in another.

²¹Stapleton (1971, p.110) wrote: "The expected rate of return of the stock depends upon the rate of interest, i , the relevant risk of earnings measured by the ratio $\sigma_{Y_j}/E[Y_j]$, and the market risk aversion factor." The terms in the ratio define themselves.

Perhaps explaining how the fundamental significance of Lintner's ratio escaped notice, the equation showing how Lintner's ratio drives beta, our (11), was not provided by Lintner, nor in the other cited papers. For empiricists using empirical asset betas, and especially when trying to explain the drivers of beta, equation (11) is the obvious starting point under CAPM. The few citations I have provided that capture Lintner's point exist like a needle in a haystack. Lintner's payoffs-based CAPM analysis is both easily overlooked and directly contradicted in literally many hundreds of research papers, some of which exist only by not recognizing Lintner's point.²² Proving the rule by the exception, a relatively recent paper that relies unusually on Lintner's ratio although not with any citation or indication of its earlier appearances, is Lambert et al. (2007, pp.392-3). They explained in detail that any change in the firm's operations or any perceived risk that affects either the firm's payoff mean or payoff covariance must alter its cost of capital, up or down (depending on whether Lintner's ratio is higher or lower). They also explained that rarely will the two payoff parameters change in proportion, and hence it is rarely ever the case that the firm's beta and cost of capital will remain unchanged when its perceived payoff mean or covariance is altered. That point says directly that we cannot do as numerous textbooks suggest and simply change the mean payoff without changing its discount rate. A change in the mean is sufficient of itself to change the equilibrium CAPM discount rate - in effect the endogeneity wired into CAPM ensures that *the numerator drives its own denominator*. Thus, when we find the asset price by the shortcut $P = E[C]/E[R]$, *the numerator $E[C]$ drives the denominator $E[R]$* .

To offset resistance to this argument, which clearly contradicts large sections of CAPM practice and teaching, the intuitive explanation put by Lambert et al. (2007) is as follows. If you take a random cash payoff C with mean $E[C] = \$10$ and add a constant number c of dollars to it, it becomes less and less risky as c increases in the sense that an expectation of \$10 plus or minus a sure \$1000 is much less risky than an expectation of \$10 plus or minus a sure \$10. The natural discount rate falls with increasing windfall c despite the payoff variance or covariance staying the same, merely because the mean payoff increases. That intuition is proved by (6) where it is clear that increasing $E[C]$ without change in covariance drives the discount rate

²²Johnstone (2016) showed that the CAPM literature in accounting on how the arrival of new or better information affects the cost of capital is built on the assumption that only the firm's payoff risk or covariance affects its cost of capital.

towards the risk-free rate.

CAPM risk must not therefore be understood in terms of only payoff (co)variance. For example, a firm-specific venture, like sending a satellite into space (Brealey et al., 2023), has zero covariance with the market but is not "risk free", especially if by taking such action the firm adds a potentially very large negative quantity to its anticipated payoff.

Supporting the theoretical analysis in Lambert et al. (2007), Babenko, Boguth and Tserlukevich (2016, p.426) recount the same simple explanation of how a cash flow becomes less risky when its mean increases relative to its covariance. They follow that explanation with an empirical study that stands on the CAPM premise that idiosyncratic risk does indeed, quite rationally, alter the firm's cost of capital:

When a positive idiosyncratic shock occurs, the size of the zero-beta asset increases... As a result, overall firm beta decreases, as do expected stock returns. Therefore, any firm characteristic correlated with the history of idiosyncratic cash flow shocks can help explain expected stock returns. (Babenko, Boguth and Tserlukevich (2016, p.426)

Lintner's exposition of the role of the mean begs the empirical question of whether changes in a firm's expected cash flow have greater effect on its return on capital than changes in the perceived market covariances of those cash flows. Simple exercises calibrated to roughly fit market conditions suggest to me that the mean is far more important than the covariance. Closely related work in portfolio optimization has led to the widely accepted conclusion that portfolio weights are far more affected by changes in mean returns than by changes in returns covariances (Chopra and Ziemba, 1993).

5.1 Firm Suffers, Market Does Not

Lintner proved that "diversifiable" cash flow risk is not costless or "unpriced" at firm level. At market level there is not the same concern, but only because the firm is typically far too small to have a material effect on the market's overall cost of capital.²³ Consider again the risk of just one firm facing a potentially huge payout from a legal

²³Here we make the usual assumption that there are very many firms in the market, and our firm adds just a negligible weight in the market's weighted average beta and cost of capital.

action or fine imposed by regulators (as in the case of Volkswagen). That chance of loss does two things at the market level. First it adds nothing to the variance of the market payoff $\text{var}(C_M)$, because it is uncorrelated (has zero correlation) with the rest of the market payoff. Secondly, it has a negative effect on the mean of the market aggregate payoff $E[C_M]$, however that effect is miniscule and negligible in a large market. Hence, in net, the firm's activity has negligible effect on the price of the market P_M and implicitly no discernible effect on the market's price-implied endogenous cost of capital, $E[r_M] = E[C_M]/P_M - 1$.

I believe that the negligible market-level effect of any single firm's firm-specific idiosyncratic risks explains why finance vernacular insists that firm-specific zero-beta risks are "unpriced". They are unpriced under CAPM *at market level* because:

(i) they are uncorrelated with the market so they leave the market payoff variance virtually²⁴ unchanged

(ii) some have positive expected payoff and others have negative expected payoff, so they tend to cancel out one another's effect on the market payoff mean $E[C_M]$. Making this net effect on expectation $E[C_M]$ all the smaller, many of these risks are of very low probability (like a satellite collision) leaving their individual expected (i.e., mean) cash payoffs or payouts very near zero.

However, while unpriced *at market level*, going by their effect on the market as a whole, they are obviously priced at firm-level, perhaps very heavily. Any zero-beta risk with negative expected payoff will bring the firm a higher cost of capital, and the increase will be greater the larger the risk. By the same CAPM algebra, a new idiosyncratic activity (e.g., a technically revolutionary low cost battery or new energy source) with high positive expected payoff will bring the firm a much lower cost of capital. In the end, therefore, the firm's revised cost of capital will depend on whether the market assesses the new (or newly considered) idiosyncratic activity's cash flow as positive or negative.

²⁴There is a small but negligible increase because the addition of a payoff with some variance (albeit zero covariance with all other elements of the total payoff) adds another term to the covariance calculation.

5.2 Defining a Twin Security

In regulation there is an issue of whether we can use the betas of firms from another market to proxy for local firms' betas. A similar problem arises in real options theory where the cost of capital applicable to an option is taken from a "twin security".

In real options literature, a twin security is usually defined as one with cash payoffs linearly related to the security or business in mind. Thus, if the asset X produces random (stochastic) payoff x then another asset Y with payoff $y = a + bx$ (where a and $b > 0$ are real constants) is a twin security. While it is true that in this case we can replicate asset X using a portfolio of risk-free assets and asset Y , the second asset Y does not have the same cost of capital as X . Its Lintner ratio is

$$\begin{aligned}\Lambda_Y &= \frac{\text{cov}(a + bx)}{E[a + bx]} \\ &= \frac{b \text{cov}(x, M)}{a + bE[x]},\end{aligned}$$

which is not the same as $\Lambda_X = \frac{\text{cov}(x, M)}{E[x]}$. For the assets to be truly twins in the sense that they have the same CAPM cost of capital (in the same market) they must have equal Lintner ratios. That would occur only with $a = 0$ and $b > 0$, meaning that the two assets have strictly proportional cash flows. That is in fact how Trigeorgis (1996) defines a "twin security".

The usual notion of a twin security agreed on by Trigeorgis (1996) and Lintner (1965) assumes that the two assets are priced in the same market. However, it may be possible to adapt Lintner's method to transform betas across markets. By mere algebraic manipulation of (9) we have

$$\beta = \frac{R_f}{\text{var}(r_M)} \left[\frac{\Lambda}{1 - \lambda\Lambda} \right].$$

Hence if we were to assume that (i) the Lintner ratios of utility stocks are the same in different international markets, and (ii) investors risk aversions are also the same, then the beta of a utility in Australia β_{Aust} could be found from the beta of a utility

in the US, β_{US} , by transformation

$$\begin{aligned}\beta_{\text{Aust}} &= \frac{R_{f \text{ Aust}}}{\text{var}_{\text{Aust}}(r_M)} \left(\frac{\text{var}_{\text{US}}(r_M)}{R_{f \text{ US}}} \beta_{\text{US}} \right) \\ &= \frac{\text{var}_{\text{US}}(r_M)}{\text{var}_{\text{Aust}}(r_M)} \frac{R_{f \text{ Aust}}}{R_{f \text{ US}}} \beta_{\text{US}}.\end{aligned}$$

Thus, the constants used in the transformation are just the ratio of the two countries' risk-free rates and the ratio of their market returns variances, both of which are easily observable. The closer these ratios to one, the more valid the use of US betas to proxy for Australian betas.

6 The Analogy With Leverage

Remarkably, it is easily proved that the effect on the cost of capital of a firm's idiosyncratic risk of loss is the same as the effect of either adding financial leverage (debt) or increasing operating leverage (fixed costs). All three constitute zero beta expected costs. The payment of fixed costs and the repayment of debt are zero beta cash flows because they are known constants, whereas the cash flow from idiosyncratic risk is zero beta because it is a random amount uncorrelated with the market. Thus, for different reasons they are all zero beta with negative mean.

Their CAPM equivalence is that all three reduce the firm's expected payoff while leaving its covariance with the market unchanged. Thus, in terms of how they affect the firm's Lintner ratio, they are indistinguishable. The difference between them is that debt and fixed costs are constants whereas an idiosyncratic loss is a random variable, but for CAPM and the cost of capital there is no difference.

It may help to understand how idiosyncratic risk can drive up the firm's cost of capital by thinking of its direct analogy with the effect of higher debt. To see this analogy we need only substitute the market value of the damages risk for the usual market value of debt. Hamada (1972) showed that the beta of a levered firm is

$$\beta_L = \beta_U \left(1 + \frac{D}{E_L} \right)$$

where β_U is the unlevered beta, D is the market value of debt and E_L is the value of equity of the levered firm. The analogous equation for an idiosyncratic risk like the

one in our numerical example is

$$\beta_{\text{new}} = \beta_{\text{old}} \left(1 + \frac{P_{\text{damages}}}{P_{\text{new}}} \right).$$

Substituting the values from our example we have

$$\begin{aligned} \beta_{\text{new}} &= 1.806 \left(1 + \frac{46.87}{39.85} \right) \\ &= 3.93, \end{aligned}$$

again exactly matching the earlier calculation of beta and completing the circle under CAPM endogeneity. Hence, just as we accept that increased debt of itself adds to the firm's cost of capital, so does a new idiosyncratic risk of loss (which in the numerical example was the risk of losing a court case).

While on leverage, it is interesting to see that a technology advance bringing higher fixed costs but also far more efficient production (lower variable costs) can increase the firm's ratio of expected payoff per unit of (co)variance, bringing a lower beta and cost of capital. To understand this point intuitively, suppose the firm's breakthrough produces large and near certain *net* cash flows. That would make the firm a virtually risk-free asset, with a cost of capital near the risk-free rate.

7 Implications for Regulation

The unspoken corollary of CAPM is that regulation can pose the most worrying cash flow risk for networks. The regulator has wide discretion over the network's mean cash flow and even its cash flow covariance.

If there is a threat that the regulator might impose settings that reduce the networks' expected cash flow, the CAPM tells us that while the regulator's decision is independent of the market it is nonetheless priced, simply because it affects the networks' mean cash flow. Of itself, a reduced mean implies higher beta. Conversely, if the forward-looking probability distribution of future cash flow shifts right, rather than left, the mean is higher and by CAPM beta is lower.

Paradoxically, therefore, the regulator "causes" the network's beta, or at least has a major effect on it, merely because regulatory settings can alter the statistical parameters of network's cash flow distribution, along with its joint distribution with the

market. This setting contrasts with a free market, where analysts observe stockmarket betas but don't control them. Instead, investors estimate the asset's exogenous cash flow parameters and, based on their estimates, they assess the required rate of return. If their cash flow forecasts turn out to be inaccurate, say too optimistic, then the realized return on the stock will be lower than was required ex ante by the investors. Such losses, or conversely unexpected gains, are frequent and part and parcel of a free market.

Regulators could in principle take the same approach. For example, they could lay down in their determinations a given ROE setting and a given return on debt (or equivalently a given equity beta and debt beta) and then leave the networks to accept a cash flow stream determined by those settings, making no after the event adjustment no matter what cash flow is realized.

However, if regulators were to do that, the networks would be exposed to major risks such as the risk of cost overruns on construction. Similarly, it could go the other way where networks achieve some windfall cash flow, as would occur for example if major construction was completed well under the amount allowed in forecasts.

By our CAPM logic, cost overrun risk (or the risk of costs being less than budgeted) is a priced risk and should have been included via a higher beta in the original regulators' setting. If it wasn't, the regulator's choice after the event is either to (i) say "sorry, we are emulating a free market", or (ii) make some ex post provision that allows recovery of reasonable unforeseen costs.

In reality, regulators use established methods to allow networks, when reasonable, to recover much of any unpredictable cost overrun, typically about 70% (and to "give up" a similar proportion of cost savings). Similarly, where there is clearly increasing risk of asset stranding, the regulator might introduce adjusted faster rates of depreciation, or some other mechanism, by which sunk costs can be recovered.

That compensation for risks not incorporated in the original equity beta setting reveals how the regulators in effect constrain the networks' beta (meaning the beta of the eventual post-adjustment cash flow). By pre-committing in their pronouncements to reasonable post hoc adjustments, the network's beta, or more simply its cash flow, is constrained from both above and below.

In the very early days of CAPM in regulation, economist Pelzman (1976) noted that this "buffering" of networks cash flows and profits tends to reduce the beta of the resulting cash flow distribution. That explanation is extended by Binder and

Norton (2015) who argue in the same way I have that a mere increase in the network's expected cash flow will of itself imply lower beta.

... if regulation *ceteris paribus* increases the firm's expected cash flows.
the value of the firm increases and beta decreases. (Binder and Norton,
2015 p.251)

While here I have tried to explain within a CAPM framework the regulators' ex post adjustments to realized cash flow, the other way to do that is to say that regulation calls for a pragmatic mix informed by the CAPM framework but also by a simpler "cost of service" framework. In any cost of service model, the networks' cash flows are tied to its costs and post hoc adjustments after costs are known are inevitable, merely because forecasting the costs of investments such as major new infrastructure is so difficult and prone to large under or over estimation.

Appendix A: Making λ Empirical

The theoretical CAPM risk aversion parameter λ stems from the investors' utility functions (as derived in Lambert et al., 2007) and manifests itself empirically in observable market observables as

$$\lambda = \frac{E[r_M] - r_f}{\text{var}(r_M)}.$$

To see this relationship, start with the CAPM pricing equation

$$P = \frac{E[C] - \lambda \text{cov}(C, r_M)}{R_f},$$

from which the total price of the market is

$$P_M = \frac{E[C_M] - \lambda \text{cov}(C_M, r_M)}{R_f},$$

where C_M is the total payoff from the sum of all assets on the market.

Using the usual statistical laws of variance and covariance, CAPM algebra reveals

$$\begin{aligned} P_M R_f &= E[C_M] - \lambda \text{cov}\left(C_M, \frac{C_M}{P_M}\right) \\ &= E[C_M] - \frac{\lambda}{P_M} \text{var}(C_M) \\ &= E[C_M] - \frac{\lambda P_M^2}{P_M} \text{var}(r_M), \end{aligned}$$

from which

$$\begin{aligned} R_f &= \frac{E[C_M]}{P_M} - \lambda \text{var}(r_M) \\ &= E[R_M] - \lambda \text{var}(r_M), \end{aligned}$$

giving

$$\lambda = \frac{E[R_M] - R_f}{\text{var}(r_M)} = \frac{E[r_M] - r_f}{\text{var}(r_M)}.$$

Appendix B: Proportionate Risk

The payoff from the activity is $C \in (V, gV)$ is a mixture distribution because C is drawn randomly from one of two distributions, depending on whether the tax is applied or not. Let $E[V] = \mu$, $\text{cov}(V, \text{market}) = \sigma$ and let the probability of being taxed equal π . The tax rate is t so $g = (1 - t)$. The mean of C is thus

$$\begin{aligned} & \pi g\mu + (1 - \pi)\mu \\ &= \mu(1 - \pi(1 - g)). \end{aligned}$$

Also, since the tax is applied or not by a decision that is independent of the market, we know that the covariance of C with the market is

$$\begin{aligned} & \pi g\sigma + (1 - \pi)\sigma \\ &= \sigma(1 - \pi(1 - g)). \end{aligned} \tag{13}$$

The Lintner ratio of C is thus

$$\frac{\text{cov}(C, \text{market})}{E[C]} = \frac{\sigma(1 - \pi(1 - g))}{\mu(1 - \pi(1 - g))} = \frac{\sigma}{\mu},$$

and is therefore equal to the Lintner ratio of a untaxed payoff V .

Some explanation of (13) might be useful. Note that the covariance of a mixture distribution is calculated using the law of total covariance. In our calculation (13) the usual second term in this formula, namely the covariance between the two means, equals zero and hence does not appear because the two population means $E[V] = \mu$ and $E[gV] = g\mu$ are independent constants.

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