THE RISK FREE RATE AND THE PRESENT VALUE PRINCIPLE

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EXECUTIVE SUMMARY

This paper has addressed a number of issues raised by the AER. The first of these issues is, in determining a regulatory rate of return, whether the use of a risk free rate averaged over a short period (10-40 business days) as close a practically possible to the start of the regulatory period is consistent with

- the Present Value principle (the present value of a regulated firm's revenue stream should match, over time, the present value of its expenditure stream plus or minus any efficiency incentive rewards or penalties), and
- the Building Block model, and
- the Cost Recovery principle (a regulated firm should have a reasonable opportunity to recover at least its efficient costs), and
- the Sharpe-Lintner model.

In relation to the Present Value principle, any present value operation requires the use of the current risk free rate, the regulatory rate of return must therefore use the risk free rate at the beginning of the regulatory period, and pragmatic considerations then lead to averaging it over a short period as close as practical to the start of the regulatory period. In relation to the Building Block model, this is a consequence of the Present Value principle and therefore the same conclusion applies. In relation to the Cost Recovery principle, this is equivalent to the Present Value principle and therefore the same conclusion also applies. In relation to the Sharpe-Lintner model, this model always requires a risk free rate prevailing at a point in time for some subsequent period rather than a historical average and application of the model to a regulatory situation would require the risk free rate prevailing at the beginning of a regulatory period.

The second issue is whether the adoption of a long term historical average risk free rate would be consistent with the four matters detailed above. The conclusions here follow from the conclusions presented in the previous paragraph: since the risk free rate at the beginning of the regulatory period is required in all cases, then the use of a long-term historical average risk free rate will not be consistent with the Present Value principle, the Building Block model, the Cost Recovery principle, or the Sharpe-Lintner model.

The third issue is whether the Present Value principle requires that the regulatory rate of return should be representative of prevailing market conditions at the start of the regulatory period or at the start of each year within the regulatory period. If the regulatory rate of return is representative of prevailing market conditions at the start of each year within the regulatory period, the Present Value principle requires that the regulatory rate of return be based upon the one year risk free rate prevailing at the beginning of each year. By contrast, if the regulatory rate of return is representative of prevailing market conditions at the start of each regulatory period, then the Present Value principle requires that the regulatory rate of return be based upon the yield to maturity prevailing at the beginning of the regulatory period on the risk free bond with a duration equal to that of the regulatory cash flows, and a very close approximation is achieved through the use of the yield to maturity on the risk free bond whose term to maturity matches that of the regulatory period. Capital expenditures within a regulatory period may worsen the approximation here but the error in the regulatory rate chosen as described above does not exceed seven basis points across the range of examples considered, and this is not material.

1. Introduction

This report seeks to address a number of issues raised by the AER, as follows:

Firstly, critically evaluate the AER's position in the Aurora final decision that the use of a risk free rate averaged over a short period (10-40 business days) as close a practically possible to the start of the regulatory period was

(a) Consistent with the use of the Building Block model, and
(b) Consistent with the economic principle that the present value of a regulated firm's revenue stream should match, over time, the present value of its expenditure stream (plus or minus any efficiency incentive rewards or penalties), and
(c) Consistent with the use of the Sharpe-Lintner CAPM, and
(d) Consistent with the economic principle that a regulated firm have a reasonable opportunity to recover at least its efficient costs.

Secondly, assess whether the adoption of a long term historical average risk free rate would be consistent with the four matters detailed above.

Thirdly, and in view of the fact that an objective that the AER seeks to achieve is that the present value of a regulated firm's revenue stream matches the present value of its expenditure stream (plus or minus any efficiency incentive rewards or penalties), assess whether the regulatory rate of return should be representative of prevailing market conditions at the start of the regulatory period or at the start of each year within the regulatory period. This assessment should take into account the fact that a regulated firm may raise equity capital throughout the regulatory period (to finance capital expenditures).

The focus here is upon the cost of equity capital and therefore it is assumed that the firm is entirely equity financed.

2. The Averaging Period for the Risk Free Rate

2.1 Consistency with the Present Value Principle

I start by considering whether the use of risk free rates that are averaged over a short period as close as practical to the start of the regulatory period, and also over a much longer historical period, are consistent with the principle that the present value of the regulated firm's revenue stream should match the present value of its expenditure stream, plus or minus any efficiency incentive rewards or penalties (the Present Value principle). The Present Value principle is fundamental to regulation; lower revenues than those that satisfy this principle will fail to entice producers to invest and higher revenues constitute the very excess profit that regulation seeks to prevent (Marshal et al, 1981).

To explore the implications of this principle for the risk free rate, it is sufficient to envisage the simplest possible regulatory scenario, in which fixed assets are purchased now, all financing is equity, a revenue or price cap is set now that yields revenues only in one year, all operating costs are incurred at the same point, the regulatory assets purchased now have a life of one year, there is no risk relating to revenues or operating costs, and there is no differential personal tax treatment across different sources of investment income. In this case the value now of the revenues received in one year (*REV*) net of operating costs received in one year (*OPEX*) is determined by discounted at the *current* one year risk free rate (R_{f1}), and the Present Value principle implies that this value should equal the purchase price of the fixed assets (*B*):[1]

$$B = \frac{REV - OPEX}{1 + R_{f1}} \tag{1}$$

It follows from this that the revenues must be as follows:[2]

$$REV = OPEX + B + BR_{f1} \tag{2}$$

So, the revenues must equal the sum of *OPEX*, the cost of the fixed assets (*B*), and the return on the investment of *B* at the current one year risk free rate R_{f1}. This analysis is a simplified version of that in Schmalensee (1989) and Lally (2004).

To illustrate the application of equation (2), suppose *OPEX* = \$10m, *B* = \$100m and R_{f1} = .06. It follows from equation (2) that *REV* must be \$116m. The intuition for this is clear.

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[1] If there is uncertainty about revenues or opex, this leads to a risk premium being added to the discount rate, and this does not otherwise affect the analysis.

[2] In this equation, regulatory depreciation equals the cost of the asset (*B*) because the asset life is only one year. When the asset life exceeds one year, as in later examples, depreciation each year is less than the purchase price of the assets.

Investors with $100m to invest could invest in the risk free asset at 6% to yield $106m in one year. Undertaking the regulatory activities and therefore purchasing the regulatory assets is an alternative investment with the same (nil) risk. Thus, undertaking the regulatory activities and therefore purchasing the regulatory assets should also yield a return of 6% on the investment of $100m, which implies net cash flow of $106m in one year, and hence revenues of $116m.

This demonstrates that the risk free rate that should be used is that prevailing at the beginning of the regulatory period. Suppose that some historic average of the one year rates had instead been used and this historic average was 7%. In that event, following equation (2), the price or revenue cap would have been set to allow revenues of $117m and therefore a rate of return of 7% on a risk free investment with a one year life. This rate of return would be too high because the one year risk free rate at the beginning of the regulatory period was 6%. Alternatively, if the historical average risk free rate had been 5%, then the price or revenue cap would have been set to allow revenues of $115m and therefore a rate of return of 5% on a risk free investment with a one year life; this rate would be too low. An inevitable critique of this sort of analysis is that it is involves simplifying assumptions. However, none of the simplifying assumptions in the above analysis change the result because present values always involve the use of the *current* risk free rate rather than an average over some historical period, and current in a regulatory context means at the beginning of the regulatory period.

In summary, the Present Value principle requires use of the risk free rate at the beginning of the regulatory period. Literally, this involves the first market price on the first day of the regulatory period. However, the use of this transaction would expose the regulatory process to reporting errors, an aberration arising from an unusually large or small transaction, and a rate arising from a transaction undertaken by a regulated firm for the purpose of influencing the regulatory decision. These pragmatic considerations imply that the rate should be averaged over a short period as close as practical to the start of the regulatory period. Rates averaged over a much longer historical period would be inconsistent with the Present Value principle, i.e., they would violate it without offering any incremental pragmatic justification.

2.2 Consistency with the Building Block Model

The next question is whether the use of risk free rates that are averaged over a short period as close as practical to the start of the regulatory period, and also over a much longer historical period, are consistent with the Building Block model. The Building Block model is equation (2), which is a consequence of the Present Value principle in equation (1). So, the answer here is exactly the same as in the previous section, i.e., the Building Block model requires use of the risk free rate at the beginning of the regulatory period and therefore the rate should be averaged over a short period as close as practical to the start of the regulatory period. Rates averaged over a much longer historical period would be inconsistent with the Building Block model.

2.3 Consistency with the Cost Recovery Principle

The next question is whether the use of risk free rates that are averaged over a short period (10-40 business days) as close a practically possible to the start of the regulatory period, and also over a much longer historical period, are consistent with the economic principle that a regulated firm have a reasonable opportunity to recover at least its efficient costs (the Cost Recovery principle). Returning to the analysis in section 2.1, the costs involved there were the purchase price of the regulatory assets, operating expenditure, and a 'fair' return on the investment (with the last of these being an opportunity cost). The Cost Recovery principle says that the revenues must cover these costs and therefore the Cost Recovery principle is equivalent to the Present Value principle. The same conclusion in section 2.1 then applies, i.e., the Cost Recovery principle requires use of the risk free rate at the beginning of the regulatory period and therefore this rate should be averaged over a short period as close as practical to the start of the regulatory period. Rates averaged over a much longer historical period would be inconsistent with the Cost Recovery principle.

2.4 Consistency with the Sharpe-Lintner CAPM

The next question is whether the use of risk free rates that are averaged over a short period (10-40 business days) as close a practically possible to the start of the regulatory period, and also over a much longer historical period, are consistent with the Sharpe-Lintner CAPM. The Sharpe-Lintner CAPM (Sharpe, 1964; Lintner, 1965; Mossin, 1966) is a model that specifies the equilibrium expected rate of return on a risky asset (i.e., the expected rate of return that just compensates for risk), and one of the parameters in this model is the risk free rate. One of the assumptions underlying this model is that investors select portfolios based on the Markowitz (1952, 1959) model, in which an investor chooses (at some point in time, *T*) that portfolio of assets that has the 'best' probability distribution of returns over a future period from time *T*. One such asset is the risk free asset and the risk free rate within the Sharpe-

Lintner model is then the risk free rate prevailing at time *T* for some future term. This model can be used to estimate the cost of equity capital for a regulated entity. Doing so requires that the Sharpe-Lintner and regulatory models be aligned. This requires that the risk free rate within the Sharpe-Lintner model must be the rate prevailing at the beginning of the regulatory period. As before, pragmatic considerations lead to choosing a risk free rate averaged over a short period as close as practical to the start of the regulatory period. Furthermore, averaging the risk free rate over a historical period would never be compatible with the Markowitz model (because an investor makes a portfolio decision at a point in time) and therefore would never be compatible with the Sharpe-Lintner model.

In summary, use of the Sharpe-Lintner model in a regulatory context requires that the risk free rate must be the rate prevailing at the beginning of the regulatory period. As before, pragmatic considerations lead to choosing a risk free rate averaged over a short period as close as practical to the start of the regulatory period. Furthermore, averaging the risk free rate over a historical period would never be compatible with the Markowitz portfolio model and therefore never with the Sharpe-Lintner model.

3. The Use of a Single or a Succession of Risk Free Rates

3.1 Setting the Regulatory Rate of Return at the Beginning of each Year

The next question to assess is whether the regulatory rate of return should be representative of prevailing market conditions at the start of the regulatory period or at the start of each year within the regulatory period. I consider the latter case in this section.

To consider this issue, it is sufficient to modify the analysis in section 2.1 only to the extent of assuming a regulatory period in excess of one year. So, suppose the regulatory period is two years and the asset life is four years. Since the asset has a life of four years then the purchase price must be allocated across the four periods; this is called regulatory depreciation and the allocation is DEP_1, … DEP_4 for years 1…..4 respectively. Thus, fixed assets are purchased now, all financing is equity, revenues are received in one, two, three and four years' time, all operating costs are incurred at the same four points, there is no risk relating to revenues or operating costs, and there is no differential personal tax treatment across different sources of investment income.[3] In addition, the regulatory rate of return (and therefore the price or revenue cap) is reset at the beginning of each year in accordance with prevailing market conditions. Other parameters would be rest every two years, because the regulatory period is two years, but they are not significant for the present purposes and therefore are treated as fixed over time. In this case, effectively, the regulatory cycle is one year.

The Present Value principle requires that the present value of the revenues net of operating costs over the four year life of the assets be equal to the purchase price of the fixed assets, and this can be achieved by using the prevailing one year risk free rate when resetting the price or revenue cap each year. To prove this, start with the situation at the end of the third year (time 3), at which point a price or revenue cap will be set to yield revenues at time 4 (REV_4). The value at time 3 of the subsequent payoffs from the regulatory assets will be the value at time 3 of these revenues at time 4 net of the operating costs at time 4 ($OPEX_4$), discounted at some rate prevailing at time 3. Since the payoffs at time 4 are certain the appropriate discount rate is the one year risk free rate prevailing at time 3 (R_{f34}). The value at time 3 of the subsequent payoffs from the regulatory assets is then as follows:

$$V_3 = \frac{REV_4 - OPEX_4}{1 + R_{f34}} \tag{3}$$

Revenues received at time 4 are set at time 3 to cover operating costs at time 4, depreciation for year 4, and the cost of capital at some rate applied to the regulatory book value at time 3 (B_3). Since the relevant rate is asserted to be the one year risk free rate prevailing at time 3 (R_{f34}) then these revenues at time 4 will be as follows:

$$REV_4 = OPEX_4 + DEP_4 + B_3 R_{f34} \tag{4}$$

Since year 4 is the last year, then $DEP_4 = B_3$. Substituting this into equation (4) and then (4) into (3) yields

$$V_3 = \frac{B_3(1 + R_{f34})}{1 + R_{f34}} = B_3 \tag{5}$$

[3] As in section 2.1, uncertainty about revenues or opex leads to a risk premium being added to the discount rate, and this does not otherwise affect the analysis.

So, the value at time 3 of the subsequent payoffs on the regulatory assets will equal the regulatory asset book value at time 3 if revenues received at time 4 are set at time 3 using the prevailing one year risk free rate. At time 2, the revenues to be received at time 3 will be set. So, at time 2, the value of the subsequent payoffs on the regulatory assets will be the value at time 2 of the revenues received at time 3 less the operating cost incurred at time 3 plus the value at time 2 of V_3, and V_3 equals B_3 as shown in equation (5), and all of these payoffs at time 3 are known at time 2. So the appropriate discount rate on these payoffs arising at time 3 will be the prevailing one year risk free rate at time 2, and therefore the value at time 2 of the subsequent payoffs on the regulatory assets will be as follows:

$$V_2 = \frac{REV_3 - OPEX_3 + B_3}{1 + R_{f23}} \tag{6}$$

Paralleling equation (4), the revenues to be received at time 3 will be set at time 2 based upon the one year risk free rate prevailing at time 2 (R_{f23}):

$$REV_3 = OPEX_3 + DEP_3 + B_2 R_{f23} \tag{7}$$

Also, $DEP_3 = B_2 - B_3$. Substituting this into equation (7) and then (7) into (6) yields

$$V_2 = \frac{B_2(1 + R_{f23})}{1 + R_{f23}} = B_2 \tag{8}$$

So, the value at time 2 of the subsequent payoffs on the regulatory assets will equal the regulatory asset book value at time 2 if revenues received at times 3 and 4 are each set one year earlier using the prevailing one year risk free rate. By continuing this process, it can be shown in the same way that $V_1 = B_1$ and then that $V_0 = B_0$. The last equation says that the value now of all future payoffs on the regulatory assets is equal to the purchase price of the assets, i.e., the Present Value principle is satisfied.

To illustrate this, suppose that the purchase price of the fixed assets is \$100m, depreciation is set at \$25m per year, operating costs are \$10m per year, the current one year risk free rate is

5%, and the subsequent one year rates in one, two and three years' time are 4%, 6% and 8% respectively. Following equation (4), in three years' time with the one year risk free rate of 8% at that time, the price or revenue cap would be set at that time to give rise to revenues one year later of:

$$REV_4 = OPEX_4 + DEP_4 + B_3 R_{f34} = \$10m + \$25m + \$25m(.08) = \$37m$$

The net cash flows at time 4 would then be these revenues of $37m less the operating costs at that time of $10m, to yield net cash flow of $27m. The value at time 3 of this net cash flow arising at time 4 would then be $27m discounted at the one year risk free rate prevailing at time 3, of 8%:

$$V_3 = \frac{\$27m}{1.08} = \$25m$$

Following equation (7), at time 2 with the one year risk free rate of 6% at that time, the price or revenue cap would be set at that time to give rise to revenues one year later of:

$$REV_3 = OPEX_3 + DEP_3 + B_2 R_{f23} = \$10m + \$25m + \$50m(.06) = \$38m$$

The cash flow at time 3 would then be these revenues of $38m less the operating costs at that time of $10m, to yield net cash flow of $28m. The value at time 2 of the subsequent payoffs on the regulatory assets would then be the value at time 2 of the net cash flow at time 3 ($28m) plus the value at time 2 of V_3 ($25m), all discounted at the one year risk free rate prevailing at time 2 of 6%:

$$V_2 = \frac{\$28m + \$25m}{1.06} = \$50m$$

Continuing in this fashion, the value now of all future payoffs on the regulatory assets would be $100m, matching the purchase price of the fixed assets and therefore satisfying the Present Value principle.

If the regulator sets the price or revenue cap, and hence revenues, in accordance with a risk free rate other than the one year rate then the Present Value principle will be violated. For example, suppose the regulator uses the ten year rate and this rate generally exceeds the one year rate by 1%.[4] In this event the regulated firm would receive revenues larger than those required to satisfy the Present Value principle, by \$0.25m at time 4 (1% of B_3), \$0.5m at time 3 (1% of B_2), \$0.75m at time 2 (1% of B_1), and \$1m at time 1 (1% of B_0). The value of these excess revenues at the purchase date of the assets would be determined by the prevailing spot rates for risk free cash flows received at these times. The current one year spot rate has already been given as 5%, and the ten year rate as 1% larger (6%). So, the spot rates for cash flows due in two, three and four years will in general be above 5%. Using 5% as a lower bound, the present value of these additional revenues will then be at least \$2.27m as follows.

$$V = \frac{\$1m}{1.05} + \frac{\$0.75m}{(1.05)^2} + \frac{\$0.50m}{(1.05)^3} + \frac{\$0.25m}{(1.05)^4} = \$2.27m$$

This example is based on an asset with a life of only four years. If the asset had a more typical life, of say 50 years, then the present value of the additional revenues would be

$$V = \frac{\$1m}{1.05} + \frac{\$0.98m}{(1.05)^2} + \frac{\$0.96m}{(1.05)^3} + ... + \frac{\$0.2m}{(1.05)^{50}} = \$12.70m$$

The present value of the additional revenues is now almost 13% of the purchase price of the asset (\$100m), i.e., 13% of the present value of the net cash flows if the correct risk free rate had been used by the regulator.

In summary, if the regulatory rate of return is representative of prevailing market conditions at the start of each year within the regulatory period, then that regulatory rate of return must be based upon the prevailing one year risk free rate at the beginning of each year in order to satisfy the Present Value principle. If any other rate were used the Present Value principle would be violated and, for every 1% that this alternative rate generally exceeds the one year rate, the present value of the additional revenues would be about 13% of the purchase price of the assets.

[4] The margin would vary over time in accordance with expectations of future one year rates, in accordance with the expectations hypothesis (van Horne, 1984). On average, this margin would be zero. However, empirically, there is a further margin called the term premium, which is always positive (McCulloch, 1975; Fama, 1984).

3.2 Setting the Regulatory Rate of Return at the Start of the Regulatory Period

I now turn to the situation in which the regulatory rate of return (and therefore the price or revenue cap) is reset at the beginning of each regulatory period (and fixed for that period), rather than reset at the beginning of each year, in accordance with prevailing market conditions. For illustrative purposes I assume the regulatory period is five years. In this case, for purposes of setting the regulatory rate of return, the risk free bond whose use will satisfy the Present Value principle is that whose duration matches that of the regulatory cash flows. If the regulatory period is five years, it might then seem that the appropriate risk free rate will be the yield to maturity on a bond maturing in five years. However the duration of this bond (something less than five years) might differ from the duration of the regulatory payoffs (something less than five years) but any difference is likely to be inconsequential.

To illustrate this point, consider the following example. Suppose the regulatory asset book value is currently \$100m, the output price is reset every five years, depreciation is 2% per year, capex is \$2m per year, operating costs are \$10m per year and incurred at year end, and revenues are certain and received annually at the end of each year.[5] In five years' time, and following the analysis in the previous section, the output price will be reset to ensure that the value at that time of the subsequent payoffs on the regulatory assets equals the regulatory asset book value prevailing at that time (of \$100m, because capex matches depreciation over the next five years). In addition, suppose the current spot interest rates for the next five years are .05 for year 1, .0525 for year 2, .055 for year 3, .0575 for year 4 and .06 for year 5.[6] Suppose the coupon interest rate on the five-year bond used to derive the five-year yield to maturity is 8%. Denoting the face value of this bond by F, the market value of this bond would be as follows:

$$B_0 = \frac{.08F}{1.05} + \frac{.08F}{(1.0525)^2} + \frac{.08F}{(1.055)^3} + \frac{.08F}{(1.0575)^4} + \frac{1.08F}{(1.06)^3} = 1.0875F$$

The yield to maturity on this bond (y) would then satisfy the following equation.

[5] As in previous sections, uncertainty about revenues or opex leads to a risk premium being added to the discount rate, and this does not otherwise affect the analysis.

[6] I have chosen an upward sloping term structure because this situation is typical.

$$1.0875F = \frac{.08F}{1+y} + \frac{.08F}{(1+y)^2} + \frac{.08F}{(1+y)^3} + \frac{.08F}{(1+y)^4} + \frac{1.08F}{(1+y)^3}$$

Accordingly, y = .0593. Using this risk free rate to set the firm's revenues, the resulting revenues for each of the next five years would be:

$$REV_1 = OPEX_1 + DEP_1 + B_0 R_f = \$10m + \$2m + \$100m(.0593) = \$17.93m$$
$$REV_2 = OPEX_2 + DEP_2 + B_1 R_f = \$10m + \$2m + \$100m(.0593) = \$17.93m$$
$$REV_3 = OPEX_3 + DEP_3 + B_2 R_f = \$10m + \$2m + \$100m(.0593) = \$17.93m$$
$$REV_4 = OPEX_4 + DEP_4 + B_3 R_f = \$10m + \$2m + \$100m(.0593) = \$17.93m$$
$$REV_5 = OPEX_5 + DEP_5 + B_4 R_f = \$10m + \$2m + \$100m(.0593) = \$17.93m$$

The net cash flows would then be \$17.93m less the opex of \$10m and the capex of \$2m, yielding \$5.93m per year. Using the spot interest rates given above, the present value of these net cash flows along with the value in five years of all subsequent payoffs on the regulatory assets (which equals the regulatory asset book value in five years, of \$100m) would then be as follows.

$$V_0 = \frac{\$5.93m}{1.05} + \frac{\$5.93m}{(1.0525)^2} + \frac{\$5.93m}{(1.055)^3} + \frac{\$5.93m}{(1.0575)^4} + \frac{\$5.93m + \$100m}{(1.06)^5} = \$99.93m$$

Since the present value of \$99.93m is marginally below the current regulatory book value of the assets, of \$100m, then the regulatory rate of return must be slightly low. In particular, a regulatory rate of return of 5.94% is required to generate a present value of \$100m, and therefore the rate used above of 5.93% is too low by 0.01%. The trivial extent of this error reflects the fact that the durations for the five year bond and the regulatory payoffs are very similar. In particular, and using Macaulay's second measure of duration (Elton et al, 2003, pp. 548-550)[7], the duration on the bond (D_B) is 4.34 years and that for the regulatory cash flows (D_R) is 4.47 years as follows.

[7] Macaulay's second rather than first measure of duration is required because the term structure of (spot) interest rates is not flat in this example.

$$D_B = \left[\frac{\frac{.08}{1.05}}{1.0875}\right](1) + \left[\frac{\frac{.08}{(1.0525)^2}}{1.0875}\right](2) + .. + \left[\frac{\frac{1.08}{(1.06)^5}}{1.0875}\right](5) = 4.34$$

$$D_R = \left[\frac{\frac{\$5.93m}{1.05}}{\$99.93m}\right](1) + \left[\frac{\frac{\$5.93m}{(1.0525)^2}}{\$99.93m}\right](2) + ... + \left[\frac{\frac{\$105.93m}{(1.06)^5}}{\$99.93m}\right](5) = 4.47$$

This close correspondence has occurred because depreciation matches capex, and therefore the regulatory asset book value is unchanged over the regulatory period. To achieve a perfect match, the coupon rate on the bond would have to be such that the duration on the bond matched that of the payoffs on the regulatory assets (and this would occur with a coupon rate on the bond of about 6%).

In summary, for purposes of setting the price or revenue cap, using a risk-free interest rate equal to the yield to maturity on the risk free bond whose duration matches that of the payoffs on the regulatory assets will satisfy the Present Value principle, and a very close approximation is achieved by using the yield to maturity on a risk free bond with a term to maturity equal to the regulatory period. Since the AER currently uses the ten year risk free rate and regulatory periods are typically less than this then the AER's current practice violates the Present Value principle and, since rates are generally higher for longer terms, will generally over compensate regulated firms. If the regulatory period is five years then the over compensation will be the excess of the ten year rate over the five year rate. Over the period since 1.1.1995, for which the five and ten year CGS rates are available, the excess of the ten year rate over the five year rate has averaged 0.23%.[8] Since the previous section shows that the present value of the excess revenues arising from using a risk free rate that is too high is about 13% of the purchase price of the assets for each 1% excess in the risk free rate used, an excess of 0.23% implies that the present value of the excess revenues would be about 3% of the purchase price of the assets.

3.3 The Effect of Capital Expenditures During the Regulatory Period

[8] The average five and ten year rates are 5.72% and 5.95% respectively, with data drawn from the RBA website (http://www.rba.gov.au/statistics/tables/index.html#interest_rates).

The previous section has dealt with a case in which capital expenditures arise during the regulatory period. However, the analysis was limited to the case in which capital expenditures (capex) exactly matched depreciation, so that the regulatory asset book value did not change. This section considers more realistic cases, in which capex exceeds depreciation (to reflect both inflation and real growth in the network) and also cases in which capex is highly volatile over time.

We start with a case in which capex exceeds depreciation. Returning to the example in the previous section, suppose instead that capex is \$6m per year, so that the regulatory asset book value grows at \$4m per year (about 4% per year). With this change to the example in the previous section, and using the yield to maturity on five year risk free bonds of 5.93%, the resulting revenues for each of the next five years would be as follows:

$$REV_1 = OPEX_1 + DEP_1 + B_0 R_f = \$10m + \$2m + \$100m(.0593) = \$17.93m$$
$$REV_2 = OPEX_2 + DEP_2 + B_1 R_f = \$10m + \$2m + \$104m(.0593) = \$18.17m$$
$$REV_3 = OPEX_3 + DEP_3 + B_2 R_f = \$10m + \$2m + \$108m(.0593) = \$18.40m$$
$$REV_4 = OPEX_4 + DEP_4 + B_3 R_f = \$10m + \$2m + \$112m(.0593) = \$18.64m$$
$$REV_5 = OPEX_5 + DEP_5 + B_4 R_f = \$10m + \$2m + \$116m(.0593) = \$18.88m$$

The net cash flows would then be the revenues net of opex and capex, yielding \$1.93m, \$2.17m, \$2.40m, \$2.64m, and \$2.88m for years 1…5 respectively. Using the spot interest rates given in the previous section, the present value of these net cash flows along with the value in five years of the subsequent payoffs on the regulatory assets (\$120m) would then be as follows:

$$V_0 = \frac{\$1.93m}{1.05} + \frac{\$2.17m}{(1.0525)^2} + ... + \frac{\$2.88m + \$120m}{(1.06)^3} = \$99.78m$$

This present value of \$99.78m is below the current regulatory book value of the assets, and therefore the allowed rate of return is too low, because capex in excess of depreciation raises the duration of the regulatory payoffs. Thus, with an upward sloping term structure, the use of a risk free rate from a bond with a lower duration than the regulatory cash flows will cause the present value of the regulatory payoffs to fall below the current regulatory asset book value. If we set the allowed rate of return at a level that would generate a present value of exactly \$100m, the rate would be 5.98%, i.e., only five basis points above the current yield to maturity on five year risk free bonds of 5.93%. This error is small.

We now consider a case in which capex is volatile. So, rather than capex of \$6m per year as in the above example, suppose that capex is \$30m at the end of one of the years and zero in all other years.[9] With this change, the biggest divergence between the present value of the payoffs on the regulatory assets and the current regulatory asset book value (\$100m) arises when the capex of \$30m occurs at the end of the second year, giving rise to a present value on the payoffs from the regulatory assets of \$99.62m. If we select the allowed rate of return that would give rise to a present value of exactly \$100m, the figure would be 6.00%, i.e., seven basis points above the current yield to maturity on five year risk free bonds. Again, this error is small.

In summary, the presence of capital expenditures within the regulatory period that exceed depreciation will tilt the regulatory cash flows towards the end of the regulatory period and raise the regulatory asset book value at the end of the regulatory period, thereby raising the duration of the payoffs on the regulatory assets above that of a bond with a term to maturity equal to that of the regulatory period. If the term structure is upward sloping, which is the normal case, the yield to maturity on a bond of this kind will be too low to ensure that the Present Value principle is satisfied. However the shortfall in the yield does not exceed 0.07% across the cases examined, and this is not significant.

4. Conclusions

This paper has addressed a number of issues raised by the AER, and the conclusions are as follows. The first of these issues is, in determining a regulatory rate of return, whether the use of a risk free rate averaged over a short period (10-40 business days) as close a practically possible to the start of the regulatory period is consistent with the Present Value principle (the present value of a regulated firm's revenue stream should match, over time, the present value of its expenditure stream plus or minus any efficiency incentive rewards or penalties), the

[9] This is sufficiently large that it could not be financed with cash flow from operations in that year, and therefore would require (for the all-equity firm assumed here) either an equity injection or retention and accumulation of cash flow from earlier years. Accordingly, this might lead to an allowance for equity raising costs under the AER's standard approach. However I do not model this effect here.

Building Block model, the Cost Recovery principle (a regulated firm should have a reasonable opportunity to recover at least its efficient costs), and the Sharpe-Lintner model. In relation to the Present Value principle, any present value operation requires the use of the current risk free rate, the regulatory rate of return must therefore use the risk free rate at the beginning of the regulatory period, and pragmatic considerations then lead to averaging it over a short period as close as practical to the start of the regulatory period. In relation to the Building Block model, this is a consequence of the Present Value principle and therefore the same conclusion applies. In relation to the Cost Recovery principle, this is equivalent to the Present Value principle and therefore the same conclusion also applies. In relation to the Sharpe-Lintner model, this model always requires a risk free rate prevailing at a point in time for some subsequent period rather than a historical average and application of the model to a regulatory situation would require the risk free rate prevailing at the beginning of a regulatory period.

The second issue is whether the adoption of a long term historical average risk free rate would be consistent with the four matters detailed above. The conclusions here follow from the conclusions presented in the previous paragraph: since the risk free rate at the beginning of the regulatory period is required in all cases, then the use of a long-term historical average risk free rate will not be consistent with the Present Value principle, the Building Block model, the Cost Recovery principle, and the Sharpe-Lintner model.

The third issue is whether the Present Value principle requires that the regulatory rate of return should be representative of prevailing market conditions at the start of the regulatory period or at the start of each year within the regulatory period. If the regulatory rate of return is representative of prevailing market conditions at the start of each year within the regulatory period, then the Present Value principle requires that the regulatory rate of return be based upon the one year risk free rate prevailing at the beginning of each year. By contrast, if the regulatory rate of return is representative of prevailing market conditions at the start of each regulatory period, then the Present Value principle requires that the regulatory rate of return be based upon the yield to maturity prevailing at the beginning of the regulatory period on the risk free bond with a duration equal to that of the regulatory cash flows, and a very close approximation is achieved through the use of the yield to maturity on the risk free bond whose term to maturity matches that of the regulatory period. Capital expenditures within a regulatory period may worsen the approximation here but the error in the regulatory rate chosen as described above does not exceed seven basis points across the range of examples considered, and this is not material.

REFERENCES

Elton, E., Gruber, M., Brown, S. and Goetzmann, W., 2003, *Modern Portfolio Theory and Investment Analysis*, 6th edition, John Wiley & Sons (New York).

Fama, E., 1984, 'The Information in the Term Structure', *Journal of Financial Economics*, vol. 13 (4), pp. 509-528.

Lally, M., 2004, Regulation and the Choice of the Risk Free Rate, *Accounting Research Journal*, vol. 17 (1), pp. 18-23.

Lintner, J., 1965, 'The Valuation of Risky Assets and the Selection of Investments in Stock Portfolios and Capital Budgets', *Review of Economics and Statistics*, vol.47, pp.13-37.

Markowitz, H., 1952, 'Portfolio Selection', *Journal of Finance*, vol. 7, pp. 77-91.

___________ 1959, *Portfolio Selection*, Yale University Press, New Haven.

Marshal, W., Yawitz, J. and Greenberg, E., 1981, 'Optimal Regulation Under Uncertainty', *The Journal of Finance*, vol. 36, pp. 909-22.

McCulloch, J., 1975, 'An Estimate of the Liquidity Premium', *Journal of Political Economy*, vol. 83, pp. 95-119.

Mossin, J., 1966, 'Equilibrium in a Capital Asset Market', *Econometrica*, vol. 24, pp. 768-783.

Schmalensee, R., 1989, 'An Expository Note on Depreciation and Profitability Under Rate-of-Return Regulation', *Journal of Regulatory Economics*, vol. 1, pp. 293-298.

Sharpe, W., 1964, 'Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk', *Journal of Finance*, vol. 19, pp. 425-442.

Van Horne, J., 1984, *Financial Market Rates and Flows*, 2nd edition, Prentice-Hall, Inc., Englewood Cliffs.