Quantonomics

QUANTITATIVE ECONOMICS

Memorandum

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From:	Michael Cunningham, Joe Hirschberg, Alice Giovani
To:	Adam Rapoport, Anthony Weir, Su Wu, Sasha Jergic, Arwin Lee, Imran Akyol
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1 Introduction

This memorandum concerns the estimation of non-reliability output index weights for the 2025 benchmarking of electricity distribution and transmission businesses. The AER's current method of constructing index weights for non-reliability outputs was developed by Denis Lawrence and Erwin Diewert (2006), and was first applied to Australian energy network benchmarking in Economic Insights (2014). These output weights were last updated in Economic Insights (2020, 124–25).

In 2024, the AER engaged the Centre for Efficiency and Productivity Analysis (CEPA) to review the method of estimating non-reliability output index weights (Peyrache 2024). CEPA concluded:

- (a) The Leontief input demand specification is more flexible than might be assumed, and this method of calculating the output weights "is substantially correct" (Peyrache 2024, 27).
- (b) The "main potential shortcoming ... comes from the fact that it is based on non-linear least squares" (NLS) which "may lead to numerically unstable results" (Peyrache 2024, 14,27). The solution may not be the global optimum and there may be several optimal solutions in terms of parameter values. Given the potential for computational problems some alternative methods were suggested by CEPA, which could be used to cross-check the results from the standard procedure.

In light of the finding (a), the current Lawrence-Diewert method will continue to be used for estimating output weights. CEPA also suggested two main alternative methods:

- to introduce the time trend variable additively rather than multiplicatively, and to estimate this model using quadratic programming.
- to choose parameters that minimize the absolute deviations rather than the squared deviations, and to estimate this model using linear programming.

These two modifications progressively introduce more linearity into the models whilst maintaining the constraint that the marginal effects of each output on cost are nonnegative.

This memorandum presents output weight estimates using each of these three methods. Data from 2006 to 2023 for Australian DNSPs and TNSPs is used. Non-inclusion of the latest year (here 2024) is consistent with Economic Insights (2020). However, the AER's latest revisions to data prior to 2024 are included.

Results for the standard Lawrence-Diewert (Economic Insights) method are presented in section 2. The results of CEPA's two approaches are presented in sections 3 and 4.



2 Lawrence-Diewert Method

This section documents the implementation of Economic Insights' method of calculating non-reliability output index weights.

2.1 Econometric model

Economic Insights' method involves estimating a separate input demand function for each input for each firm, using the Leontief specification. *For a given firm*, ¹ the estimating equations are (plus a random disturbance):

$$x_i = \sum_{j=1}^{N} a_{ij}^2 y_j (1 + b_i t) + \varepsilon$$
 for each $i = 1 \dots M$ (2.1)

where x_i is the quantity of input i, with $i = 1 \dots M$ inputs, and y_j is the quantity of output j, with $j = 1 \dots N$ outputs; and t is a measure of time; and ε is a normally distributed disturbance. The a's and b's are parameters to be estimated. This is a set of M equations. This set of equations needs to be estimated for each firm (ie, DNSP or TNSP as applicable).

Model (2.1) is estimated using nonlinear least squares (NLS) using Stata's nl command. The nonlinearity is due to both the squaring of the a_{ij} coefficients, used to impose non-negativity on these coefficients, and the way that the time trend enters into the equation.

Initial parameter values are generally required for NLS. Three alternative approaches to setting starting values for parameters in the NLS regression routine have been considered.

- (a) Economic Insights used 0.001 for all a_{ij} and b_i as starting values.
- (b) CEPA used the previously estimated parameter values $\{\hat{a}_{ij}, \hat{b}_i\}$, with the *a*'s expressed in absolute terms.
- (c) Alternatively, previously estimated parameter values could be used without changing the sign of \hat{a}_{ij} and subject to a minimum absolute value of 0.001.

We have elected to use Economic Insights' method of selecting starting values.

2.2 Method of deriving output weights

2.2.1 Output cost shares

The output shares of costs can be computed from the parameter estimates, given data on input prices, denoted w_i for input i. The input demand equations (2.1) are consistent with a multi-output Leontief cost function. This functional form essentially assumes that firms use inputs

¹ The output weights are calculated for each firm and averaged over all firms. To simplify the notation, we consider a single firm in the equations we present here.



in fixed proportions for each output. The cost function for an individual firm, with M inputs and N outputs is given by:

$$C(y, w, t) = \sum_{i=1}^{M} w_i \left[\sum_{j=1}^{N} a_{ij}^2 y_j (1 + b_i t) \right]$$
 (2.2)

where w_i is the price of input i, and the other variables and parameters have been defined. The estimated parameters can be used in equation (2.2) to calculate total cost for each DNSP in each year. The DNSP sample has 18 years and 13 firms, and 234 observations overall. The TNSP sample has 18 years and 5 firms, and 65 observations overall. We denote these observations $k = 1 \dots K$. Hence, we have K observations on the predicted cost, \hat{C} .

The cost associated with each *input* is $w_i \hat{x}_i$, where \hat{x}_i is derived from (2.1) multiplied by w_i , using the estimated parameters and noting that $E(\varepsilon) = 0$. However, we are interested in the cost associated with each *output*. We find this by separating the input cost into the parts attributable to each output, recalling that there are M input demand equations for each firm, and M = 4.

$$\begin{split} w_1 \hat{x}_1 &= w_1 \hat{a}_{11}^2 y_1 \big(1 + \hat{b}_1 t \big) + w_1 \hat{a}_{12}^2 y_2 \big(1 + \hat{b}_1 t \big) + w_1 \hat{a}_{13}^2 y_3 \big(1 + \hat{b}_1 t \big) + w_1 \hat{a}_{14}^2 y_4 \big(1 + \hat{b}_1 t \big) \\ w_2 \hat{x}_2 &= w_2 \hat{a}_{21}^2 y_1 \big(1 + \hat{b}_2 t \big) + w_2 \hat{a}_{22}^2 y_2 \big(1 + \hat{b}_2 t \big) + w_2 \hat{a}_{23}^2 y_3 \big(1 + \hat{b}_2 t \big) + w_2 \hat{a}_{24}^2 y_4 \big(1 + \hat{b}_2 t \big) \\ w_3 \hat{x}_3 &= w_3 \hat{a}_{31}^2 y_1 \big(1 + \hat{b}_3 t \big) + w_3 \hat{a}_{32}^2 y_2 \big(1 + \hat{b}_3 t \big) + w_3 \hat{a}_{33}^2 y_3 \big(1 + \hat{b}_3 t \big) + w_3 \hat{a}_{34}^2 y_4 \big(1 + \hat{b}_3 t \big) \\ w_4 \hat{x}_4 &= w_4 \hat{a}_{41}^2 y_1 \big(1 + \hat{b}_4 t \big) + w_4 \hat{a}_{42}^2 y_2 \big(1 + \hat{b}_4 t \big) + w_4 \hat{a}_{43}^2 y_3 \big(1 + \hat{b}_4 t \big) + w_4 \hat{a}_{44}^2 y_4 \big(1 + \hat{b}_4 t \big) \end{split}$$

The input cost components associated with each output can be summed columnwise over each output rather than rowwise over each input. Thus, the components corresponding to output 1 can be summed over all 4 input cost equations, and likewise for each of the other three outputs. Since the cost of each input is fully attributed to each output, total cost is also fully attributable to each output. This procedure yields estimated cost component attributable to output j, as expressed in (2.3):

$$\hat{C}_{j} = \sum_{i=1}^{M} w_{i} \hat{a}_{ij}^{2} y_{j} (1 + \hat{b}_{i} t)$$
(2.3)

Note that: $\hat{C} = \sum_j \hat{C}_j$; ie, the costs are fully attributable to the outputs. The estimated cost share of output j at a particular observation k is: $\hat{s}_j = \hat{C}_j/\hat{C}$. We now define the average estimated costs over all K observations as: $\bar{C}_j = \sum_k \hat{C}_j/K$; and $\bar{C} = \sum_k \hat{C}/K$. The weighted average cost share used by Economic Insights is derived as the ratio of the average cost share attributable to output j to total cost:



$$\bar{s}_j = \frac{\bar{C}_j}{\bar{C}} = \frac{\sum_k \hat{C}_j}{\sum_k \hat{C}} \tag{2.4}$$

These weighted average cost shares sum to 1 and are used as the non-reliability output weights prior to rescaling to incorporate the reliability output weight.

2.3 Econometric results

2.3.1 DNSPs

The estimation results for the 4 input demand equations (2.1) are presented for each of the 13 DNSPs in tables 2.1 to 2.13. Each regression has 18 observations.

Table 2.1 EVO Leontief cost function regression results

	<u>Real O</u>	<u>pex</u>	<u>O/H L</u> i	ines	<u>U/G (</u>	<u>Cables</u>	<u>Transfor</u>	mers .
	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat
Energy	4.071	21.15	0.000		0.000	•	0.497	2.99
RMD	0.000		5.977	4.61	0.000	•	0.000	
Cust. No.	0.000		0.000	0.00	0.000	•	0.000	
Circ. Len.	0.000		3.082	8.85	1.344	105.27	0.716	9.99
Time	0.003	0.31	-0.005	-1.94	0.018	8.93	0.006	2.92
R^2	0.018		0.853		0.962		0.985	

Table 2.2 AGD Leontief cost function regression results

	<u>Real O</u>	<u>nex</u>	<u>O/H L</u> i	ines	<u>U/G (</u>	<u>Cables</u>	<u>Transfo</u>	rmers
	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat
Energy	0.000		0.000		0.000		0.000	
RMD	8.535	3.12	4.948	2.17	0.000	•	2.061	81.08
Cust. No.	0.000	0.00	0.000		0.000		0.000	
Circ. Len.	0.000		2.324	2.95	2.030	167.96	0.000	
Time	-0.023	-4.62	-0.004	-3.47	0.003	2.56	0.013	4.98
R^2	0.460		0.316		0.827		0.800	

Table 2.3 CIT Leontief cost function regression results

	<u>Real C</u>	<u>pex</u>	<u>O/H I</u>	<u>Lines</u>	<u>U/G Cal</u>	<u>U/G Cables</u>		<u>Transformers</u>	
	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat	
Energy	2.551	38.51	0.000		0.292	0.59	0.334	2.39	
RMD	0.000		0.000		0.000	0.00	1.007	5.12	
Cust. No.	0.000		0.134	4.11	0.000	0.00	0.000		
Circ. Len.	0.000		0.995	3.05	1.422	8.67	0.993	11.60	
Time	0.027	4.36	-0.009	-12.94	0.015	3.16	0.007	3.57	
R^2	0.530		0.068		0.999		0.984		



Table 2.4 END Leontief cost function regression results

	<u>Real C</u>	<u>pex</u>	<u>O/H L</u> i	ines	<u>U/G C</u>	'ables	<u>Transfor</u>	mers .
	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat
Energy	0.000		0.000		0.000		0.000	•
RMD	-7.426	-18.25	7.969	7.17	-2.581	-64.89	-1.756	-7.48
Cust. No.	0.000	0.00	0.000		0.000		0.000	
Circ. Len.	0.000		1.639	2.53	0.000	•	0.264	1.40
Time	-0.013	-4.15	-0.010	-9.78	0.071	14.74	0.016	8.10
R^2	0.141		0.429		0.984		0.971	

Table 2.5 ENX Leontief cost function regression results

	<u>Real O</u>	<u>pex</u>	<u>O/H L</u> i	ines	<u>U/G C</u>	<u>ables</u>	<u>Transfo</u>	rmers
	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat
Energy	0.000		0.000	0.00	0.000	•	0.000	•
RMD	7.286	52.96	-2.469	-1.76	3.296	68.80	1.396	5.82
Cust. No.	0.000	•	0.000	0.00	0.000	•	0.000	
Circ. Len.	0.000	•	2.300	4.03	0.000	•	0.494	7.22
Time	-0.001	-0.41	-0.003	-0.34	0.028	8.36	0.013	11.07
R^2	0.539		0.886		0.942		0.975	

Table 2.6 ERG Leontief cost function regression results

	<u>Real Op</u>	<u>vex</u>	O/H Lines		<u>U/G C</u>	ables	<u>Transformers</u>	
	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat
Energy	0.000		0.000		0.000	•	0.000	
RMD	10.643	2.35	-0.002	0.00	1.931	59.16	0.000	
Cust. No.	0.054	0.01	0.000	0.00	0.000		0.131	112.09
Circ. Len.	0.000		1.914	2.69	0.000		0.000	
Time	-0.014	-2.39	0.000	0.31	0.042	9.64	0.014	7.85
R^2	0.461		NA		0.947		0.957	

Table 2.7 ESS Leontief cost function regression results

	Real O	<u>pex</u>	<u> </u>	ines .	<u>U/G Cables</u>		<u>Transformers</u>	
	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat
Energy	0.000		1.614	0.53	0.000	•	1.150	96.44
RMD	0.000	•	8.939	7.08	1.665	1.30	0.000	
Cust. No.	0.670	30.58	0.000	•	0.000		0.000	
Circ. Len.	0.000		1.437	7.19	0.186	1.29	0.000	
Time	-0.019	-4.04	0.002	1.63	0.034	1.82	0.015	6.80
R^2	0.127		0.940		0.800		0.853	



Table 2.8 JEN Leontief cost function regression results

	Real O _l	<u>vex</u>	<u>O/H</u>	<u>Lines</u>	<u>U/G (</u>	<u>Cables</u>	<u>Transfor</u>	mers .
	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat
Energy	0.000	•	0.000	•	0.000		0.000	0.00
RMD	-7.302	-24.90	0.000	•	0.000	•	1.133	5.23
Cust. No.	0.000	•	0.000	•	0.155	18.05	0.000	
Circ. Len.	0.000	•	2.829	227.10	0.000	0.00	0.608	8.28
Time	0.003	0.45	-0.004	-5.17	0.041	20.65	0.018	9.09
R^2	0.162		0.845		0.993		0.980	

Table 2.9 PCR Leontief cost function regression results

	<u>Real O</u>	<u>vex</u>	<u>O/H L</u> 1	ines	<u>U/G Ca</u>	<u>ıbles</u>	<u>Transfo</u>	rmers
	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat
Energy	1.958	0.34	0.000		0.000	0.00	0.000	•
RMD	-4.455	-1.18	0.000		1.456	3.81	0.651	3.76
Cust. No.	0.000		0.590	7.62	-0.071	-1.70	0.000	
Circ. Len.	0.763	0.37	1.693	6.83	0.000		0.309	26.75
Time	0.008	1.10	-0.010	-6.23	0.059	3.59	0.024	28.84
R^2	0.488		0.852	•	0.975	•	0.997	

Table 2.10 SAP Leontief cost function regression results

	<u>Real (</u>	<u> Opex</u>	<u> </u>	<u>ines</u>	<u>U/G C</u>	<u>ables</u>	<u>Transfo</u>	rmers
	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat
Energy	-3.180	-29.35	1.649	2.46	0.667	1.18	0.000	0.00
RMD	0.000		0.000		0.000		1.422	5.47
Cust. No.	0.000		-0.294	-2.87	0.146	2.13	0.000	
Circ. Len.	0.000		1.092	4.27	-0.103	-0.11	0.248	4.85
Time	0.053	5.38	-0.002	-0.78	0.020	1.93	0.017	5.48
R^2	0.689		0.693		0.967		0.962	

Table 2.11 AND Leontief cost function regression results

	Real C	<u> Opex</u>	<u>O/H L</u>	<u>ines</u>	U/G Ca	<u>bles</u>	Transf	ormers
	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat
Energy	0.000	•	0.000	0.00	0.000		0.000	•
RMD	8.132	33.82	2.098	0.94	0.000		1.236	4.54
Cust. No.	0.000	0.00	0.000	•	0.133	5.37	0.000	•
Circ. Len.	0.000		2.254	3.18	-0.254	-1.43	-0.322	-7.36
Time	0.007	1.21	-0.004	-2.86	0.048	7.26	0.014	8.45
R^2	0.540		NA		0.997		0.971	



Table 2.12 TND Leontief cost function regression results

	<u>Real O</u>	<u>pex</u>	<u>O/H L</u>	<u>ines</u>	U/G Ca	ables	<u>Transfor</u>	mers_
	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat
Energy	-2.003	-0.82	2.253	13.05	0.000	•	0.000	
RMD	0.000		0.000		0.000		0.000	
Cust. No.	0.394	1.84	0.000		0.000		0.115	170.00
Circ. Len.	0.000		1.857	42.95	0.674	242.45	0.000	
Time	0.002	0.24	0.003	8.46	0.013	15.86	0.011	9.83
R^2	0.231		0.970		0.977		0.969	

Table 2.13 UED Leontief cost function regression results

	<u>Real O</u>	<u>pex</u>	<u> 0/H L</u>	<u>ines</u>	<u>U/G Ca</u>	<u>bles</u>	<u>Transforr</u>	ners
	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat
Energy	1.993	1.35	0.000	0.00	0.000		-0.083	-0.08
RMD	5.269	2.27	3.615	3.80	1.136	8.46	1.115	8.21
Cust. No.	0.000		0.323	6.22	0.000	•	0.079	6.76
Circ. Len.	0.000		0.000	•	0.734	22.10	0.000	
Time	-0.005	-0.69	0.000	-0.04	0.025	30.11	0.016	4.67
R^2	0.232		0.847		0.994		0.989	

2.3.2 TNSPs

The estimation results for 4 input demands (equation (2.1)) for each of the 5 TNSPs are presented in tables 2.14 to 2.18. Each regression has 18 observations.

Table 2.14 ENT Leontief cost function regression results

	<u>Real O</u>	<u>pex</u>	<u>O/H</u>	<u>Lines</u>	<u>U/G Ca</u>	<u>bles</u>	<u>Transfor</u>	mers .
	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat
Energy	0.652	1.179	-0.000	•	0.278	0.576	0.271	1.657
RMD	0.524	0.093	0.000		0.700	0.853	1.362	12.115
Cust. No.	0.225	2.899	-0.000		0.000		0.000	•
Circ. Len.	-0.000		17.815	216.932	0.000	0.000	0.000	
Time	0.014	2.222	0.005	5.335	0.310	1.258	0.020	8.063
R^2	0.898		0.861		0.734		0.932	

Table 2.15 PLK Leontief cost function regression results

	<u>Real C</u>	<u> Ppex</u>	<u>O/H</u> .	<u>Lines</u>	<u>U/G C</u>	<u>Cables</u>	<u>Transfo</u>	ormers
	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat
Energy	-0.835	-1.884	0.000		0.000		-0.000	•
RMD	-0.000		-0.000		-0.000		-0.000	
Cust. No.	0.000		-0.000		0.000	0.000	-0.000	
Circ. Len.	2.701	4.787	24.309	118.880	0.727	8.139	1.402	72.454
Time	-0.002	-0.569	0.003	1.707	-0.002	-0.825	0.019	6.556
R^2	0.443		0.914		0.774		0.932	



Table 2.16 ANT Leontief cost function regression results

	<u>Real (</u>	<u> Opex</u>	<u> </u>	<u>ines</u>	<u>U/G (</u>	<u>Cables</u>	<u>Transfo</u>	ormers
	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat
Energy	1.042	4.636	2.993	3.808	0.339	98.220	-0.000	•
RMD	-0.650	-0.331	0.000	•	-0.000		0.559	3.207
Cust. No.	0.060	0.838	-0.683	-1.295	0.000		0.000	0.000
Circ. Len.	-0.000		26.363	5.287	0.000		1.744	21.322
Time	-0.007	-1.080	-0.003	-0.707	-0.014	-7.949	0.012	12.057
R^2	0.541		0.157		0.881		0.959	

Table 2.17 TNT Leontief cost function regression results

	<u>Real Opex</u>		<u> 0/H L</u>	<u>O/H Lines</u>		<u>U/G Cables</u>		<u>Transformers</u>	
	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat	
Energy	1.816	4.748	-0.000		-0.000		-0.000	-0.000	
RMD	0.000	•	-0.000		-0.000		-0.000		
Cust. No.	-0.099	-0.292	0.959	3.410	0.000		0.156	72.718	
Circ. Len.	0.000		12.242	7.975	0.556	9.662	-0.000		
Time	-0.035	-12.875	0.008	2.539	0.156	3.110	0.006	2.343	
R^2	0.886		0.874		0.787		0.818		

Table 2.18 TRG Leontief cost function regression results

	<u>Real C</u>	<u> Ppex</u>	<u>O/H1</u>	<u>Lines</u>	<u>U/G Ca</u>	<u>bles</u>	<u>Transfor</u>	mers .
	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat
Energy	0.000		0.000	•	0.184	0.246	-0.000	•
RMD	-1.023	-0.191	20.775	86.140	0.000	0.000	1.470	43.992
Cust. No.	0.000	0.000	0.000	•	0.007	0.011	-0.000	
Circ. Len.	2.823	0.981	-0.000	•	1.356	1.387	0.000	
Time	-0.004	-1.316	0.011	4.816	0.075	1.551	0.022	4.377
R^2	0.016		0.762		0.986		0.732	

2.4 Non-reliability output index weights

The non-reliability output index weights, calculated as shown in equation (2.4), are presented in Tables 2.19 and 2.20, for DNSPs and TNSPs respectively. Also shown in these tables are the Economic Insights estimates produced in 2020 using 13 years of data from 2006 to 2018. The current estimates use 5 extra years of data, and there have been some revisions to earlier data (including a redefinition of opex inputs, and associated changes in the prices of capital inputs). It should therefore be unsurprising that there are substantial differences in the output cost shares estimated in this analysis and those estimated in 2020.

For DNSPs, the main changes are the large increase in the weight of RMD, and an almost as large decrease in the weight of circuit length. For TNSPs, the weight of energy throughput has decreased whereas the weights of RMD and end-customer numbers has increased.



Table 2.19 DNSP output cost shares: Economic Insights method (%)

Output	2006-2023	2006-2018	Difference
Energy throughput	10.793	8.58	2.21
Ratcheted max. demand	47.827	33.76	14.07
Customer numbers	15.230	18.52	-3.29
Circuit length	26.151	39.14	-12.99
Total*	100.000	100.00	0.00

^{*} Figures may not sum to 100.000 due to rounding.

Table 2.20 TNSP output cost shares: Economic Insights method (%)

	Economic Insights					
Output	2006-2023	2006-2018	Difference			
Energy throughput	9.445	14.91	-5.47			
Ratcheted max. demand	28.685	24.71	3.98			
End-Customer numbers	9.329	7.59	1.74			
Circuit length	52.540	52.79	-0.25			
Total*	100.000	100.00	0.00			

^{*} Figures may not sum to 100.000 due to rounding.

It is not a simple matter to pin down the reasons for the changes in the output cost shares. They depend on the estimated coefficients of input demand functions, relative input prices and the relative outputs of businesses. In relation to DNSPs, changes in relative outputs and relative input prices do not appear to be important reasons for changes in output cost shares.² Changes in the average parameters of the input demand functions, which reflect the importance of the different outputs as demand drivers, are the main cause of changes in the output cost shares. On average across DNSPs:

- the demands for opex inputs appear to be driven more by RMD and less by circuit length than was the case in Economic Insights' 2020 estimates.
- the demands for transformer inputs appear to be driven more by energy throughput and RMD, and less by customers than was the case in the 2020 study.
- the influence of outputs on the demands for overhead lines and underground cables do not appear to have changed to a large extent.

² There were no significant changes in the average relative outputs of DNSPs between the 2006-2023 and 2006-2018 data samples. The average input prices are not greatly different between the two samples. In the updated sample, the average opex input price is 6.1 per cent higher and the average underground cables price is 6.9 per cent lower than in the 2006-2018 sample, with little change in the other input prices.



3 Additive time trend & Quadratic programming

3.1 Quantitative method

CEPA suggested some model specification refinements to establish whether there are estimation issues with NLS. The two most important suggestions are outlined in this and the following sections.

CEPA suggested modifying equation (2.1) by replacing the multiplicative time trend with an additive trend:

$$x_i = \sum_{j=1}^{N} a_{ij}^2 y_j + \beta_i t \tag{3.1}$$

An important issue to be clarified is the method of calculating output weights when the time trend is additive rather than multiplicative. This is discussed in Appendix A.

If (3.1) is estimated using regression, NLS is required because of the squared expression (a_{ij}^2), which is used to ensure that the coefficient on each output is positive. This would be straightforward and could be tested. An alternative way of implementing this model, proposed by CEPA, is to solve it using quadratic programming. In this case the model should be expressed as:

$$x_i = \sum_{j=1}^{N} \alpha_{ij} y_j + \beta_i t , \qquad \alpha_{ij} \ge 0$$
 (3.2)

where $\alpha_{ij} = a_{ij}^2$. Quadratic programming chooses, for each i, parameter values $\{\alpha_{ij}, \beta_i\}$ that minimise the squared deviations between x_i and $(\sum_{j=1}^N \alpha_{ij}y_j + \beta_i t)$ subject to $\alpha_{ij} \ge 0$. The program is:

$$\min_{\alpha,\beta} \sum_{k} \left[x_i - \beta_i t - \sum_{j=1}^{N} \alpha_{ij} y_j \right]^2$$
 (3.3)

$$s.t.$$
 $\alpha_{ij} \ge 0$ for each j

CEPA indicates that quadratic programming should produce more numerically stable results. For this reason, we have estimated the coefficients of the additive time trend model using quadratic programming. Quadratic programming is not available in Stata. We have used the *Mathematica* routine QuadraticOptimization.³

 $^{^3\} https://reference.wolfram.com/language/ref/QuadraticOptimization.html.$



3.2 Quadratic programming results

3.2.1 DNSPs

The estimation results for the 4 input demand equations (3.2) are presented for each of the 13 DNSPs in tables 3.1 to 3.13. In these tables the estimated coefficients applying to outputs are the $\hat{\alpha}_{ij}$'s, unlike the tables in section 2, which report the $\hat{\alpha}_{ij}$'s, which are the square roots of the $\hat{\alpha}_{ij}$'s. Because they are produced using quadratic programming, there are no standard errors for these parameters.

Table 3.1 EVO Additive time trend model

	Opex	OH lines	UG cables	Transf.
Energy	9.73	0.00	0.00	0.28
RMD	0.00	32.33	0.00	0.00
Cust. No.	0.00	0.00	0.00	0.00
Circ. Len.	4.24	10.00	1.77	0.49
Time (β)	-114.25	-391.89	190.00	23.33

Table 3.2 AGD Additive time trend model

	Opex	OH lines	UG cables	Transf.
Energy	0.00	0.00	0.00	0.00
RMD	72.37	26.71	5.63	4.24
Cust.	0.00	0.00	0.00	0.00
Circ.Len.	0.00	5.05	3.20	0.00
Time (β)	-10752.72	-1643.85	613.22	352.44

Table 3.3 CIT Additive time trend model

	Opex	OH lines	UG cables	Transf.
Energy	6.33	0.00	0.00	0.11
RMD	0.00	0.00	0.00	0.99
Cust.	0.00	0.02	0.00	0.00
Circ.Len.	0.58	1.01	2.14	0.99
Time (β)	859.48	-96.01	135.70	45.00

Table 3.4 END Additive time trend model

	Орех	OH lines	UG cables	Transf.
Energy	0.00	0.00	0.00	0.00
RMD	54.53	0.00	0.00	3.62
Cust.	0.00	0.00	0.00	0.00
Circ.Len.	0.00	10.02	0.34	0.00
Time (β)	-2746.34	-3808.56	3593.11	284.26

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Table 3.5 ENX Additive time trend model

	Opex	OH lines	UG cables	Transf.
Energy	0.00	0.00	0.00	0.00
RMD	53.11	6.88	10.70	2.81
Cust.	0.00	0.00	0.00	0.01
Circ.Len.	0.00	5.04	0.00	0.00
Time (β)	-403.95	-266.45	1708.07	295.91

Table 3.6 ERG Additive time trend model

	Opex	OH lines	UG cables	Transf.
Energy	0.00	0.00	0.00	0.00
RMD	114.17	0.00	3.71	0.00
Cust.	0.00	0.00	0.00	0.02
Circ.Len.	0.00	3.44	0.00	0.00
Time (β)	-5230.35	25.47	514.74	194.84

Table 3.7 ESS Additive time trend model

	Opex	OH lines	UG cables	Transf.
Energy	12.06	2.18	0.00	1.07
RMD	0.00	81.08	1.89	0.00
Cust.	0.27	0.00	0.00	0.00
Circ.Len.	0.00	2.07	0.05	0.00
Time (β)	-6739.35	1437.89	555.02	224.13

Table 3.8 JEN Additive time trend model

	Opex	OH lines	UG cables	Transf.
Energy	0.00	0.00	0.00	0.00
RMD	52.98	0.00	0.00	0.99
Cust.	0.00	0.00	0.02	0.00
Circ.Len.	0.00	8.03	0.00	0.41
Time (β)	217.76	-220.22	384.17	69.87

Table 3.9 PCR Additive time trend model

	Opex	OH lines	UG cables	Transf.
Energy	0.00	0.19	0.00	0.00
RMD	55.21	0.00	0.36	0.26
Cust.	0.00	0.13	0.01	0.00
Circ.Len.	0.00	4.86	0.00	0.10
Time (β)	480.00	-2938.64	599.08	209.68



Table 3.10 SAP Additive time trend model

	Орех	OH lines	UG cables	Transf.
Energy	0.00	1.84	0.84	0.00
RMD	41.41	0.00	0.00	1.99
Cust.	0.00	0.11	0.01	0.00
Circ.Len.	0.00	1.10	0.02	0.06
Time (β)	3170.86	-740.74	621.59	207.85

Table 3.11 AND Additive time trend model

	Opex	OH lines	UG cables	Transf.
Energy	0.00	0.00	0.00	0.00
RMD	65.14	0.39	0.00	1.68
Cust.	0.00	0.00	0.02	0.00
Circ.Len.	0.00	4.73	0.00	0.10
Time (β)	1087.16	-664.73	811.88	112.85

Table 3.12 TND Additive time trend model

	Opex	OH lines	UG cables	Transf.
Energy	4.21	5.16	0.00	0.00
RMD	0.00	0.00	0.00	0.00
Cust.	0.15	0.00	0.00	0.01
Circ.Len.	0.00	3.43	0.45	0.03
Time (β)	154.49	315.86	140.71	50.71

Table 3.13 UED Additive time trend model

	Opex	OH lines	UG cables	Transf.
Energy	3.97	0.00	0.00	0.00
RMD	27.83	13.25	1.08	1.12
Cust.	0.00	0.10	0.00	0.01
Circ.Len.	0.00	0.00	0.57	0.00
Time (β)	-445.64	-1.64	251.09	115.21

3.2.2 TNSPs

The estimation results for 4 input demands (equation (3.2)) for each of the 5 TNSPs are presented in tables 3.14 to 3.18. In these tables the estimated coefficients applying to outputs are the $\hat{\alpha}_{ij}$'s. Because they are produced using quadratic programming, there are no standard errors for these parameters.



Table 3.14 ENT Additive time trend model

	Орех	OH lines	UG cables	Transf.
Energy	0.55	0.00	0.09	0.09
RMD	0.00	0.00	0.34	1.80
Cust. No.	0.05	0.00	0.00	0.00
Circ. Len.	0.40	317.56	0.00	0.00
Time (β)	833.19	8223.49	1029.40	187.17

Table 3.15 PLK Additive time trend model

	Opex	OH lines	UG cables	Transf.
Energy	0.66	0.00	0.00	0.00
RMD	0.00	0.00	0.00	0.00
Cust.	0.00	0.00	0.00	0.00
Circ.Len.	7.47	590.96	0.50	1.95
Time (β)	-387.38	22210.48	12.91	567.75

Table 3.16 ANT Additive time trend model

	Opex	OH lines	UG cables	Transf.
Energy	1.02	9.70	0.11	0.00
RMD	0.59	0.00	0.01	0.30
Cust.	0.00	1.14	0.00	0.00
Circ.Len.	0.00	441.12	0.04	3.06
Time (β)	-475.06	-46139.66	-68.80	272.72

Table 3.17 TNT Additive time trend model

	Opex	OH lines	UG cables	Transf.
Energy	2.81	0.00	0.00	0.00
RMD	0.00	0.00	0.00	0.00
Cust.	0.03	0.93	0.00	0.02
Circ.Len.	0.00	148.94	0.33	0.00
Time (β)	-1649.62	6088.24	160.30	54.29

Table 3.18 TRG Additive time trend model

	Opex	OH lines	UG cables	Transf.
Energy	0.00	0.00	0.32	0.00
RMD	1.21	431.12	0.00	2.16
Cust.	0.00	0.00	0.00	0.00
Circ.Len.	7.62	0.00	0.00	0.00
Time (β)	-497.43	95787.12	2245.59	934.52

3.3 Non-reliability output index weights

The non-reliability output index weights associated with the additive time trend specification are presented in Table 3.19, for both DNSPs and TNSPs.



Table 3.19	Output cost shares: QP method (%)

Output	DNSP	TNSP
Energy throughput	8.433	11.259
Ratcheted max. demand	53.650	26.321
Customer numbers	14.129	11.274
Circuit length	23.787	51.146
Tota1*	100.000	100.000

^{*} Figures may not sum to 100.000 due to rounding.

4 Least absolute deviations & Linear programming

4.1 Quantitative method

CEPA's second suggestion is to fit the input demand functions using least absolute deviations (LAD) rather than least squares. The model can then be transformed into a linear program (LP), as set out in equation (17) of the CEPA paper, and reproduced as (4.1) below.

$$\min_{\alpha,\beta,u} \sum_{t} u_{t}$$

$$s.t. \quad \alpha_{ij} \geq 0 \quad \text{for each j}$$

$$x_{i} - \beta_{i}t - \sum_{j=1}^{N} \alpha_{ij}y_{j} \leq u_{t}$$

$$\beta_{i}t + \sum_{j=1}^{N} \alpha_{ij}y_{j} - x_{i} \leq u_{t}$$

$$(4.1)$$

Quantile regression is another way of estimating a model using LAD. If quantile regression were used, the constraint $a_{ij} \ge 0$ is imposed by using a_{ij}^2 as in equation (3). Hence, nonlinear quantile regression would be needed. However, CEPA suggests that the LP approach should yield more reliable results than quantile regression. Further, Stata does not have a routine for nonlinear quantile regression.

Stata's Mata matrix programming functionality includes LP, and we have used this to carry out the LP analysis. There is a separate linear program for each input for each DNSP.

4.2 Linear programming results

4.2.1 DNSPs

The results for the linear programs (4.1) are presented for each of the 13 DNSPs in tables 4.1 to 4.13. In these tables the estimated coefficients applying to outputs are the $\hat{\alpha}_{ij}$'s, similar to



those in section 3, but unlike the tables in section 2. Because they are produced using linear programming, there are no standard errors for the parameter estimates.

Table 4.1 EVO LP model

	Opex	OH lines	UG cables	Transf.
Energy	13.95	0.40	0.00	0.10
RMD	0.00	34.20	0.00	0.00
Cust. No.	0.00	0.00	0.00	0.00
Circ. Len.	0.00	9.42	1.72	0.61
Time (β)	502.49	-362.68	201.89	14.08

Table 4.2 AGD LP model

	Opex	OH lines	UG cables	Transf.
Energy	0.00	0.00	0.00	0.00
RMD	75.11	45.58	0.00	4.17
Cust.	0.00	0.00	0.00	0.00
Circ.Len.	0.00	1.95	4.11	0.00
Time (β)	-12887.61	-1183.77	521.01	383.08

Table 4.3 CIT LP model

	Opex	OH lines	UG cables	Transf.
Energy	6.58	0.00	0.00	0.06
RMD	0.00	0.00	0.00	0.61
Cust.	0.00	0.01	0.00	0.00
Circ.Len.	0.00	1.34	2.13	1.20
Time (β)	940.81	-88.81	141.75	36.63

Table 4.4 END LP model

	Орех	OH lines	UG cables	Transf.
Energy	0.00	0.00	0.00	0.00
RMD	53.37	72.38	6.61	2.81
Cust.	0.00	0.00	0.00	0.00
Circ.Len.	0.00	1.63	0.00	0.10
Time (β)	-2806.27	-3752.19	2044.04	259.44

Table 4.5 ENX LP model

	Opex	OH lines	UG cables	Transf.
Energy	0.00	0.00	0.00	0.00
RMD	51.90	11.13	10.47	2.24
Cust.	0.00	0.03	0.00	0.00
Circ.Len.	0.00	3.89	0.00	0.21
Time (β)	72.79	-755.94	1867.27	361.23

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Table 4.6 ERG LP model

	Opex	OH lines	UG cables	Transf.
Energy	0.00	7.43	0.00	0.00
RMD	0.00	14.14	3.67	0.00
Cust.	0.27	0.00	0.00	0.02
Circ.Len.	1.10	2.47	0.00	0.00
Time (β)	-5026.18	392.15	528.47	202.31

Table 4.7 ESS LP model

	Opex	OH lines	UG cables	Transf.
Energy	19.88	0.00	0.00	0.00
RMD	0.00	84.68	3.61	0.16
Cust.	0.00	0.00	0.00	0.00
Circ.Len.	0.71	2.17	0.02	0.08
Time (β)	-6117.57	1301.25	587.22	228.15

Table 4.8 JEN LP model

	Opex	OH lines	UG cables	Transf.
Energy	0.00	0.00	0.00	0.00
RMD	30.32	0.73	0.00	0.90
Cust.	0.00	0.00	0.02	0.00
Circ.Len.	3.46	7.92	0.00	0.43
Time (β)	403.84	-217.50	401.92	64.57

Table 4.9 PCR LP model

-	Орех	OH lines	UG cables	Transf.
Energy	0.00	1.70	0.00	0.00
RMD	0.00	0.00	0.00	0.14
Cust.	0.00	0.13	0.01	0.00
Circ.Len.	1.71	4.68	0.00	0.10
Time (β)	1462.95	-2904.11	587.16	211.94

Table 4.10 SAP LP model

	Opex	OH lines	UG cables	Transf.
Energy	0.00	1.76	0.74	0.00
RMD	40.81	0.00	0.77	3.43
Cust.	0.00	0.19	0.01	0.00
Circ.Len.	0.00	0.40	0.05	0.01
Time (β)	3445.61	-1311.60	533.91	180.77



Table 4.11 AND LP model

	Opex	OH lines	UG cables	Transf.
Energy	0.00	3.87	0.00	0.07
RMD	63.10	3.07	0.00	2.66
Cust.	0.00	0.00	0.02	0.00
Circ.Len.	0.00	3.89	0.00	0.04
Time (β)	985.12	-517.99	801.57	97.08

Table 4.12 TND LP model

	Opex	OH lines	UG cables	Transf.
Energy	4.37	4.57	0.00	0.00
RMD	0.00	0.00	0.00	0.00
Cust.	0.13	0.00	0.00	0.01
Circ.Len.	0.00	3.53	0.45	0.05
Time (β)	523.44	343.78	139.36	47.21

Table 4.13 UED LP model

	Opex	OH lines	UG cables	Transf.
Energy	2.92	0.00	0.00	0.00
RMD	34.16	14.73	1.35	1.23
Cust.	0.00	0.01	0.00	0.01
Circ.Len.	0.00	4.41	0.53	0.03
Time (β)	-841.97	201.88	246.79	116.37

4.2.2 TNSPs

The LP estimation results for the 4 input demands for each of the 5 TNSPs are presented in tables 4.14 to 4.18.

Table 4.14 ENT LP model

	Opex	OH lines	UG cables	Transf.
Energy	2.18	0.00	0.17	0.28
RMD	3.19	0.00	0.00	1.11
Cust. No.	0.00	0.00	0.00	0.00
Circ. Len.	0.28	323.95	0.00	0.00
Time (β)	1230.85	3238.70	1000.91	210.98

Table 4.15 PLK LP model

	Opex	OH lines	UG cables	Transf.
Energy	0.57	0.00	0.00	0.00
RMD	0.00	0.00	0.00	0.00
Cust.	0.00	0.00	0.00	0.00
Circ.Len.	7.71	588.88	0.51	2.03
Time (β)	-573.62	37873.62	6.29	496.77



Table 4.16 ANT LP model

	Opex	OH lines	UG cables	Transf.
Energy	0.60	11.47	0.11	0.01
RMD	1.05	0.00	0.00	0.21
Cust.	0.01	0.53	0.00	0.00
Circ.Len.	0.00	657.73	0.00	3.10
Time (β)	-973.12	-24342.31	-59.05	277.24

Table 4.17 TNT LP model

	Opex	OH lines	UG cables	Transf.
Energy	3.55	0.00	0.00	0.00
RMD	0.00	0.00	0.00	0.00
Cust.	0.00	1.52	0.00	0.02
Circ.Len.	0.00	107.74	0.32	0.22
Time (β)	-1613.95	3897.80	156.16	81.31

Table 4.18 TRG LP model

	Opex	OH lines	UG cables	Transf.
Energy	0.00	21.05	0.32	0.00
RMD	0.00	334.30	0.00	2.40
Cust.	0.00	0.00	0.00	0.00
Circ.Len.	9.44	0.00	0.00	0.00
Time (β)	-308.76	127881.88	2142.64	619.87

4.3 Non-reliability output index weights

The non-reliability output index weights associated with the LP method and LAD are presented in Table 4.19, for both DNSPs and TNSPs.

Table 4.19 Output cost shares: LP method (%)

Output	DNSP	TNSP
Energy throughput	8.950	14.846
Ratcheted max. demand	46.605	23.450
Customer numbers	11.003	7.304
Circuit length	33.441	54.399
Total*	100.000	100.000

^{*} Figures may not sum to 100.00 due to rounding.

5 Conclusions

Figures 5.1 and 5.2 compare the non-reliability output weights produced by the three methods applied in this report. The results indicate a reasonable degree of consistency between the methods. This provides confidence in the reliability of the results. Tables 5.1 and 5.2 compare



the estimated output cost shares using the three methods presented here for DNSPs and TNSPs respectively. The consistency of the results obtained using the QP and LP methods lend support to the reliability of the output cost shares obtained using the Economic Insights method.

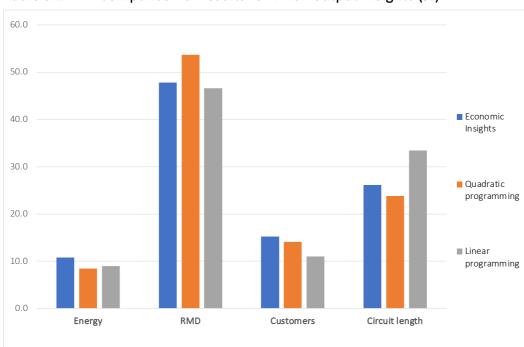


Table 5.1 Comparison of results for DNSP output weights (%)



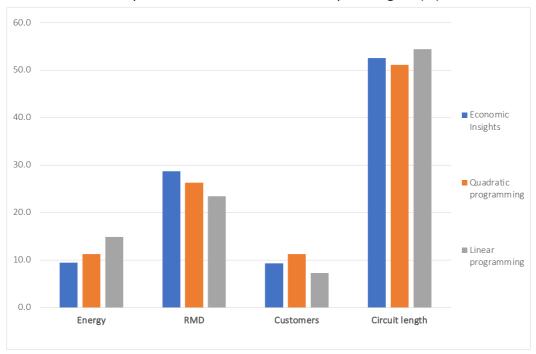




Table 5.1 DNSP output cost shares: Comparison of methods (%)

	Economic Insights		
Output	method	QP method	LP method
Energy throughput	10.793	8.433	8.950
RMD	47.827	53.650	46.605
Customer numb.	15.230	14.129	11.003
Circuit length	26.151	23.787	33.441
Total*	100.000	100.000	100.000

^{*} Figures may not sum to 100.000 due to rounding.

Table 5.2 TNSP output cost shares: Comparison of methods (%)

	Economic Insights		
Output	method	QP method	LP method
Energy throughput	9.445	11.259	14.846
RMD	28.685	26.321	23.450
End-cust.	9.329	11.274	7.304
Circuit length	52.540	51.146	54.399
Total*	100.000	100.000	100.000

^{*} Figures may not sum to 100.000 due to rounding.

The alternative methods proposed by CEPA can be used as cross-checks against the Economic Insights method. Specifically, to check whether the NLS estimates of output weights are unduly affected by the problem of non-unique solution values for the estimated coefficients, as raised by CEPA. The reasonable degree of consistency between the methods tends to support the continued use of the Economic Insights method for the purposes of the 2025 electricity network productivity index analysis.



Appendix A: Calculating output weights in the additive trend model

CEPA's approach of separating out the time trend term as shown in equation (3.1) would no longer provide for the input costs to be fully attributed to each output. Instead, relative weights would need to be derived for each output.

An alternative way of calculating output weights is to use elasticities of cost with respect to output. When elasticities are used, the output weights are defined as: $\mu_j = \epsilon_j / \sum_j \epsilon_j$, where ϵ_j is the cost-elasticity with respect to output j, and division by the sum of the elasticities ensures they must sum to unity. Each elasticity is defined as:

$$\epsilon_j \equiv \frac{\partial C}{\partial y_i} \cdot \frac{y_j}{C} \tag{A.1}$$

where $\partial C/\partial y_j$ is the marginal cost of producing output j, which serves as the shadow price of output j.

In the Lawrence-Diewert method, the elasticities of cost with respect to each output are.

$$\epsilon_{j} = \frac{\partial C}{\partial y_{j}} \frac{y_{j}}{C} = \frac{1}{C} \sum_{i=1}^{M} w_{i} \frac{\partial x_{i}}{\partial y_{j}} y_{j}$$

$$= \frac{1}{C} \sum_{i=1}^{M} w_{i} \hat{a}_{ij}^{2} y_{j} (1 + \hat{b}_{i}t) = \frac{C_{j}}{C}$$
(A.2)

Hence, the method of calculating weights using elasticities in the Lawrence-Diewert method yields the same weights as their method of full cost allocation and calculation of cost shares.

In the CEPA additive time trend approach, the elasticities of cost with respect outputs are:

$$\epsilon_j = \frac{\partial C}{\partial y_j} \frac{y_j}{C} = \frac{1}{C} \sum_{i=1}^M w_i \hat{a}_{ij}^2 y_j \tag{A.3}$$

The output weight for output *j* can then derived by the elasticities' method:

$$\mu_j = \frac{\sum_{i=1}^{M} w_i \hat{a}_{ij}^2 y_j}{\sum_{j=1}^{N} \sum_{i=1}^{M} w_i \hat{a}_{ij}^2 y_j}$$
(A.4)

In summary, in CEPA's additive time-trend variation, costs are not fully allocated to outputs and the elasticities method needs to be used to calculate output weights. The formula for calculating output weights is given by (A.2).



Appendix B: Standard Errors of output weights in Economic Insights' method

We have derived expressions for standard errors of the average output cost shares, that serve as non-reliability output weights. These are based on the standard errors of estimated parameters in the Economic Insights input demand specification, and an application of the delta method. The delta method is only an approximation. The method is implemented in our Stata programs used to estimate that specification. An explanation of the mathematical derivation of these standard errors of the output weights can be provided on request. However, there are problems with applying the method.

Tables 2.1 to 2.18 show that, for many coefficients, the t-statistics are missing, since the associated standard errors are missing. This typically arises when the estimated coefficient is zero. To calculate the standard errors of output cost shares, it is necessary to treat these missing standard errors as zero. This is a strong assumption, which implies the confidence intervals for the output weights are likely unreliable. They are likely to substantially understate the true standard errors. For these reasons, we have not presented the results for standard errors of the output weights.

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