Estimating β

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1. Introduction: The Capital Asset Pricing Model

This report builds upon the material discussed in the preliminary report on β estimation provided to the ACCC by the Consultant in 2008.[1]

The Capital Asset Pricing Model, CAPM predicts that the expected return to the i^{th} asset, $E(r_i)$, is given by

$$E(r_i) = r_f + \beta_i \left[E\left(r_m - r_f\right) \right], \tag{1}$$

Where r_f is the rate of return to the riskless security and $\beta_i = \frac{Cov[r_i, r_m]}{Var[r_m]}$.

Essentially the CAPM describes the excess expected return to the i^{th} asset, $E(r_i) - r_f$ as a risk premium. This risk premium may be written as a fixed price per unit of risk, $\lambda_i = \left[E\left(r_m - r_f\right) \right] / Var[r_m]$, multiplied by a quantity of risk, $Cov[r_i, r_m]$.

$$E(r_i) - r_f = \lambda_i Cov[r_i, r_m], \tag{2}$$

2. Estimation of the Capital Asset Pricing Model

Using raw returns, estimates of $\beta_i = \frac{Cov[r_i, r_m]}{Var[r_m]}$ may be obtained from the regression

$$r_{i,t} = \alpha_i + \beta_i r_{m,t} + \varepsilon_{i,t}, \tag{3}$$

Where, the residual is $\varepsilon_{i,t} = r_{i,t} - \alpha_i + \beta_i r_{m,t}$. Assuming that the risk free rate does not vary substantially with time the data may be transformed to excess returns $R_{i,t} = r_{i,t} - r_{f,t}$; $R_{m,t} = r_{m,t} - r_{f,t}$ and estimates of β_i may be obtained from the regression

$$R_{i,t} = \beta_i R_{m,t} + \varepsilon_{i,t} \tag{3'}$$

2.1 Ordinary Least Squares

[1] Henry, O.T., *Econometric advice and beta estimation*, 28 November 2008, Available at: www.aer.gov.au/content/index.phtml/itemId/724620.

Typically (3) and (3’) are estimated using the method of Ordinary Least Squares, OLS. This approach obtains estimates of the parameters of interest α_i and β_i by minimizing the sum of the squared residuals:

$$\sum_{t=1}^{T}\varepsilon_{i,t}^2 = \sum_{t=1}^{T}\left(r_{i,t} - \hat{r}_{i,t}\right)^2 = \sum_{t=1}^{T}\left(r_{i,t} - \hat{\alpha}_i - \hat{\beta}_i r_{m,t}\right)^2 \tag{4}$$

3. The relationship between R^2 and the OLS estimate of β.

The SFG Consulting report[2] of February 2009, hereafter referred to as "The SFG report" presents a simulation study designed to illustrate a relationship between the R^2 statistic and the estimate of β. The SFG report assumes a sample period of four years and a monthly sampling frequency, implying a total of 48 observations to be generated. The data generating process used is

$$r_{i,t} = 1.0 r_{m,t} + \varepsilon_{i,t} \tag{5}$$

Where $r_{i,t}, r_{m,t}$ and $\varepsilon_{i,t}$ represent the total return to i^{th} asset, the return to the market portfolio and the idiosyncratic component of the return to the i^{th} asset, respectively. The SFG consulting report then assumes that the volatility of asset specific and market specific risk are uniformly distributed in the range 1% - 10% Values for $r_{m,t}$ and $\varepsilon_{i,t}$ are obtained as random draws from normal distributions with mean 1% and 0, respectively where the variance of these normal distributions is obtained by draws from the uniform distribution. In table 3.1, the SFG Consulting results are replicated.

[2] SFG Consulting, *The reliability of empirical beta estimates: Response to AER proposed revision of WACC parameters*, 1 February 2009.

Table 3.1: Simulation results illustrating the relationship between R-squared and β estimates for sample size N=48

Decile	Mean R-Squared (%)	Mean Beta Estimate	Standard deviation of the beta estimate	Proportion in which estimates are below 1 (%)	Proportion in which estimate is reported as significantly below 1 (%)	Proportion in which estimate is reported as significantly above 1 (%)
1	4	0.66	0.50	80	13	0
2	15	1.06	0.42	55	5	1
3	25	1.07	0.34	51	5	4
4	36	1.05	0.24	49	4	5
5	46	1.04	0.18	46	4	5
6	56	1.04	0.15	43	3	6
7	65	1.04	0.12	42	3	7
8	75	1.02	0.10	43	4	8
9	86	1.01	0.07	45	4	7
10	95	1.00	0.04	46	4	6
	50	1.00	0.29	49.9	5	4.9

The SFG report argues that in cases where the regression the R^2 statistic is very low, then the associated estimate of β is biased downwards from the true value of 1.0 given by the data generating process. This conclusion is warranted based on the assumptions underlying the simulation experiment being valid. However no guidance is given as to the robustness of the conclusions to variations in these assumptions.

The SFG report assumes that monthly data are available over a four year period. Henry (2008) argues that the use of data sampled at a weekly frequency is appropriate for the purpose of estimating β. Over a four year period 208 weekly observations are available to the researcher. The experiment in the SFG report was repeated using identical assumptions to those used to obtain the results in Table 3.1, save for a single change, an increase in sample size to 208 observations.

The results for this second experiment are reported in Table 3.2. It is clear that the relationship between the R^2 statistic the associated estimate of β is very much reduced given the increased sample size. In short, the evidence suggests that the OLS estimator is unbiased across both table 3.1 and table 3.2 as $E\left(\hat{\beta}\right) = 1.0$. Moreover, the tendency for the estimate of β to be low when the R^2 statistic is low is dramatically reduced as the sample size increases within each replication.

Table 3.2: Simulation results illustrating the relationship between R-squared and β estimates for sample size N=208

Decile	Mean R-Squared (%)	Mean Beta Estimate	Standard deviation of the beta estimate	Proportion in which estimates are below 1 (%)	Proportion in which estimate is reported as significantly below 1 (%)	Proportion in which estimate is reported as significantly above 1 (%)
1	5	0.92	0.31	65	8	1
2	15	1.03	0.19	51	5	4
3	26	1.01	0.13	50	5	5
4	36	1.01	0.1	50	5	5
5	46	1.01	0.08	49	5	5
6	55	1.01	0.07	46	4	6
7	64	1.01	0.05	46	4	6
8	75	1.01	0.04	47	4	6
9	85	1	0.03	47	4	6
10	95	1	0.02	48	5	5
	50	1.00	0.102	50	5	5

Why then does the estimate of β still appear to display a relationship with the R^2 statistic when N=248? The most likely cause of this distortion is the assumption that both asset specific and market specific risk are uniformly distributed in the range 1% to 10%. This assumption may have been made to guarantee a positive definite draw for volatility, but is not justified on empirical or theoretical grounds in the SFG report. It is just as likely, or indeed unlikely, that the asset specific and market volatilities are normally distributed.

Figure 3.1, drawn for illustrative purposes only, allows visual comparison of the assumption of uniformly distributed volatility with that of, for example, normally distributed volatility. The figure is drawn assuming a common average level of σ^2 = 5.5% as assumed in the SFG study.

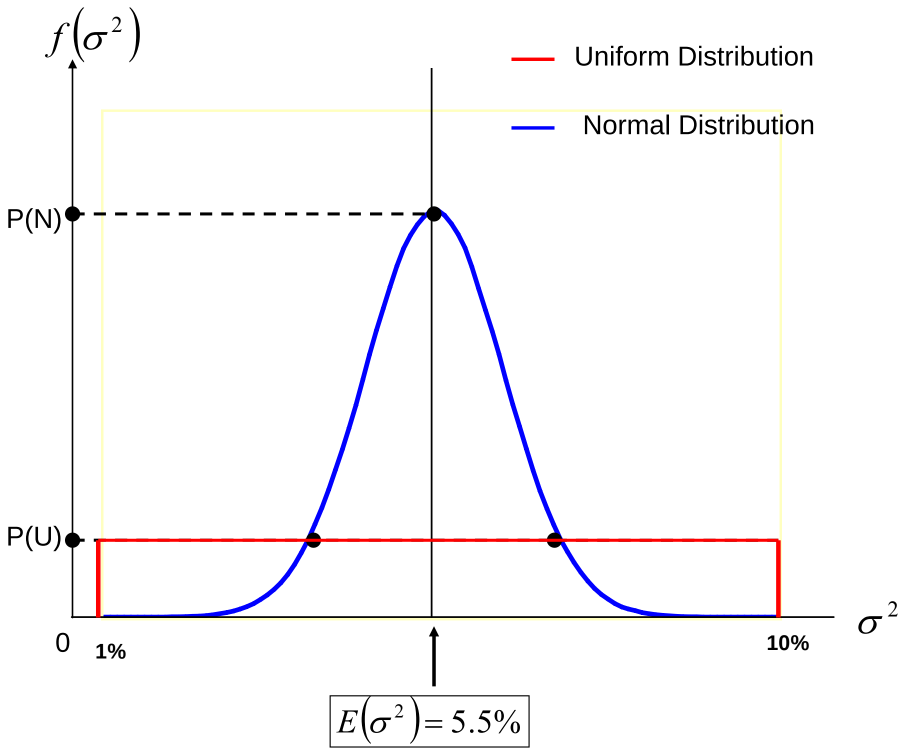

Figure 3.1: Uniform distribution and normal distribution of volatility

Comparing the uniform distribution as a candidate distribution for σ^2 with the normal distribution is very informative. The implications of the uniform distribution are very strong. The volatilities of $r_{m,t}$ and $\varepsilon_{i,t}$ are assumed to take a range of values in the range 1% to 10% with equal probability, P(U). Hence, average levels of volatility are just as likely to occur as very volatile or very calm returns. The probability of an average level of volatility being drawn from a normal distribution is much higher at P(N). Relative to the normal distribution, average levels of σ^2 will be under-represented in the simulations based upon the uniform. Similarly, in comparison with the normal distribution, high and low levels of σ^2 are likely to be over represented as a result of the assumption that volatility is uniformly distributed. The design of experiment over-represents data which exhibits extremely low or high signal to noise ratios. Moreover, this experimental design may also under-represent draws with the average level of volatility for each factor.

In order to make any strong conclusions about a relationship between R^2 and the estimate of β from the SFG study, the results of the experiment should be reasonably robust to deviations from the assumptions from the experiment. This robustness is not achieved as it is clear from table 3.2 above that any relationship weakens as the sample size increases. Furthermore, it must be possible to justify the assumptions underlying the experiment. No explanation is given as to why σ^2 should be discretely normally distributed, nor is the robustness of the results to deviations from this assumption examined. An empirical exploration of whether the uniform distribution is a suitable candidate distribution for σ^2 is beyond the scope of this study. However, without a justification for this choice of distribution and an examination of the impact of deviations from the assumptions underlying the experiment the conclusions drawn in the SFG study should be regarded as tenuous.

4. Re-Levered/Delevered estimates of β.

This report examines data sampled at weekly and monthly frequency over the period January 1st 2002 to 1st September 2008. This sample period was chosen to avoid potential issues associated with the technology bubble. The consultant advised the ACCC that the events associated with the Global Financial Crisis after September 2008 mitigate against extending the sample post September 2008. The Capital Asset Pricing Model is an equilibrium asset pricing model. Events in the period post-2008:9 are unlikely to be consistent with equilibrium and are consequently excluded from the sample under consideration. The robustness of the estimation results are examined by varying the chosen sample period and sampling frequency.

The data on each asset were sourced from Datastream as was the proxy for the market portfolio, in this case the All Ordinaries Index. Where there are less than 80 monthly or 348 weekly observations in the sample, the firm began trading after January 1st 2002. The exceptions to this are AGK (sample end date 31st October 2006) and AAN (sample end date August 17th 2007 and GAS (sample end date November 17th 2006). The AAN and GAS data are price index data sourced from Bloomberg and provided to the consultant by the ACCC.

There are some concerns about the validity of the OLS estimator of α_i and β_i in the presence of outliers. In such circumstances the estimates of α_i and β_i may vary with time. It is also possible that estimates of σ_i^2, the variance of the residual, $\varepsilon_{i,t}$, may be affected by the presence of outliers.

4.1 Least Absolute Deviations

There are a range of possible approaches that may be followed in order to allow for outliers, the most popular of which is the Least Absolute Deviations, LAD, approach given by

$$\sum_{t=1}^{T} \left| \varepsilon_{i,t} \right| = \sum_{t=1}^{T} \left| r_{i,t} - \tilde{r}_{i,t} \right| = \sum_{t=1}^{T} \left| r_{i,t} - \tilde{\alpha}_i - \tilde{\beta}_i r_{m,t} \right| \tag{6}$$

Here the estimates are obtained by minimizing the absolute value of the residuals. By focusing on minimizing the sum of the absolute values of the residuals rather than the sum of the squared residuals, the effect of the LAD estimator is to reduce the influence of outlying observations.

4.2 De-levering / Re-Levering β.

Let β_A and β_E represent the asset and equity β, respectively. Assuming a debt β of zero, the de-levering/re-levering equation is

$$\beta_A = \beta_E \frac{E}{V} \tag{7}$$

Here E/V is the proportion of equity in the firm's capital structure. The average gearing level is calculated for the sample period used obtain estimates of the firm or portfolio β using data obtained from Bloomberg. The level of gearing is usually defined as the book value of debt divided by the value of the firm as represented by the sum of the market value of equity and the book value of debt. Define the average level of gearing as $\bar{G}$, then

$$\bar{G} = \frac{\bar{D}}{\bar{D} + \bar{E}} \tag{8}$$

Where D is the book value of net debt and E is the market value of equity. It is possible to show that the appropriate re-levering factor that should be applied to the raw beta estimates is:

$$\omega = \frac{1 - \bar{G}}{1 - 0.60} \tag{9}$$

If it is assumed that ω is constant and that the $\bar{G}$ is independent of $\hat{\beta}$ then, the re-levered β, $\hat{\beta}_r$ has a mean of $\omega\hat{\beta}$ and a variance of $\omega^2\sigma^2_{\hat{\beta}}$. The results of the delevering/relevering process for individual stocks are reported in table 4.1 for the monthly sampling frequency, while table 4.2 presents results for the weekly sampling frequency. It is important to note that the Gearing assumptions used in this report have been changed at the instruction of the ACCC from those in the initial report.

Table 4.1: De-Levered/Relevered estimates of β
Australian Companies 2002.1 – 2008.9, Sampled monthly

	AGK	ENV	APA	GAS	DUE	HDF	SPA	SKI	AAN
$\bar{G}$	0.3017	0.7079	0.5737	0.6617	0.7619	0.4657	0.5673	0.3620	0.4133
ω	1.7457	0.7302	1.0658	0.8457	0.5953	1.3357	1.0818	1.5951	1.4667
$\hat{\beta}$	0.4299	0.2948	0.6212	0.1883	0.4077	0.8467	0.3665	1.1060	0.8394
s.e	0.2785	0.0988	0.1898	0.1780	0.1205	0.3016	0.1685	0.2807	0.3593
$\hat{\beta}_u$	0.9758	0.4884	0.9932	0.5372	0.6438	1.4378	0.6968	1.6563	1.5437
$\hat{\beta}_1$	-0.1160	0.1012	0.2492	-0.1607	0.1717	0.2556	0.0362	0.5558	0.1351
$\tilde{\beta}$	0.1835	0.1524	0.7039	0.3177	0.1890	0.6535	0.1869	0.8219	0.8725
s.e	0.2824	0.1002	0.1901	0.1789	0.1249	0.3036	0.1821	0.2896	0.3612
$\tilde{\beta}_u$	0.7370	0.3488	1.0765	0.6682	0.4338	1.2486	0.5439	1.3896	1.5805
$\tilde{\beta}_1$	-0.3699	-0.0439	0.3314	-0.0329	-0.0558	0.0585	-0.1701	0.2542	0.1645
N	57	80	80	59	48	44	32	18	68

Considering the re-levered/de-levered β estimates for the equities, the OLS point estimates range from 0.1883 to 1.1060. In 3 out of 9 cases, HDF, SKI and AAN the 95% confidence interval around the OLS estimate admits 1 as a plausible value for β. The corresponding de-levered LAD estimates range from 0.1524 to 0.8725. In this case 4 out of 9 confidence intervals for the LAD estimator admit 1, those for APA, HDF, SKI and AAN.

Table 4.2 presents re-levered/de-levered estimates of β obtained using data sampled at the weekly frequency, the OLS point estimates range from 0.2522 to 1.0103. In 4 out of 9 cases, AGK, HDF, SKI and AAN the 95% confidence interval around the OLS estimate admits 1 as a plausible value for β. The corresponding de-levered LAD estimates range from 0.1023 to 1.0375. However, only 1 out of the 9 confidence intervals for the LAD estimator contains 1, that of SKI.

Table 4.2: De-Levered/Relevered estimates of β

Australian Companies 2002.01 – 2008.9, Sampled weekly

	AGK	ENV	APA	GAS	DUE	HDF	SPA	SKI	AAN
$\bar{G}$	0.3017	0.7079	0.5737	0.6617	0.7619	0.4657	0.5673	0.3620	0.4133
ω	1.7457	0.7302	1.0658	0.8457	0.5953	1.3357	1.0818	1.5951	1.4667
$\hat{\beta}$	0.7192	0.2522	0.6910	0.3151	0.3550	1.0103	0.2828	0.7865	0.9401
s.e	0.1698	0.0526	0.1011	0.0885	0.0676	0.1750	0.1260	0.3020	0.1863
$\hat{\beta}_u$	1.0520	0.3553	0.8892	0.4885	0.4874	1.3534	0.5297	1.3785	1.3052
$\hat{\beta}_l$	0.3864	0.1491	0.4928	0.1417	0.2225	0.6673	0.0359	0.1945	0.5749
$\tilde{\beta}$	0.5264	0.1023	0.5976	0.2341	0.2519	0.4888	0.2432	1.0375	0.5974
s.e	0.1703	0.0532	0.1013	0.0888	0.0679	0.1791	0.1264	0.3035	0.1876
$\tilde{\beta}_u$	0.8603	0.2066	0.7962	0.4082	0.3850	0.8398	0.4910	1.6323	0.9650
$\tilde{\beta}_l$	0.1925	-0.0020	0.3990	0.0601	0.1187	0.1378	-0.0046	0.4427	0.2298
N	252	348	348	255	211	193	141	78	294

The balance of the evidence from tables 4.1 and 4.2 suggests that the OLS point estimates lie largely in the range 0.2 to 1.0, with the majority of these estimates lying in the range 0.3 to 0.8.

We note that the point rather than interval estimate of β is the correct comparison across firms. For example the point estimate of the de-levered/re-levered β for AGK obtained using OLS and weekly data over the period January 2001 to September 2008 is 0.7266 with a standard error of 0.1700. The 95% confidence interval for AGK is therefore [0.3933, 1.0598]. In repeat sampling 95% of all confidence intervals constructed in this fashion will contain the true value of β for AGK. Figure 4.1 illustrates the relationship between the point estimate, $\hat{\beta}$ and the corresponding confidence interval.

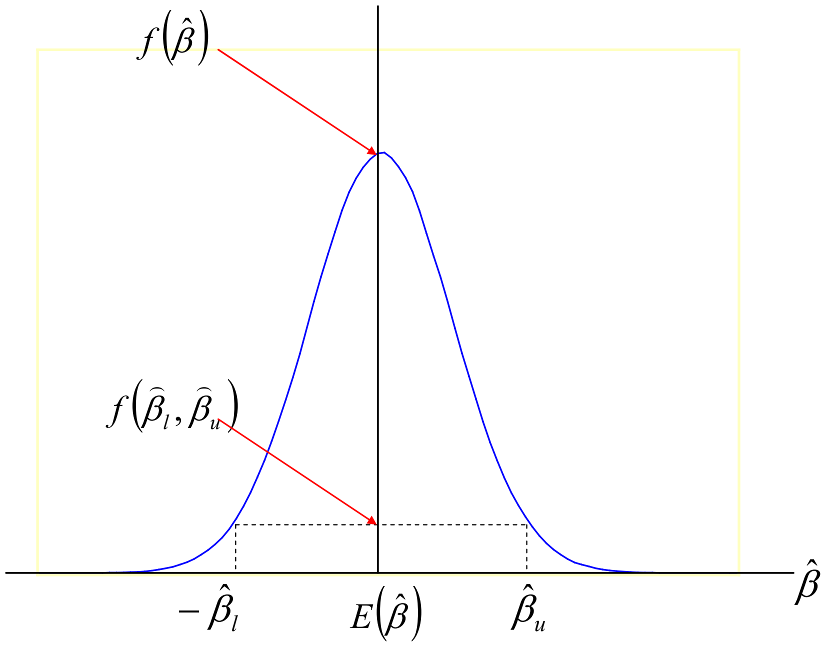

Figure 4.1: Relationship between point estimates and confidence intervals

In repeat sampling, the point estimate, the midpoint of the confidence interval converges to the true value of β, given the unbiased nature of OLS. Within the range of possible values for β given a particular sample, $\hat{\beta}$ has a unique probability of occurrence, $f(\hat{\beta})$. Furthermore, within this range of possible values for β given a particular sample, $\hat{\beta}$ has the highest probability of occurrence The upper bound on the interval, $\hat{\beta}_u$, which depends on the arbitrarily chosen level of confidence, is as likely to obtain as the lower bound, $\hat{\beta}_l$. When comparing the exposure to market risk of two companies, $\hat{\beta}$ is therefore the most reasonable point of comparison

In relation to the notion of confidence intervals, the ACG report of January 2009[3] states:

> *… the AER asserted that 'confidence intervals' were not relevant to the decision that it had to make, but also stated that if it was to have regard to confidence intervals that it would look at both the upper limit and the lower limit of the intervals.*

[3] Allen Consulting Group, "Australian Energy Regulator's draft conclusions on the weighted average cost of capital parameters: Commentary on the AER's analysis of the equity beta", January 2009.

> *Central to the notion of a confidence interval is that estimates of parameters like the beta are imprecise. This means that, even though we may obtain a 'best estimate' for a parameter, from the evidence being examined, the true value could easily be higher or lower, given the imprecision of the estimate. A confidence interval, in broad terms, describes the limit of our confidence about the true value given the evidence that has been considered – on the strength of the evidence examined, the true value could lie anywhere within the outer bounds of the confidence interval, but in contrast, we are confident that the true value cannot lie outside of those bounds. Thus, confidence intervals are a succinct statement from statistical theory about the degree of 'persuasiveness' of the relevant piece of empirical evidence.*
>
> (ACG, January 2009 report, p. 15)

This comment by ACG is incorrect. While it is true that a confidence interval is arrived at via the notion of uncertainty about the parameter estimate it is completely incorrect to state: " *A confidence interval, in broad terms, describes the limit of our confidence about the true value given the evidence that has been considered – on the strength of the evidence examined, the true value could lie anywhere within the outer bounds of the confidence interval, but in contrast, we are confident that the true value cannot lie outside of those bounds.*"

The reason the above statement is incorrect lies in a misinterpretation of the concept of a confidence intervals. In the estimation of an interval we construct two functions $f_1\left(r_{i,1}, r_{i,2}, \ldots., r_{i,n}\right)$ and $f_2\left(r_{i,1}, r_{i,2}, \ldots., r_{i,n}\right)$ using the sample observations such that

$$\Pr\left(f_1 < \beta < f_2\right) = \text{ a given level of probability, say 95\%}$$

This results in a 95% confidence interval (f_1,f_2). Since β is a parameter and is therefore an unknown constant (which we estimate as $\hat{\beta}$), the confidence interval is a statement about f_1 and f_2 and not about β. What this implies is that if we use the functions $f_1\left(r_{i,1}, r_{i,2}, \ldots., r_{i,n}\right)$ and $f_2\left(r_{i,1}, r_{i,2}, \ldots., r_{i,n}\right)$ repeatedly with different samples then we may be confident that 95% of these confidence intervals will contain the true value, β. It follows therefore that we cannot say " *the true value could lie anywhere within the outer bounds of the confidence interval, but in contrast, we are confident that the true value cannot lie outside of those bounds.*" In the case of any particular interval we cannot say anything about the true value because (i) the confidence interval is a statement about f_1 and f_2 and not about β and (ii) the value of β is a parameter and is therefore unknown. In the case of a 95% confidence interval 5% of the intervals constructed in repeat sampling will not contain the true value which contradicts the statement "*, we are confident that the true value cannot lie outside of those bounds*".

Tables 4.3 and 4.4 present $\hat{\beta}$ and $\tilde{\beta}$ for the Australian data sampled over the period 2003:09:01 to 2008:09:01 on a monthly and weekly basis. The results are broadly consistent with those presented in tables 4.1 and 4.2.

Table 4.3: De-Levered/Relevered estimates of β

Australian Companies 2003.09 – 2008.9, Sampled monthly

	AGK	ENV	APA	GAS	DUE	HDF	SPA	SKI	AAN
$\bar{G}$	0.2719	0.7006	0.5851	0.6617	0.7619	0.4657	0.5673	0.3620	0.4133
ω	1.8203	0.7485	1.0372	0.8457	0.5953	1.3357	1.0818	1.5951	1.4667
$\hat{\beta}$	0.6193	0.3908	0.7400	0.2829	0.4077	0.8467	0.3665	1.1060	1.0749
s.e	0.4019	0.1166	0.2231	0.2694	0.1205	0.3016	0.1685	0.2807	0.4528
$\hat{\beta}_u$	1.4071	0.6193	1.1772	0.8109	0.6438	1.4378	0.6968	1.6563	1.9623
$\hat{\beta}_l$	-0.1685	0.1623	0.3027	-0.2451	0.1717	0.2556	0.0362	0.5558	0.1875
$\tilde{\beta}$	1.1188	0.4202	0.9172	0.4323	0.1890	0.6535	0.1869	0.8219	1.0042
s.e	0.4172	0.1169	0.2246	0.2705	0.1249	0.3036	0.1821	0.2896	0.4529
$\tilde{\beta}_u$	1.9365	0.6493	1.3574	0.9626	0.4338	1.2486	0.5439	1.3896	1.8919
$\tilde{\beta}_l$	0.3012	0.1912	0.4771	-0.0979	-0.0558	0.0585	-0.1701	0.2542	0.1165
N	37	60	60	38	48	44	32	18	48

The monthly OLS estimates for β lie in the range 0.2829 to 1.1060. Of the corresponding confidence intervals, 5 out of 9 include unity. The LAD estimates lie in the range 0.1869 – 1.1188 with 5 out of 9 confidence intervals containing unity.

Using data with weekly sampling frequency over the period 2003:09:1 – 2008:09:01 yields $\hat{\beta}$ values in the range 0.2828 – 1.2569, with 4 of the 9 confidence intervals containing unity. The LAD estimates in Table 4.4 lie in the range 0.1615 – 1.1826 with 3 of the 9 confidence intervals containing 1.

Table 4.4: De-Levered/Relevered estimates of β									
Australian Companies 2003.09 – 2008.9, Sampled weekly									
	AGK	ENV	APA	GAS	DUE	HDF	SPA	SKI	AAN
$\bar{G}$	0.2719	0.7006	0.5851	0.6617	0.7619	0.4657	0.5673	0.3620	0.4133
ω	1.8203	0.7485	1.0372	0.8457	0.5953	1.3357	1.0818	1.5951	1.4667
$\hat{\beta}$	1.2438	0.2959	0.7612	0.3805	0.3550	1.0103	0.2828	0.7865	1.2569
s.e	0.2313	0.0616	0.1186	0.1270	0.0676	0.1750	0.1260	0.3020	0.2291
$\hat{\beta}_u$	1.6971	0.4166	0.9937	0.6294	0.4874	1.3534	0.5297	1.3785	1.7060
$\hat{\beta}_1$	0.7905	0.1753	0.5287	0.1316	0.2225	0.6673	0.0359	0.1945	0.8079
$\tilde{\beta}$	1.1826	0.1615	0.6220	0.3543	0.2519	0.4888	0.2432	1.0375	0.9284
s.e	0.2320	0.0621	0.1190	0.1284	0.0679	0.1791	0.1264	0.3035	0.2308
$\tilde{\beta}_u$	1.6373	0.2833	0.8554	0.6060	0.3850	0.8398	0.4910	1.6323	1.3809
$\tilde{\beta}_1$	0.7280	0.0397	0.3887	0.1027	0.1187	0.1378	-0.0046	0.4427	0.4760
N	166	261	261	168	211	193	141	78	207

Again, it is not unreasonable to suggest that the evidence in Tables 4.3 and 4.4 is consistent with the majority of β estimates lying in the range 0.3 – 0.8.

It is also useful to note that very low R^2 values are the exception rather than the rule for the Australian data. While no R^2 exceeds 50%, this is typical of asset return regression. Table 4.5 presents the R^2 values from the regressions reported in tables 4.1-4.4.

Table 4.5: R^2 values for the regressions reported in tables 4.1 – 4.4									
Monthly sampling frequency									
Start	AGK	ENV	APA	GAS	DUE	HDF	SPA	SKI	AAN
2002:01	0.0415	0.1025	0.1208	0.0196	0.1994	0.1580	0.1362	0.4924	0.0764
2003:09	0.0635	0.1623	0.1594	0.0297	0.1994	0.1580	0.1362	0.4924	0.1092
Weekly sampling frequency									
Start	AGK	ENV	APA	GAS	DUE	HDF	SPA	SKI	AAN
2002:01	0.0670	0.0623	0.1189	0.0477	0.1166	0.1485	0.0350	0.0819	0.0802
2003:09	0.1499	0.0819	0.1372	0.0513	0.1166	0.1485	0.0350	0.0819	0.1280

SFG comment:

> *simulation analysis is used to create a data set in which the true value of beta is known, but which otherwise has characteristics that are similar to real data. We can then apply estimation techniques to the simulated data set and compare the resulting estimate to the known true value.*
>
> SFG 2009 (p. 30)

SFG argue that the simulation approach provides a measure of the reliability of the estimate and suggest that their simulation analysis "demonstrates that in circumstances where the R-Squared statistic is low it is difficult to obtain statistically reliable estimates".

Leaving aside any discussion of the assumptions underlying the SFG report, it is well known that a high R^2 value is neither a necessary nor a sufficient condition for statistically reliable estimates. Regressions with high R^2 values may exhibit non-spherical residuals, or in the extreme may simply be "spurious" regressions. Moreover, regressions used to explain asset returns typically exhibit relatively low coefficients of determination.

SFG recommend that the R^2 value for each regression is reported. This is done in Table 4.5. Even allowing for the caveats discussed in Section 3 above regarding the robustness and generality of the SFG simulation results, it is clear that relatively few of the regressions are associated with low R^2 values. Accounting for sample overlap, 5 out of 14 monthly cases exhibit an R^2<10%, AGK and GAS in both sample periods and AAN in the January 2002 – September 2008 sample period. In only two of these cases was the sample size less than 48 observations, that of AGK and GAS in the post 2003:09 sample.

SFG conclude that there is evidence of bias in regressions with R^2<10% in samples of 48 observations. In 8 out of 14 cases the OLS R^2 was less than 10% for data sampled at the weekly frequency. However, in all cases the sample size is well in excess of the 48 observations considered by SFG, with the smallest sample containing 78 observations and the largest containing 348 obsetvations. The results in table 3.2 demonstrate that the apparent bias is reduced by an increase in the sample size. Given the larger sample sizes and then fact that only 3 of 13 cases exhibit R^2<5% there are unlikely to be concerns regarding the potential for a downward bias in $\hat{\beta}$ unless one accepts the generality and robustness of the SFG simulation results completely.

4.3 Thin Trading: Individual Stocks

Thin trading can create issues with the magnitude of the estimate of β. In effect, if the stock does not trade regularly, the OLS estimate of β tends to be biased towards zero. In the literature there are 2 popular approaches to adjusting for thin trading. The Scholes-Williams[4] approach constructs a measure of β as:

$$\beta_i^{SW} = \frac{\left(\hat{\beta}_i^{-1} + \hat{\beta}_i + \hat{\beta}_i^{+1}\right)}{\left(1 + 2_1\hat{\rho}_m\right)} \tag{12}$$

Where $\hat{\beta}_i^{-1}$ is the estimated slope when $r_{i,t}$ is regressed on $r_{m,t-1}$, $\hat{\beta}_i$ is the estimated slope when $r_{i,t}$ is regressed on $r_{m,t}$, $\hat{\beta}_i^{+1}$ is the estimated slope when $r_{i,t}$ is regressed on $r_{m,t+1}$, and ${}_1\hat{\rho}_m$ is the estimated first order serial correlation coefficient of $r_{m,,t}$. While the Scholes-Williams measure of β has the advantage of simplicity, it relies on estimates of $\hat{\beta}_i^{-1}$ and $\hat{\beta}_i^{+1}$ that are obtained from regressions whose theoretical foundation suggests a potential for omitted variable bias. Moreover, calculation of a standard error for (12) is a non-trivial task.

The Dimson[5] approach involves estimation of the regression

$$r_{i,t} = \alpha_i + \beta_{i-1} r_{m,t-1} + \beta_i r_{m,t} + \beta_{i+1} r_{m,t+1} + \varepsilon_{i,t} , \tag{13}$$

The Dimson estimate of β, β_i^D is obtained from sum of the coefficients of the independent variables in equation (13). If the CAPM is the correct model of equilibrium returns then the lag and lead of $r_{m,,t}$ are irrelevant variables. Inclusion of these variables may lead to inefficient estimates of β, but there is little danger of the potential for bias underlying β_i^{SW} . Additionally, calculation of a standard error for β_i^D is straightforward.

Tables 4.6 and 4.7 report OLS estimates of β adjusted using the Dimson approach for weekly data, sampled over the periods 2002:09 – 2008:09 and 2003:09 – 2008:9, respectively.

[4] Scholes, M. and J Williams (1977) "Estimating betas from nonsynchronous data" *Journal of Financial Economics*, 5, 309-327
[5] Dimson, E. and P. Marsh (1983) "The stability of UK risk measures and the problem in thin trading", *Journal of Finance*, 38 (3) 753-784

Table 4.6 Dimson's β – Firms

Weekly data: 2002:01 – 2008:09

	AGK	ENV	APA	GAS	DUE	HDF	SPA	SKI	AAN
β_{i-1}	-0.0374	-0.0047	-0.2162	-0.1010	-0.1197	-0.0996	-0.0063	-0.0077	-0.1875
s.e	0.0982	0.0732	0.0961	0.1055	0.1162	0.1337	0.1184	0.1962	0.1315
β_i	0.4245	0.3434	0.6726	0.3847	0.6038	0.7327	0.2608	0.4672	0.6535
s.e	0.0984	0.0728	0.0955	0.1057	0.1166	0.1342	0.1192	0.1977	0.1280
β_{i+1}	-0.0708	0.0509	-0.0425	-0.0823	0.0496	0.1660	-0.1125	0.1997	-0.0143
s.e	0.0982	0.0725	0.0952	0.1056	0.1149	0.1320	0.1167	0.1917	0.1282
β_i^D	0.3163	0.3896	0.4138	0.2014	0.5337	0.7991	0.1421	0.6592	0.4516
s.e	0.1604	0.1181	0.1549	0.1724	0.1877	0.2166	0.1921	0.3146	0.2088
β_i^{OLS}	0.4120	0.3454	0.6483	0.3725	0.5962	0.7564	0.2614	0.4931	0.6410
s.e	0.0973	0.0720	0.0949	0.1046	0.1135	0.1310	0.1165	0.1894	0.1270
$\beta_i^{OLS} = \beta_i^D$	0.9834	-0.6139	2.4719	1.6359	0.5511	-0.3259	1.0248	-0.8777	1.4907

Table 4.6 presents OLS estimates of β for each firm adjusted for thin trading following the Dimson approach using weekly data sampled over the period 2002:01 – 2008:09. Only in the case of APA is there statistically significant evidence against the hypothesis that $\beta_i^{OLS} = \beta_i^D$. However, in this case there is only weak evidence of thin trading. While β_{i-1} is statistically significant, β_{i+1} is not significant. These t-statistics are constructed as

$$t = \frac{\hat{\beta}_i - \beta_i^D}{s.e.\left(\hat{\beta}_i\right)}$$

The statistics are constructed in this fashion to allow the use of the smaller OLS standard errors in the construction of the t-statistic. Given the absence of evidence of thin trading, the Dimson estimator is inefficient relative to OLS and so $s.e.\left(\hat{\beta}_i\right) < s.e.\left(\beta_i^D\right)$. This approach gives the greatest chance of rejecting $H_0 : \hat{\beta}_i = \beta_i^D$.

An alternative approach would be to calculate the t-statistic as:

$$t = \frac{\beta_i^D - \hat{\beta}_i}{s.e.\left(\beta_i^D\right)}$$

In this situation, any t-statistic constructed will be opposite in sign and smaller in magnitude if $s.e.\left(\hat{\beta}_i\right) < s.e.\left(\beta_i^D\right)$. The approach followed maximizes the chance of finding evidence against $H_0 : \hat{\beta}_i = \beta_i^D$

Looking at the results for the shorter sample period 2003:09 – 2008:09, the results are consistent. Again there is statistically significant evidence against the hypothesis that $\beta_i^{OLS} = \beta_i^D$ for APA. As before this evidence is weak because while β_{i-1} is statistically significant, β_{i+1} is not significant. Similarly the hypothesis $\beta_i^{OLS} = \beta_i^D$ for AAN is on the borderline of statistical significance at the 5% level. However this is unlikely to be as a result of thin trading as neither β_{i-1} nor β_{i+1} are significant.

In summary, there is an absence of evidence for thin trading in the weekly data on the individual firms.

Table 4.7 Dimson's β – Firms

Weekly data: 2003:09 – 2008:09

	AGK	ENV	APA	GAS	DUE	HDF	SPA	SKI	AAN
β_{i-1}	-0.1240	-0.0405	-0.2212	-0.2984	-0.1197	-0.0996	-0.0063	-0.0077	-0.3288
s.e	0.1250	0.0837	0.1152	0.1500	0.1162	0.1337	0.1184	0.1962	0.1634
β_i	0.6725	0.3938	0.7515	0.4392	0.6038	0.7327	0.2608	0.4672	0.8593
s.e	0.1250	0.0830	0.1142	0.1501	0.1166	0.1342	0.1192	0.1977	0.1564
β_{i+1}	-0.0189	0.0977	-0.1079	0.0671	0.0496	0.1660	-0.1125	0.1997	0.0207
s.e	0.1251	0.0828	0.1139	0.1502	0.1149	0.1320	0.1167	0.1917	0.1572
β_i^D	0.5297	0.4510	0.4224	0.2080	0.5337	0.7991	0.1421	0.6592	0.5512
s.e	0.2157	0.1364	0.1878	0.2590	0.1877	0.2166	0.1921	0.3146	0.2648
β_i^{OLS}	0.6833	0.3953	0.7339	0.4499	0.5962	0.7564	0.2614	0.4931	0.8570
s.e	0.1270	0.0822	0.1144	0.1502	0.1135	0.1310	0.1165	0.1894	0.1562
$\beta_i^{OLS} = \beta_i^D$	1.2093	-0.6773	2.7236	1.6109	0.5511	-0.3260	1.0248	-0.8777	1.9579

5 Portfolio analysis

5.1 Constant Portfolio Weights

Consider a portfolio, *P*, containing two assets, *X* and *Y*, paying returns R_x and R_y, respectively. This portfolio has a constant proportion, *a*, of wealth invested in asset *X* and the remaining 1-a of wealth invested in *Y*. The expected return to *P* is given by

$$E(R_p) = aE(R_x) + (1-a)E(R_y) \tag{10}$$

It is straightforward to show that the variance of return to *P* is given by

$$Var(R_p) = a^2 Var(R_x) + (1-a)^2 Var(R_y) + 2a(1-a)Cov(R_x R_y) \tag{11}$$

Two sets of portfolios were constructed using continuously compounded returns to the accumulation indices. The first set of portfolios was constructed assuming equal weights, while the second set was based on value weights. These weights were calculated using the average market capitalization over the sample period. OLS and LAD estimates of β are reported and in an appendix to this report we describe recursive estimation and present recursive estimates of β.

The first portfolio, P1', contains ENV and APA. Data is available for this portfolio over the period 1st January 2002 - 1st September 2008. P1 also contains ENV and APA, in this case sampled over the period 1st September 2003 - 1st September 2008. P2 adds DUE to P1 using data sampled over the period 13th August 2004 - 1st September 2008. Adding HDF to the constituents of P2 yields the third portfolio sampled over the interval 17th December 2004 - 1st September 2008. The fourth portfolio is estimated over the period 16th December 2005 - 1st September 2008 and contains ENV, APA, DUE, HDF, and SPA. The fifth portfolio adds SKI to the constituents of the fourth portfolio. Data over the period 2nd March 2007 - 1st September 2008 is available for the fifth portfolio. Table 5.1 reports de-levered/relevered OLS and LAD estimates of β for equal weighted portfolios using monthly data. Table 5.2 presents evidence for the corresponding value weighted portfolios

Table 5.1: De-Levered/Relevered estimates of β						
Equal Weight Portfolios, Sampled monthly						
	P1'	P1	P2	P3	P4	P5
Sample	1 Jan 2002 – 1 Sep 2008	1 Sep 2003 – 1 Sep 2008	13 Aug 2004 – 1 Sep 2008	17 Dec 2004 – 1 Sep 2008	16 Dec 2005 – 1 Sep 2008	2 Mar 2007 – 1 Sep 2008
Firms	ENV, APA,	ENV, APA,	ENV, APA, DUE	ENV, APA, DUE, HDF	ENV, APA, DUE, HDF SPA	ENV, APA, DUE, HDF SPA, SKI
$\bar{G}$	0.6408	0.6428	0.6825	0.6283	0.6191	0.6243
ω	0.8980	0.8929	0.7937	0.9292	0.9523	0.9392
$\hat{\beta}$	0.4434	0.5464	0.5001	0.5853	0.5870	0.6178
s.e	0.1119	0.1305	0.1143	0.1402	0.1593	0.2034
$\hat{\beta}_u$	0.6627	0.8022	0.7241	0.8602	0.8993	1.0164
$\hat{\beta}_l$	0.2242	0.2905	0.2760	0.3104	0.2748	0.2192
$\tilde{\beta}$	0.4478	0.5980	0.6966	0.5716	0.6243	0.8141
s.e	0.1119	0.1307	0.1187	0.1419	0.1600	0.2097
$\tilde{\beta}_u$	0.6671	0.8543	0.9293	0.8497	0.9379	1.2252
$\tilde{\beta}_l$	0.2285	0.3418	0.4640	0.2936	0.3107	0.4030
N	81	61	46	44	32	18

The OLS estimates lie in the range 0.4434 to 0.6178 for the equal weight portfolios. Only one of the interval estimates for $\hat{\beta}$, that relating to P5, contains unity. The corresponding LAD estimates lie in the range 0.4478 to 0.8141, and again only the LAD interval estimate for P5 contains unity.

Table 5.2 presents $\hat{\beta}$ and $\tilde{\beta}$ for the value weighted portfolios constructed using monthly data. The OLS estimates lie in the range 0.4729 – 0.6113 with none of the confidence intervals for $\hat{\beta}$ containing unity. Similarly, the range for $\tilde{\beta}$ was 0.4915 – 0.9383 while two of the LAD intervals contain unity, P1 and P5.

Table 5.2: De-Levered/Relevered estimates of β

Value Weight Portfolios, Sampled monthly

	P1'	P1	P2	P3	P4	P5
Sample	1 Jan 2002 – 1 Sep 008	1 Sep 2003 – 1 Sep 2008	13 Aug 2004 – 1 Sep 2008	17 Dec 2004 – 1 Sep 2008	16 Dec 2005 – 1 Sep 2008	2 Mar 2007 – 1 Sep 2008
Firms	ENV, APA,	ENV, APA,	ENV, APA, DUE	ENV, APA, DUE, HDF	ENV, APA, DUE, HDF SPA	ENV, APA, DUE, HDF SPA, SKI
$\bar{G}$	0.6408	0.6428	0.6825	0.6283	0.6191	0.6243
ω	0.8980	0.8929	0.7937	0.9292	0.9523	0.9392
$\hat{\beta}$	0.4729	0.5766	0.5230	0.6113	0.5501	0.5952
s.e	0.1241	0.1469	0.1188	0.1428	0.1484	0.1931
$\hat{\beta}_u$	0.7162	0.8645	0.7559	0.8912	0.8410	0.9736
$\hat{\beta}_1$	0.2296	0.2887	0.2902	0.3315	0.2592	0.2168
$\tilde{\beta}$	0.5670	0.7498	0.5164	0.5488	0.4915	0.9383
s.e	0.1246	0.1488	0.1197	0.1433	0.1490	0.2114
$\tilde{\beta}_u$	0.8112	1.0414	0.7510	0.8297	0.7836	1.3527
$\tilde{\beta}_1$	0.3228	0.4582	0.2819	0.2679	0.1995	0.5238
N	81	61	46	44	32	18

Using data sampled at the weekly frequency, Table 5.3 presents $\hat{\beta}$ and $\tilde{\beta}$ for the value weighted portfolios. The OLS point estimates lie in the range 0.4529 – 0.6157 and unity is not an element of any of the associated 95% confidence intervals. The LAD point estimates range from a minimum of 0.3539 to a maximum of 0.6398. Again none of the associated 95% confidence intervals contain unity.

Table 5.3: De-Levered/Relevered estimates of β

Equal Weight Portfolios, Sampled weekly

	P1'	P1	P2	P3	P4	P5
Sample	1 Jan 2002 – 1 Sep 008	1 Sep 2003 – 1 Sep 2008	13 Aug 2004 – 1 Sep 2008	17 Dec 2004 – 1 Sep 2008	16 Dec 2005 – 1 Sep 2008	2 Mar 2007 – 1 Sep 2008
Firms	ENV, APA,	ENV, APA,	ENV, APA, DUE	ENV, APA, DUE, HDF	ENV, APA, DUE, HDF SPA	ENV, APA, DUE, HDF SPA, SKI
$\bar{G}$	0.6408	0.6428	0.6825	0.6283	0.6191	0.6243
ω	0.8980	0.8929	0.7937	0.9292	0.9523	0.9392
$\hat{\beta}$	0.4529	0.5105	0.4588	0.5846	0.5886	0.6157
s.e	0.0558	0.0657	0.0580	0.0703	0.0761	0.1010
$\hat{\beta}_u$	0.5623	0.6394	0.5725	0.7223	0.7377	0.8136
$\hat{\beta}_1$	0.3434	0.3817	0.3450	0.4469	0.4395	0.4177
$\tilde{\beta}$	0.3539	0.4204	0.4239	0.5121	0.5409	0.6398
s.e	0.0561	0.0660	0.0581	0.0705	0.0766	0.1012
$\tilde{\beta}_u$	0.4638	0.5497	0.5378	0.6503	0.6910	0.8382
$\tilde{\beta}_1$	0.2439	0.2910	0.3099	0.3739	0.3908	0.4414
n	349	262	211	193	141	78

Using weekly data, table 5.4 presents $\hat{\beta}$ and $\tilde{\beta}$ for value weighted portfolios. The OLS estimates lie between 0.4891 and 0.6007. None of the OLS 95% confidence intervals encompass unity. Similarly, none of the 95% confidence intervals around the LAD estimates contain unity, while the point estimates range from 0.4489 to 0.6065.

Table 5.4: De-Levered/Relevered estimates of β

Value Weight Portfolios, Sampled weekly

	P1'	P1	P2	P3	P4	P5
Sample	1 Jan 2002 – 1 Sep 2008	1 Sep 2003 – 1 Sep 2008	13 Aug 2004 – 1 Sep 2008	17 Dec 2004 – 1 Sep 2008	16 Dec 2005 – 1 Sep 2008	2 Mar 2007 – 1 Sep 2008
Firms	ENV, APA,	ENV, APA,	ENV, APA, DUE	ENV, APA, DUE, HDF	ENV, APA, DUE, HDF SPA	ENV, APA, DUE, HDF SPA, SKI
$\bar{G}$	0.6408	0.6428	0.6825	0.6283	0.6191	0.6243
ω	0.8980	0.8929	0.7937	0.9292	0.9523	0.9392
$\hat{\beta}$	0.5101	0.5743	0.4891	0.6007	0.5215	0.5554
s.e	0.0634	0.0755	0.0630	0.0734	0.0761	0.1020
$\hat{\beta}_u$	0.6344	0.7223	0.6125	0.7446	0.6707	0.7554
$\hat{\beta}_l$	0.3859	0.4264	0.3657	0.4568	0.3723	0.3554
$\tilde{\beta}$	0.4489	0.5096	0.5120	0.5274	0.5701	0.6065
s.e	0.0635	0.0757	0.0630	0.0737	0.0764	0.1027
$\tilde{\beta}_u$	0.5734	0.6580	0.6354	0.6719	0.7198	0.8077
$\tilde{\beta}_l$	0.3244	0.3612	0.3885	0.3828	0.4205	0.4052
N	349	262	211	193	141	78

5.2 Time Varying Portfolio Weights

The consultant was instructed to calculate β estimates for a set of portfolios constructed using average and median returns, which exhibit time variation in the portfolio weights. Recall that P1 to P5 described in section 5.1 had fixed weights, *a* and *1-a*. The consultant was instructed to calculate the portfolio returns as

> "where individual businesses 'drop in' and 'drop out' of the portfolio sample based on data availability, as per ACG's 'mean' portfolio or the AER's 'portfolio 6' in the explanatory statement. Portfolios are to be equal-weighted during each sub-period, as per ACG's approach (i.e. where 3 businesses are in the portfolio, each has a 1/3 weight; after a 4^{th} business 'drops into' the portfolio, each has a 1/4 weight)."

Technically, a portfolio is defined using a fixed vector of weights. If the vector of weights changes a new portfolio is defined. Moreover, when a new business "drops in" and/ or "drops out" of the portfolio, both the investment opportunity set and/or the market portfolio may change as a result of takeovers and IPO activity. In short, great caution should be exercised when interpreting the β estimates from the resulting 'portfolios'.

Two sets of 'portfolios' are constructed, average 'portfolios' and median 'portfolios'. Average 'portfolios' use the equally weighted average returns to the n_t firms that are held in the 'portfolio' in period t. Median 'portfolios' use the median of the n_t firms that are held in the 'portfolio' in period t The periods are defined as follows

Period	Firms	Weight $1/n_t$
1 Jan 2002 – 12 Aug 2004	ENV APA GAS AAN AGK	1/5
13 Aug 2004 – 16 Dec 2004	ENV APA GAS AAN AGK DUE	1/6
17 Dec 2004 – 15 Dec 2005	ENV APA GAS AAN AGK DUE HDF	1/7
16 Dec 2005 – 30 Oct 2006	ENV APA GAS AAN AGK DUE HDF SPA	1/8
31 Oct 2006 – 16 Nov 2006	ENV APA GAS AAN DUE HDF SPA	1/7
17 Nov 2006 – 2 Mar 2007	ENV APA AAN DUE HDF SPA	1/6
3 Mar 2007 – 16 Aug 2007	ENV APA AAN DUE HDF SPA SKI	1/7
17 Aug 2007 – 1 Sep 2008	ENV APA DUE HDF SPA SKI	1/6

It is very important to recall that equation (10) is written assuming that the weight $a=1/n_t$ is constant, which is clearly not the case for the results presented below. As a consequence there is very likely to be substantial measurement error in the returns data as the return to the portfolio may vary because the asset values in the portfolio vary, or the weights in the portfolio vary, or both. Moreover, is very likely that equation (11) will provide a very poor guide as to the variance of this second set of 'portfolios' as terms such as $\text{Var}(1/n_t)$ and $\text{Cov}(r_{it}, 1/n_t)$ will be omitted from the measurement of variance of return. The resulting estimates and any associated inference difficult to interpret. In particular, it is not clear whether $\text{Cov}(r_{mt}, r_{pt})$ will be affected by this measurement error, and what the impact of the measurement error could be. Any issues with bias in the β estimates obtained using this data are as a result of the particular approach used to construct the 'portfolio' returns and not due to problems with the OLS or LAD estimator

Table 5.5 displays β estimates for the average and median 'portfolios' using data sampled on a monthly basis for 2 sample periods, January 2002 – September 2008 and September 2003 - September 2008.

Table 5.5: De-Levered/Relevered estimates of β

Average and Median Portfolios, Sampled Monthly

Sample Period	Average Jan 2002 – Sep 2008	Average Oct 2003 – Sep 2008	Median Jan 2002 – Sep 2008	Median Oct 2003 – Sep 2008
D/E	0.5615	0.5599	0.5615	0.5599
ω	1.0963	1.1002	1.0963	1.1002
$\hat{\beta}$	0.5511	0.6670	0.5407	0.6769
s.e	0.1070	0.1243	0.1075	0.1227
$\hat{\beta}_u$	0.7607	0.9107	0.7514	0.9174
$\hat{\beta}_l$	0.3414	0.4234	0.3300	0.4364
$\tilde{\beta}$	0.5699	0.7826	0.4112	0.6319
s.e	0.1070	0.1253	0.1094	0.1231
$\tilde{\beta}_u$	0.7796	1.0282	0.6256	0.8732
$\tilde{\beta}_l$	0.3602	0.5371	0.1967	0.3905
N	80	60	80	60

The OLS slope estimates lie in the range 0.5407 to 0.6769 with none of the associated confidence intervals containing unity. The LAD slope estimates lie in the range 0.4112 to 0.7826. The 95% confidence associated with the LAD for the Average ‘portfolio’ estimated for the period 2003:10 – 2008:09 contains unity.

Table 5.6 displays β estimates for the average and median ‘portfolios’ using data sampled on a weekly basis for 2 sample periods, January 2002 – September 2008 and September 2003 - September 2008. The OLS slope estimates lie in the range 0.5093 to 0.6433 with none of the associated confidence intervals containing unity. The LAD slope estimates lie in the range 0.4295 to 0.6596. None of the 95% confidence intervals associated with the LAD estimates contain unity.

Table 5.6: De-Levered/Relevered estimates of β

Average and Median Portfolios, Sampled Weekly

Sample Period	Average Jan 2002 – Sep 2008	Average Oct 2003 – Sep 2008	Median Jan 2002 – Sep 2008	Median Oct 2003 – Sep 2008
D/E	0.5615	0.5599	0.5615	0.5599
ω	1.0963	1.1002	1.0963	1.1002
$\hat{\beta}$	0.5559	0.6433	0.5093	0.5819
s.e	0.0526	0.0615	0.0479	0.0550
$\hat{\beta}_u$	0.6590	0.7638	0.6033	0.6897
$\hat{\beta}_l$	0.4529	0.5228	0.4154	0.4741
$\tilde{\beta}$	0.5524	0.6596	0.4295	0.5151
s.e	0.0526	0.0615	0.0481	0.0553
$\tilde{\beta}_u$	0.6554	0.7801	0.5239	0.6234
$\tilde{\beta}_l$	0.4493	0.5391	0.3352	0.4068
N	348	261	348	261

5.3 Thin Trading: Portfolios

Tables 5.7 – 5.9 report estimates of β obtained using monthly data and adjusted for thin trading using the Dimson approach described in section 4.3. The estimates were obtained using OLS for the various equal weight portfolios described in sections 5.1 and 5.2.

The tables also report a t-statistic for the null hypothesis of equality between the Dimson and OLS estimate of β, $H_0 : \hat{\beta}_i = \beta_i^D$. The test is constructed as:

$$t = \frac{\hat{\beta}_i - \beta_i^D}{s.e.\left(\hat{\beta}_i\right)}$$

Table 5.7 Dimson's β – Equal Weighted Portfolios

Monthly data: 2002:01 – 2008:09

	P1'	P2	P3	P4	P5	Average	Median
β_{i-1}	-0.1503	-0.2404	-0.2702	-0.3237	-0.4349	-0.0796	-0.0210
s.e	0.1257	0.1503	0.1462	0.1690	0.2617	0.0497	0.0373
β_i	0.4316	0.5944	0.5799	0.6146	0.7121	0.4962	0.3435
s.e	0.1251	0.1492	0.1436	0.1692	0.2504	0.0500	0.0374
β_{i+1}	0.3797	0.3666	0.4381	0.3709	0.2448	0.0258	0.0339
s.e	0.1206	0.1417	0.1371	0.1591	0.2395	0.0493	0.0369
β_i^D	0.6610	0.7206	0.7478	0.6617	0.5220	0.4424	0.3563
s.e	0.1877	0.2135	0.2117	0.2391	0.3562	0.0803	0.0601
β_i^{OLS}	0.4938	0.6301	0.6299	0.6165	0.6578	0.4942	0.3395
s.e	0.1246	0.1440	0.1509	0.1673	0.2165	0.0490	0.0366
$\beta_i^{OLS} = \beta_i^D$	-1.3424	-0.6284	-0.7810	-0.2705	0.6272	1.0568	-0.4585

Table 5.8 Dimson's β – Equal Weighted Portfolios

Monthly data: 2003:09 – 2008:09

	P1	P2	P3	P4	P5	Average	Median
β_{i-1}	-0.1926	-0.2404	-0.2702	-0.3237	-0.4349	-0.1184	-0.0652
s.e	0.1471	0.1503	0.1462	0.1690	0.2617	0.0579	0.0406
β_i	0.5543	0.5944	0.5799	0.6146	0.7121	0.5723	0.3818
s.e	0.1450	0.1492	0.1436	0.1692	0.2504	0.0581	0.0407
β_{i+1}	0.4223	0.3666	0.4381	0.3709	0.2448	0.0491	0.0650
s.e	0.1403	0.1417	0.1371	0.1591	0.2395	0.0572	0.0401
β_i^D	0.7840	0.7206	0.7478	0.6617	0.5220	0.5030	0.3817
s.e	0.2199	0.2135	0.2117	0.2391	0.3562	0.0946	0.0663
β_i^{OLS}	0.6119	0.6301	0.6299	0.6165	0.6578	0.5698	0.3752
s.e	0.1462	0.1440	0.1509	0.1673	0.2165	0.0573	0.0403
$\beta_i^{OLS} = \beta_i^D$	-1.1772	-0.6284	-0.7810	-0.2705	-0.6272	1.1650	-0.1625

Table 5.9 Dimson's β – Value Weighted Portfolios Monthly data: 2002:01 – 2008:09						
	P1'	P1	P2	P3	P4	P5
β_{i-1}	-0.1246	-0.1804	-0.2708	-0.2933	-0.3458	-0.5287
s.e	0.1383	0.1654	0.1575	0.1518	0.1647	0.2521
β_i	0.4337	0.5614	0.6299	0.6187	0.6182	0.7707
s.e	0.1376	0.1631	0.1563	0.1492	0.1648	0.2412
β_{i+1}	0.3783	0.4008	0.3269	0.3769	0.2180	0.0521
s.e	0.1327	0.1578	0.1484	0.1425	0.1550	0.2307
β_i^D	0.6874	0.7818	0.6860	0.7023	0.4904	0.2940
s.e	0.2065	0.2472	0.2236	0.2199	0.2329	0.3431
β_i^{OLS}	0.5266	0.6458	0.6590	0.6579	0.5777	0.6338
s.e	0.1382	0.1645	0.1497	0.1536	0.1559	0.2056
$\beta_i^{OLS} = \beta_i^D$	-1.1629	-0.8270	-0.1803	-0.2887	0.5600	1.6529

Looking across Tables 5.7 – 5.9 there is an absence of evidence of thin trading for all cases considered. None of the t-tests of the hypothesis $\beta_i^{OLS} = \beta_i^D$ provide evidence against this null. Similarly, there is little or no evidence that any of the β_{i-1} or β_{i+1} estimates are significant.

Tables 5.10 – 5.12 report estimates of β using weekly data adjusted for thin trading using the Dimson approach and a t-statistic for the null hypothesis of equality between the Dimson and OLS estimate of β, $H_0 : \hat{\beta}_i = \beta_i^D$.

Table 5.10 Dimson's β – Equal Weighted Portfolios

Weekly data: 2002:01 – 2008:09

	P1'	P2	P3	P4	P5	Average	Median
β_{i-1}	-0.1101	-0.1282	-0.1291	-0.1299	-0.0950	-0.0837	-0.0456
s.e	0.0637	0.0745	0.0770	0.0820	0.1127	0.0491	0.0449
β_i	0.5059	0.5828	0.6268	0.6219	0.6585	0.5095	0.4682
s.e	0.0635	0.0748	0.0773	0.0825	0.1135	0.0490	0.0448
β_{i+1}	0.0095	0.0148	0.0528	0.0165	0.0078	0.0261	0.0231
s.e	0.0626	0.0737	0.0760	0.0808	0.1101	0.0483	0.0442
β_i^D	0.4053	0.4694	0.5504	0.5085	0.5713	0.4520	0.4456
s.e	0.1019	0.1204	0.1248	0.1330	0.1806	0.0786	0.0719
β_i^{OLS}	0.5043	0.5780	0.6291	0.6181	0.6555	0.5071	0.4646
s.e	0.0622	0.0731	0.0756	0.0799	0.1075	0.0479	0.0437
$\beta_i^{OLS} = \beta_i^D$	1.5919	1.4845	1.0410	1.3721	0.7831	1.1503	0.4333

Table 5.11 Dimson's β – Equal Weighted Portfolios

Weekly data: 2003:09 – 2008:09

	P1	P2	P3	P4	P5	Average	Median
β_{i-1}	-0.1224	-0.1282	-0.1291	-0.1299	-0.0950	-0.1220	-0.0905
s.e	0.0748	0.0745	0.0770	0.0820	0.1127	0.0571	0.0512
β_i	0.5839	0.5828	0.6268	0.6219	0.6585	0.5871	0.5336
s.e	0.0744	0.0748	0.0773	0.0825	0.1135	0.0567	0.0508
β_{i+1}	0.0022	0.0148	0.0528	0.0165	0.0078	0.0460	0.0520
s.e	0.0740	0.0737	0.0760	0.0808	0.1101	0.0559	0.0501
β_i^D	0.4637	0.4694	0.5504	0.5085	0.5713	0.5111	0.4950
s.e	0.1201	0.1204	0.1248	0.1330	0.1806	0.0922	0.0827
β_i^{OLS}	0.5718	0.5780	0.6291	0.6181	0.6555	0.5847	0.5289
s.e	0.0736	0.0731	0.0756	0.0799	0.1075	0.0559	0.0500
$\beta_i^{OLS} = \beta_i^D$	1.4686	1.4845	1.0410	1.3721	0.7831	1.3165	0.6780

Table 5.12 Dimson's β – Value Weighted Portfolios Weekly data: 2002:01 – 2008:09						
	P1'	P1	P2	P3	P4	P5
β_{i-1}	-0.1487	-0.1463	-0.1448	-0.1461	-0.1127	-0.0791
s.e	0.0720	0.0859	0.0807	0.0803	0.0819	0.1139
β_i	0.5653	0.6607	0.6187	0.6459	0.5509	0.5988
s.e	0.0717	0.0853	0.0810	0.0807	0.0825	0.1148
β_{i+1}	0.0024	-0.0238	0.0039	0.0296	-0.0283	-0.0398
s.e	0.0707	0.0850	0.0798	0.0793	0.0808	0.1113
β_i^D	0.4191	0.4907	0.4778	0.5294	0.4099	0.4800
s.e	0.1151	0.1378	0.1304	0.1302	0.1330	0.1826
β_i^{OLS}	0.5681	0.6432	0.6162	0.6465	0.5476	0.5913
s.e	0.0706	0.0846	0.0793	0.0790	0.0799	0.1086
$\beta_i^{OLS} = \beta_i^D$	2.1114	1.8042	1.7447	1.4818	1.7225	1.0250

In only 1 case is there evidence against $H_0 : \hat{\beta}_i = \beta_i^D$, that is for P1' in Table 5.12. As the estimate of β_{i-1} is only very marginally significant in this case, this evidence must be considered very weak at best.

In summary, there is no convincing evidence of thin trading in the portfolio data presented in tables 5.7-5.12.

5.4 Structural Stability: Portfolios

The appendix presents recursive estimates of β_i for each of the portfolios discussed in 5.1 and 5.2 above. The estimates are produced using either a moving window with a fixed width of 1 year or an expanding window with initial width of 1 year. First, irrespective of the construction of the recursion, the evidence for each portfolio is consistent. Second, there is only very weak visual evidence of time variation in the estimates of β_i across the plots in the appendix. That is, there are no occasions when the recursive estimates display sudden substantial jumps across all the cases considered. Moreover, there is no systematic evidence of regression to unity in the estimates of β. For example, figures A1 and A2 suggest that the β for P1 lies somewhere between 0.15 and 0.6 (the OLS estimate in Table 5.10 is 0.5043). Similarly, figures A17 and A17 suggest that the β for P4 lies in the region 0.2 to 0.6 (the OLS estimate in Table 1 is 0.6181). In short, the recursive estimation provides no systematic evidence of parameter instability in the OLS estimates of β for the portfolios considered in this second report.

Since the recursive and sequential estimates are only visual guides to the stability of the estimates, we also report Hansen's (1992) test for parameter stability.[6] This test examines the regression model (3) for evidence of instability in the residual variance, σ_i^2, the intercept, α_i, the slope coefficient β_i, and then a joint test for instability in all three measures. In performing the Hansen test it is not necessary to impose an arbitrary sample splitting, or to choose forecast intervals. Rather it is necessary to estimate the model of interest a single time using the full sample of data available to the researcher. The null hypothesis of the Hansen (1992) test is that there is no instability in the parameter of interest, while the alternative is that there is instability in the parameter of interest. A joint test of the null hypothesis of no instability in α_i, β_i and σ_i^2 can be interpreted as a test for parameter stability in the model (3). Rejection of the joint null hypothesis indicates that the model suffers from parameter instability.

H_0 : The paramter (model) of interest is stable
H_1 : The paramter (model) of interest is not stable

The test has a nonstandard asymptotic distribution which depends upon the number of coefficients being tested for stability. The decision rule is straightforward; in the absence of a significant test statistic, then the investigator may be reasonably confident that either the model has not displayed parameter instability over the sample or that the data is not sufficiently informative to reject this hypothesis. In the presence

[6] Hansen, B.E. (1992) "Parameter Instability in Linear Models", Journal of Policy Modeling, 14 (4), 1992, pp. 517-533.

of a significant test statistic, the investigator may confidently conclude that the model is misspecified and prone to parameter instability.

In reference to tests of structural stability the ACG report of January 2009 comments

> *Secondly, given the imprecision with which betas are estimated, the odds are stacked against finding evidence of statistically insignificant instability in those estimates – an alternative explanation for the finding of no statistically significant instability in the true beta reflects the poor precision of the underlying beta estimates.* (ACG, January 2009 report, p. 17)

This statement is erroneous, confusing the concepts of parameter instability and precision of estimation. These concepts are, in fact, independent. Recall that estimates of $\beta_i = \frac{Cov[r_i, r_m]}{Var[r_m]}$ may be obtained from the regression

$$r_{i,t} = \alpha_i + \beta_i r_{m,t} + \varepsilon_{i,t}, \quad (3)$$

The OLS estimator for β in this case is given by

$$\hat{\beta} = \frac{\sum_{t=1}^{T}\left(r_{m,t} - E\left(r_{m,t}\right)\right)\left(r_{i,t} - E\left(r_{i,t}\right)\right)}{\sum_{t=1}^{T}\left(r_{m,t} - E\left(r_{m,t}\right)\right)^2} \quad (12)$$

while the OLS estimator for α is

$$\bar{\alpha} = E\left(r_{m,t}\right) - \hat{\beta}E\left(r_{i,t}\right) \quad (13)$$

The test for parameter instability, in effect, searches for instability in these expressions. OLS not only produces estimates of the intercept and slope in the population regression function, but also produces an estimate of the error variance in this function, σ^2. The variance of the error term is estimated as

$$\hat{\sigma}^2 = \frac{\sum_{t=1}^{T}\left(\varepsilon_t - E\left(\varepsilon_t\right)\right)^2}{T-k-1} \quad (13)$$

The majority of the evidence is that there is instability in the estimate of σ^2. but no evidence of instability in the estimates of α or β.

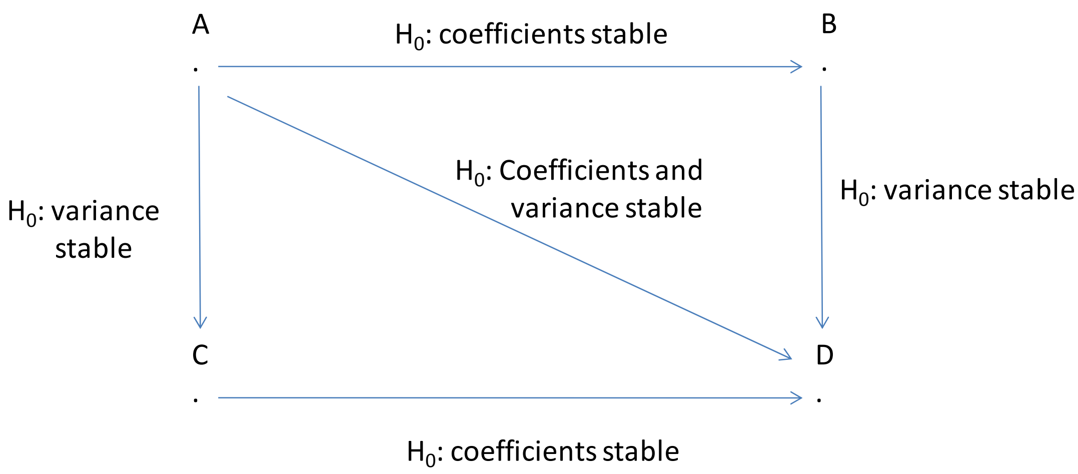

Figure 5.1 Testing for model instability

In testing for model instability on may test for stability in the coefficient estimates, α and β and then test for instability in the estimated residual variance. This strategy is summarised by starting at A in Fig 5.1 and then contingent of the outcome of the test on AB, proceeding to test BD. An equivalent approach is to test for stability in the estimated residual variance first, AC, and then test for stability in the coefficient estimates, CD. A third strategy is to jointly test for stability in α,β and σ^2, depicted as AD in Figure 5.1.

The precision, or imprecision, of the estimate of β is determined by the standard error of β, calculated as

$$V(\hat{\beta}) = \frac{\hat{\sigma}^2}{\sum_{t=1}^{T}\left(r_{m,t} - E\left(r_{m,t}\right)\right)^2} \qquad (14)$$

Note that the denominator in (14) enters (12). The evidence of instability in this report overwhelmingly concerns (13). This will affect the precision of the estimates in the sense that the confidence intervals may be wider or narrower depending on whether $\hat{\sigma}^2$ increases or decreases due to the instability. However the statement " *given the imprecision with which betas are estimated, the odds are stacked against finding evidence of statistically insignificant instability in those estimates*" is erroneous. The finding of statistically significant instability or lack thereof is independent of the precision of the estimate. Stability in β depends on the stability in (12) which require that the denominator of (14) be stable. Both of these conditions are satisfied for our data unless the instability in the denominator of (12) is offset exactly by instability in the numerator of (12), which is a stochastic singularity that is unlikely to occur. Therefore, the observed instability in (13) will directly affect the precision of the estimates without impact on the unbiased nature of the OLS estimator.

It follows that the statement "*an alternative explanation for the finding of no statistically significant instability in the true beta reflects the poor precision of the underlying beta estimates*" is incorrect. The absence of evidence against stability in the coefficients is independent of the precision of the estimates. While one might attempt to mount an argument that the Hansen test has poor size or power properties in this situation, such an argument is unlikely to carry much weight given the consistency of the evidence across different sample periods and sampling frequencies for data on stock and portfolio returns.

There is no convincing evidence of instability regarding the estimates of α and β in this data. There is evidence of instability in σ2, which may affect the estimated standard errors for β. Investigation of the source or impact of this instability (whether the standard errors are inflated or deflated) is beyond the scope of this report.

It is also important to note that instability in the estimate of σ^2 will likely be a reasonably common event with asset returns. Instability in the estimate of σ^2 simply reflects change in the estimated level of asset-specific risk.

5.4.1 Constant Portfolio Weights

Table 5.13 presents marginal significance levels, also referred to as p-values, for the Hansen (1992) test for structural stability applied to OLS estimates of (3) using continuously compounded returns to the accumulation indices.

Table 5.13: Hansen (1992) Structural Stability Tests

Weekly Data

	Equal Weight Portfolios					
	P1'	***P1***	***P2***	***P3***	***P4***	***P5***
Sample	1 Jan 2002 – 1 Sep 008	1 Sep 2003 – 1 Sep 2008	13 Aug 2004 – 1 Sep 2008	17 Dec 2004 – 1 Sep 2008	16 Dec 2005 – 1 Sep 2008	2 Mar 2007 – 1 Sep 2008
Joint	0.0010	0.0061	0.0044	0.0007	0.0008	0.0161
σ^2	0.0001	0.0005	0.0005	0.0001	0.0001	0.0056
α	0.1747	0.4683	0.7938	0.8783	0.8470	0.7563
β	0.0433	0.3168	0.2016	0.0561	0.4105	0.7318
	Value Weight Portfolios					
	P1'	***P1***	***P2***	***P3***	***P4***	***P5***
Joint	0.0014	0.0308	0.0043	0.0016	0.0010	0.0669
σ^2	0.0002	0.0055	0.0009	0.0006	0.0001	0.0116
α	0.4191	0.6731	0.9760	0.9689	0.9357	0.7670
β	0.0173	0.1295	0.1397	0.0609	0.5230	0.6835

Table 5.13 presents results for the Hansen stability test for data sampled at the weekly frequency. There is some evidence of parameter instability in the estimates across the securities. In 9 out of 12 cases there is evidence against the joint null of no structural instability at the 1% level of confidence or better. However in only 2 of the 12 cases considered is there is evidence against the null hypothesis of no instability in β at the 10% level of confidence or better and neither of these rejections are significant at the 1% level of confidence or better. Many of the rejections of the joint null appear to be as a result of instability in σ^2 rather than in β. There is no evidence of time variation in α at the 5% level of confidence. In short, where there is evidence of instability in the model, it appears that much of this instability is associated with the variance of the error term, and not with the estimates of the coefficients of the model. That is, the evidence suggests that any parameter instability detected in Table 5.13 is associated with shifts in the variance of the asset specific return.

Table 5.14: Hansen (1992) Structural Stability Tests

Monthly Data						
Equal Weight Portfolios						
	P1'	***P1***	***P2***	***P3***	***P4***	***P5***
Sample	1 Jan 2002 – 1 Sep 008	1 Sep 2003 – 1 Sep 2008	13 Aug 2004 – 1 Sep 2008	17 Dec 2004 – 1 Sep 2008	16 Dec 2005 – 1 Sep 2008	2 Mar 2007 – 1 Sep 2008
Joint	0.0001	0.0006	0.0059	0.0281	0.0728	0.1169
σ^2	0.0005	0.0021	0.0091	0.0136	0.0220	0.0296
α	0.1112	0.4476	0.7469	0.7964	0.7181	0.5621
β	0.3475	0.9345	0.8803	0.8522	0.9887	0.9807
Value Weight Portfolios						
	P1'	***P1***	***P2***	***P3***	***P4***	***P5***
Joint	0.0001	0.0023	0.0200	0.0455	0.0667	0.1305
σ^2	0.0003	0.0015	0.0073	0.0141	0.0097	0.0318
α	0.2624	0.5897	0.9282	0.9076	0.8095	0.4724
β	0.2981	0.8716	0.7784	0.7890	0.9701	0.9624

Table 5.14 presents results of the Hansen test for data sampled at the monthly frequency. As before the majority of the Joint tests suggest that there is parameter instability in the model. However, this instability is attributable to movements in the residual variance. In no case is there evidence of instability in β, at the 29% level of confidence or better, or α, at the 11% level of confidence or better.

5.4.2 Time-Varying Portfolio Weights

Table 5.15 presents the results of Hansen stability tests for weekly and monthly data.

Table 5.15: Hansen (1992) Structural Stability Tests

Time Varying Portfolio Weights

	Average’	Median’	Average	Median
Monthly Data				
	Average’	Median’	Average	Median
Sample	1 Jan 2002 –1 Sep 008		1 Sep 2003 –1 Sep 2008	
Joint	0.0200	0.0182	0.0418	0.0374
σ^2	0.0056	0.0098	0.0070	0.0116
α	0.1673	0.0830	0.8554	0.6063
β	0.3074	0.1539	0.9702	0.7633
Weekly Data				
	Average’	Median’	Average	Median
Joint	0.0005	0.0042	0.0035	0.0168
σ^2	0.0002	0.0016	0.0002	0.0011
α	0.2325	0.2529	0.8856	0.5825
β	0.0101	0.0100	0.5583	0.2646

There is evidence against the null hypothesis of the Joint test at the 5% level of confidence or better in all cases. In only 2 instances is the null hypothesis of no instability in β rejected at the 5% level of confidence, with no rejections occurring at the 1% level. There is no evidence of instability in α at the 5% or 1% level of confidence. The observed parameter instability may be a result of changes in the residual variance, or as a result of time variation in the constituents and/or portfolio weights used in the construction of the ‘portfolio’ returns underlying Table 5.15.

6. Robustness: US Data

Weekly and monthly samples of comparable stocks and the Standard and Poor's Composite Index for the USA over the period 1st January 1990 - 1st September 2008 were collected and used to obtain estimates of β using the OLS and LAD estimators. Table 6.1 and 6.2 present de-levered/re-levered estimates of β for the period 2002:01:01 to 2008:09:01 for the weekly and monthly sampling frequencies. Tables 6.3 and 6.4 present similar estimates for the period 2003:09:01 to 2008:09:01. Tables 6.5 and 6.6 examine the entire sample period 1990:01:01 to 2008:09:01 omitting the period of the technology boom, namely 1998:06:30 to 2001:12:31.

The list of companies examined in this section was provided to the consultant by the ACC, as were the selected sample dates. All data for POM was excluded prior to October 1st 2002 due to M&A activity concerning this stock. No data was available for SRP and NST prior to August 23rd 1999 on a weekly basis, or September 1999 on a monthly basis.

Table 6.1: De-Levered/Relevered estimates of β

2002:01:01 – 2008:09:01 Sampled Weekly

	CHG	CNP	EAS	NI	NJR	NST	NU	SRP	UIL	POM	PORT
$\bar{G}$	0.3223	0.7324	0.5417	0.5803	0.3176	0.4705	0.6034	0.7425	0.4342	0.5904	0.5335
ω	1.6943	0.6690	1.1457	1.0492	1.7059	1.3238	0.9914	0.6436	1.4146	1.0240	1.1662
$\hat{\beta}$	1.0359	0.3345	0.5440	0.7138	0.9909	0.6029	0.5518	0.6494	0.7308	0.6100	0.6919
s.e	0.0883	0.0916	0.0656	0.0558	0.0796	0.0625	0.0565	0.0876	0.1018	0.0667	0.0507
$\hat{\beta}_u$	1.2090	0.5140	0.6725	0.8232	1.1469	0.7253	0.6626	0.8210	0.9303	0.7407	0.7914
$\hat{\beta}_l$	0.8628	0.1550	0.4154	0.6044	0.8348	0.4804	0.4410	0.4777	0.5314	0.4793	0.5925
$\tilde{\beta}$	1.1211	0.5286	0.5609	0.7179	1.0571	0.7119	0.5474	0.5744	0.9436	0.5669	0.7421
s.e	0.0885	0.0923	0.0657	0.0560	0.0797	0.0628	0.0565	0.0879	0.1024	0.0669	0.0509
$\tilde{\beta}_u$	1.2944	0.7095	0.6897	0.8277	1.2134	0.8349	0.6582	0.7467	1.1443	0.6980	0.8419
$\tilde{\beta}_l$	0.9477	0.3477	0.4321	0.6081	0.9008	0.5888	0.4366	0.4022	0.7428	0.4358	0.6422
N	347	347	347	347	347	347	347	347	347	309	347

Table 6.2: De-Levered/Relevered estimates of β

2002:01:01 – 2008:09:01 Sampled Monthly

	CHG	CNP	EAS	NI	NJR	NST	NU	SRP	UIL	POM	PORT
$\bar{G}$	0.3223	0.7324	0.5417	0.5803	0.3176	0.4705	0.6034	0.7425	0.4342	0.5904	0.5335
ω	1.6943	0.6690	1.1457	1.0492	1.7059	1.3238	0.9914	0.6436	1.4146	1.0240	1.1662
$\hat{\beta}$	0.7458	0.9835	0.4190	0.6446	0.4005	0.6167	0.5165	1.1562	1.6499	0.6368	0.9048
s.e	0.2509	0.2271	0.1722	0.1672	0.2068	0.1701	0.1680	0.2603	0.2966	0.1870	0.1370
$\hat{\beta}_u$	1.2375	1.4287	0.7565	0.9723	0.8058	0.9501	0.8457	1.6665	2.2313	1.0033	1.1734
$\hat{\beta}_1$	0.2540	0.5383	0.0815	0.3170	-0.0047	0.2832	0.1872	0.6460	1.0685	0.2703	0.6362
$\tilde{\beta}$	0.7992	0.6829	0.0728	0.7056	0.2605	0.7511	0.4757	0.8848	1.4911	0.4210	0.5110
s.e	0.2512	0.2303	0.1722	0.1683	0.2079	0.1708	0.1681	0.2622	0.3060	0.1888	0.1467
$\tilde{\beta}_u$	1.2916	1.1343	0.4103	1.0355	0.6679	1.0859	0.8053	1.3987	2.0908	0.7909	0.7987
$\tilde{\beta}_1$	0.3069	0.2315	-0.2647	0.3758	-0.1469	0.4163	0.1462	0.3709	0.8913	0.0510	0.2234
N	80	80	80	80	80	80	80	80	80	72	80

Table 6.3: De-Levered/Relevered estimates of β

2003:09:01 – 2008:09:01 Sampled Weekly

	CHG	CNP	EAS	NI	NJR	NST	NU	SRP	UIL	POM	PORT
$\bar{G}$	0.3391	0.6764	0.5198	0.5842	0.3139	0.4568	0.5737	0.6345	0.3897	0.5450	0.5033
ω	1.6523	0.8089	1.2006	1.0395	1.7153	1.3580	1.0658	0.9138	1.5257	1.1376	1.2417
$\hat{\beta}$	1.3180	0.6100	0.4770	0.7415	1.2125	0.7516	0.6021	0.8319	1.0606	0.8615	0.8506
s.e	0.1239	0.0659	0.0980	0.0711	0.1096	0.0839	0.0784	0.0958	0.1410	0.0801	0.0596
$\hat{\beta}_u$	1.5607	0.7392	0.6691	0.8809	1.4273	0.9160	0.7558	1.0196	1.3369	1.0185	0.9674
$\hat{\beta}_l$	1.0752	0.4808	0.2848	0.6022	0.9976	0.5871	0.4483	0.6443	0.7843	0.7045	0.7338
$\tilde{\beta}$	1.3002	0.6249	0.4979	0.7106	1.2826	0.7960	0.6201	0.7866	1.1207	0.8208	0.7632
s.e	0.1239	0.0659	0.0981	0.0715	0.1097	0.0840	0.0785	0.0959	0.1410	0.0804	0.0601
$\tilde{\beta}_u$	1.5430	0.7542	0.6901	0.8506	1.4977	0.9607	0.7739	0.9744	1.3971	0.9783	0.8809
$\tilde{\beta}_l$	1.0574	0.4957	0.3057	0.5705	1.0676	0.6313	0.4663	0.5987	0.8443	0.6632	0.6454
N	262	262	262	262	262	262	262	262	262	262	262

Table 6.4: De-Levered/Relevered estimates of β

2003:09:01 – 2008:09:01 Sampled Monthly

	CHG	CNP	EAS	NI	NJR	NST	NU	SRP	UIL	POM	PORT
$\bar{G}$	0.3391	0.6764	0.5198	0.5842	0.3139	0.4568	0.5737	0.6345	0.3897	0.5450	0.5033
ω	1.6523	0.8089	1.2006	1.0395	1.7153	1.3580	1.0658	0.9138	1.5257	1.1376	1.2417
$\hat{\beta}$	1.4746	0.5321	0.0903	0.2240	0.8682	0.7250	0.6260	1.2461	1.6390	0.7106	0.8110
s.e	0.3241	0.1958	0.2301	0.2094	0.2990	0.2525	0.2551	0.2454	0.3538	0.2360	0.1674
$\hat{\beta}_u$	2.1098	0.9159	0.5413	0.6345	1.4543	1.2198	1.1260	1.7271	2.3325	1.1731	1.1391
$\hat{\beta}_l$	0.8393	0.1482	-0.3607	-0.1864	0.2821	0.2302	0.1260	0.7650	0.9456	0.2481	0.4829
$\tilde{\beta}$	1.4040	0.6906	-0.0555	0.0840	0.8747	0.4830	0.7725	1.1967	1.6081	0.5173	0.5629
s.e	0.3243	0.1973	0.2309	0.2154	0.3006	0.2545	0.2577	0.2498	0.3653	0.2374	0.1730
$\tilde{\beta}_u$	2.0396	1.0772	0.3971	0.5061	1.4640	0.9818	1.2775	1.6862	2.3241	0.9826	0.9019
$\tilde{\beta}_l$	0.7684	0.3039	-0.5080	-0.3381	0.2855	-0.0158	0.2675	0.7071	0.8922	0.0520	0.2239
N	60	60	60	60	60	60	60	60	60	60	60

Table 6.5: De-Levered/Relevered estimates of β

1990:01:01-1998:06:30 and 2002:01:01 – 2008:09:01

Sampled Monthly

	CHG	CNP	EAS	NI	NJR	NST	NU	SRP	UIL	POM	PORT
$\bar{G}$	0.4054	0.6273	0.5184	0.5129	0.3972	0.4705	0.6204	0.7425	0.5882	0.5904	0.5473
ω	1.4866	0.9318	1.2040	1.2177	1.5070	1.3238	0.9491	0.6436	1.0295	1.0240	1.1317
$\hat{\beta}$	0.6230	0.8525	0.6970	0.6277	0.5435	0.6166	0.4207	1.1562	0.8902	0.6368	0.7080
s.e	0.1379	0.1604	0.1315	0.1141	0.1418	0.1701	0.1269	0.2603	0.1222	0.1870	0.0854
$\hat{\beta}_u$	0.8933	1.1669	0.9547	0.8514	0.8214	0.9501	0.6693	1.6665	1.1296	1.0033	0.8753
$\hat{\beta}_l$	0.3527	0.5382	0.4393	0.4040	0.2657	0.2832	0.1720	0.6460	0.6507	0.2703	0.5407
$\tilde{\beta}$	0.7219	0.5997	0.5733	0.5337	0.5380	0.7511	0.2632	0.8848	0.6123	0.4210	0.4692
s.e	0.1383	0.1615	0.1318	0.1144	0.1418	0.1708	0.1275	0.2622	0.1248	0.1888	0.0879
$\tilde{\beta}_u$	0.9929	0.9163	0.8316	0.7578	0.8158	1.0859	0.5131	1.3987	0.8569	0.7909	0.6415
$\tilde{\beta}_l$	0.4509	0.2831	0.3150	0.3096	0.2601	0.4163	0.0132	0.3709	0.3677	0.0510	0.2968
N	181	181	181	181	181	80	181	80	181	72	181

Table 6.6: De-Levered/Relevered estimates of β

1990:01:01-1998:06:30 and 2002:01:01 – 2008:09:01

Sampled Weekly

	CHG	CNP	EAS	NI	NJR	NST	NU	SRP	UIL	POM	PORT
$\bar{G}$	0.4054	0.6273	0.5184	0.5129	0.3972	0.4705	0.6204	0.7425	0.5882	0.5904	0.5473
ω	1.4866	0.9318	1.2040	1.2177	1.5070	1.3238	0.9491	0.6436	1.0295	1.0240	1.1317
$\hat{\beta}$	0.7516	0.4766	0.5725	0.6995	0.6759	0.6028	0.4607	0.6494	0.4772	0.6100	0.5810
s.e	0.0519	0.0682	0.0515	0.0441	0.0588	0.0625	0.0471	0.0876	0.0444	0.0667	0.0320
$\hat{\beta}_u$	0.8533	0.6103	0.6735	0.7859	0.7911	0.7253	0.5529	0.8211	0.5641	0.7407	0.6438
$\hat{\beta}_l$	0.6499	0.3429	0.4715	0.6132	0.5607	0.4804	0.3684	0.4777	0.3902	0.4793	0.5183
$\tilde{\beta}$	0.6578	0.5763	0.5351	0.6586	0.6873	0.7119	0.4536	0.5745	0.4669	0.5669	0.5204
s.e	0.0520	0.0684	0.0516	0.0441	0.0588	0.0628	0.0471	0.0879	0.0443	0.0669	0.0321
$\tilde{\beta}_u$	0.7597	0.7103	0.6362	0.7450	0.8026	0.8350	0.5459	0.7467	0.5538	0.6980	0.5833
$\tilde{\beta}_l$	0.5560	0.4423	0.4339	0.5722	0.5721	0.5889	0.3612	0.4022	0.3800	0.4358	0.4574
n	790	790	790	790	790	347	790	347	790	309	790

The evidence in tables 6.1-6.6 suggests that the choice of sampling frequency is not important when estimating β for the sample of US stocks. The majority of the estimates are clustered in the 0.5 to 0.9 range, although several estimates exceed 1. The estimated β for the time-varying portfolio ranges from 0.4692 to 0.9048. It is also useful to note that several of the β estimates are insignificantly different to zero in the post-2003 monthly sample.

7. Summary of advice

The following is a brief set of conclusions that the consultant has drawn from working with the data described in this document and in the initial report.

7.1 Sampling Frequency

A reasonable compromise is to sample the data at a weekly frequency. Given the sparse nature of the data there are too few monthly observations available for many of the stocks to produce statistically reliable estimates of β. For some of the stocks and portfolios considered in this report there are less than 30 monthly observations meaning that statistical inference is unlikely to be reliable. There is a tradeoff between the noisy nature of the daily data and the lack of degrees of freedom in the monthly data. The best compromise would appear to be the use of data sampled at the weekly frequency.

7.2 Construction of Returns

While it is usual to employ continuously compounded returns there is no evidence that β estimates obtained from discretely compounded data are manifestly different.

7.3 Parameter Instability

There is no overwhelming issue with instability. It is the case that the OLS and LAD estimates of β differ. However as the estimators are maximizing very different functions, this difference is somewhat unsurprising.

Neither of the recursive least squares estimators appears to demonstrate convincing evidence of parameter instability. It is important to note that these estimators are not sufficient in the sense that they do not employ all available information. The use of the Hansen (1992) test for parameter instability produces systematic evidence of instability in the regression models. Where this instability is detected it is almost uniformly due to a change in the error variance in the regression model. There is no evidence of parameter instability associated with the coefficients of the regression models themselves. This evidence is largely consistent with the view that asset specific volatility may have been unstable during the period examined by the consultant.

7.4 Standard Errors

There is evidence of structural instability in the variance of the errors in the estimated model. If there is instability in σ^2 it should be possible to date this instability and adjust the model appropriately. Dating this instability and appropriate adjustment are beyond the scope of this report. However, both the White (1980) and Newey-West (1987) estimators can correct for heteroscedasticity of unknown form. If the instability can dated a simple correction for heteroscedasticity of a known form can be made.

It is not clear what the implications of this instability are for the performance of the White (1980) and Newey-West (1987) approaches to calculating standard errors. Given the difficulties associated with choosing the optimal lag length in the non-parametric estimator of the long run variance, the Newey-West approach appears most fragile. While the confidence intervals obtained using the White and Newey-West corrections can differ from than those obtained using the OLS standard errors, there is no evidence of systematic bias in the standard errors.

7.5 Alternative data

Re-estimation of the various regression models using US data does not manifestly alter the conclusions one would draw about the magnitude of the point estimates of the de-levered/re-levered β. Rather it is the case that the balance of the evidence points towards the point estimate of β lying in the range 0.4 to 0.7.

7.6 *The Relationship between R^2 and β*

The SFC report argues that in regressions which exhibit low R^2 values the estimate of β exhibits a downward bias. This report demonstrates such an assertion is very strong and is likely to be affected by the assumptions underlying the experiment. Firstly, we show that increasing the sample size by moving from an assumed monthly to weekly sampling frequency reduces the size of any bias substantially. Secondly, the results presented by SFC depend critically on the assumption that volatility follows a uniform distribution, an assumption which is likely to over-emphasise the presence of extreme draws for volatility in the sample and under-emphasise the presence of average draws for volatility when compared to a normal distribution. Further, this assumption of a uniform distribution is not justified by SFC on theoretical or empirical grounds, nor is the impact of deviation from this assumption examined.

Finally, if their simulation evidence is taken at face value, then it is the case that very few of the results presented in this report would be subject to the criticisms of the SFC report. Most of the samples exceed 48 observations and exhibit R^2 values far in excess of 5%.

7.7 *Confidence Intervals*

While the confidence interval around β is a useful measure of the uncertainty about any particular point estimate, it is not a particularly useful method of comparison across estimates of β. The most appropriate point of comparison of the exposure to market risk across firms is the point estimate of β. Any other value chosen from the confidence interval (e.g. the upper bound) has an corresponding value in the other tail of the interval that obtains with equal probability.

Appendix:
Recursive Least Squares Estimates

This appendix presents recursive estimates of β_i for each of the portfolios discussed in Section 5. Two estimation strategies are employed using a moving window with a fixed width of 1 year of data and an expanding window with initial width of 1 year of data, whether weekly or monthly in sampling frequency. The results are, in general, remarkably similar. First, irrespective of the construction of the recursion, the evidence for each portfolio is consistent. Second, there is only weak visual evidence of time variation in the estimates of β_i across the plots in the appendix . That is, there are no occasions when the recursive estimates display sudden substantial jumps across all the cases considered. Moreover, there is no systematic evidence of regression to unity. In short, there is no strong evidence of instability in the estimate of β.

Recursive Least Squares Estimates:

Weekly Data

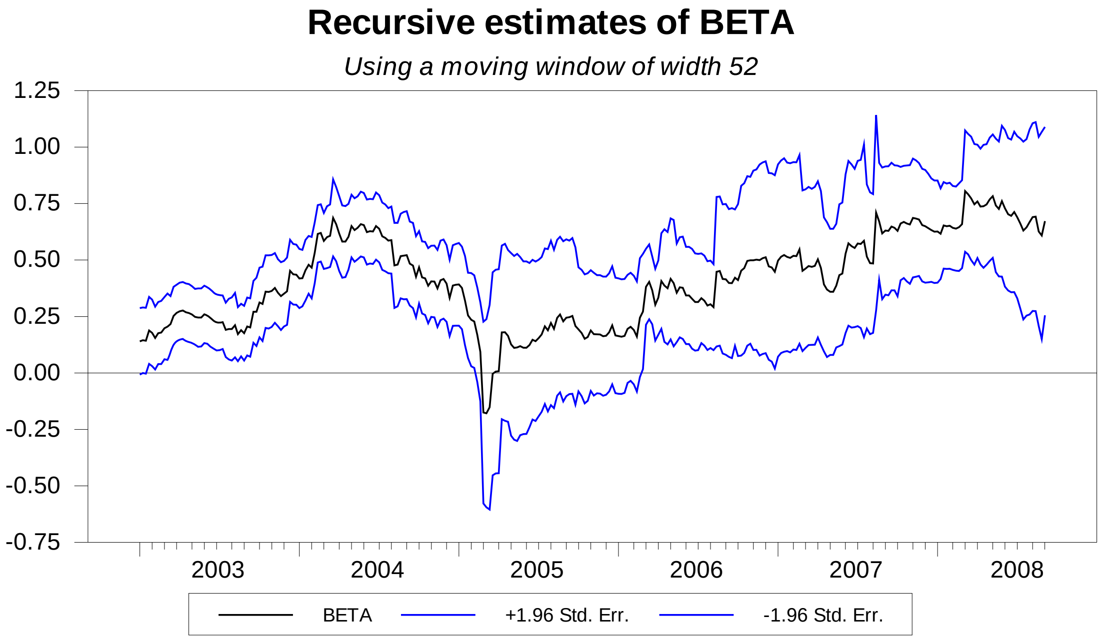

Figure A1: P1: 2002:01:01 – 2008:09:01, equal weights

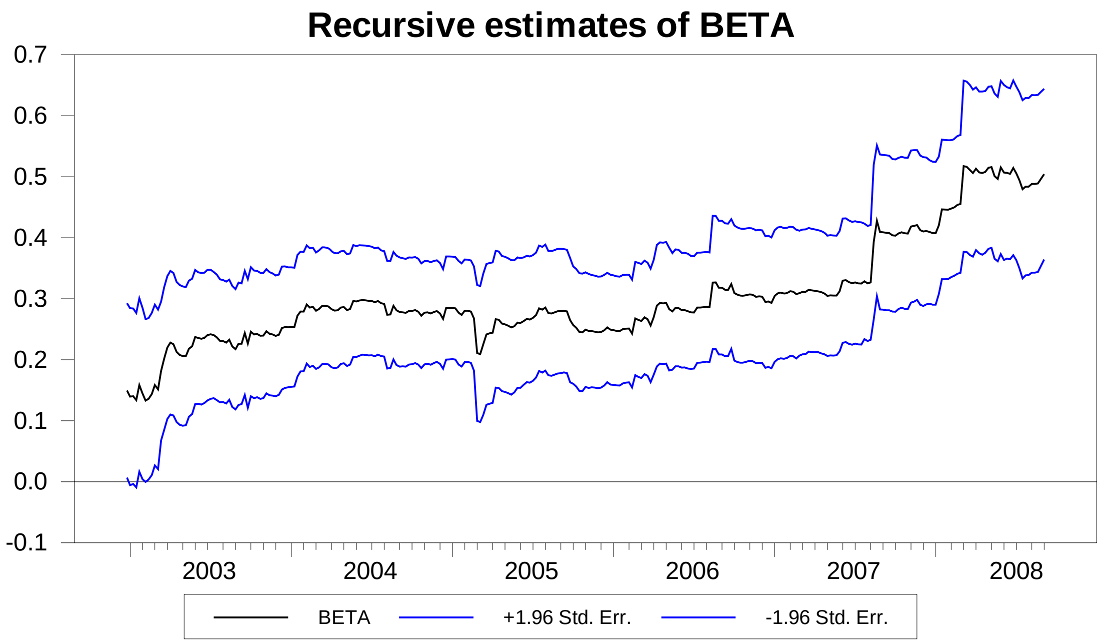

Figure A2: P1: 2002:01:01 – 2008:09:01, equal weights

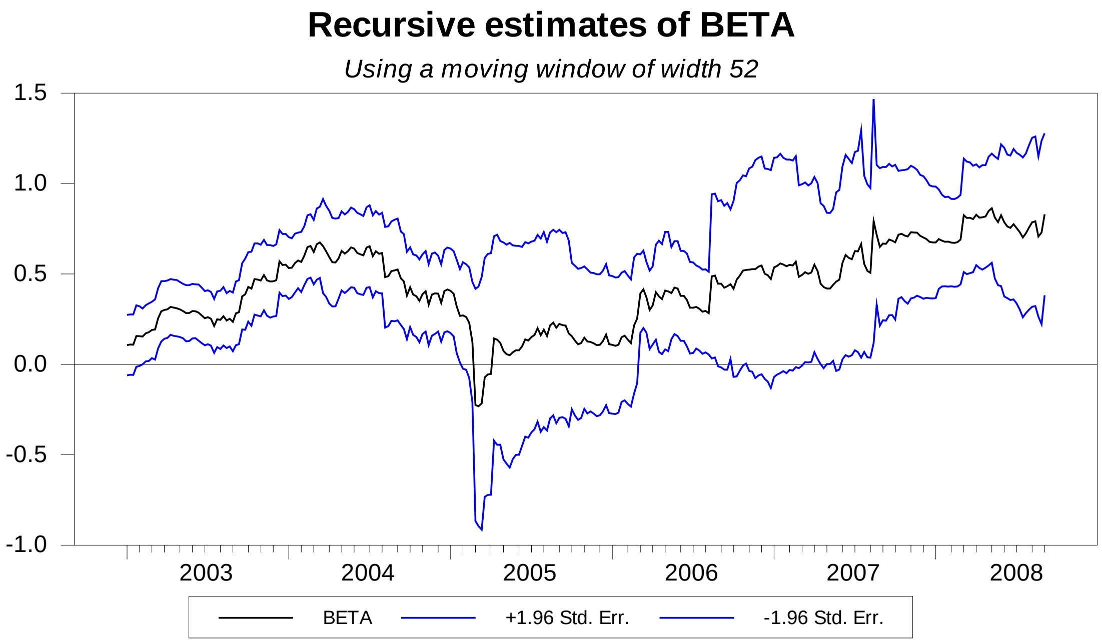

Figure A3: P1: 2002:01:01 – 2008:09:01, value weights

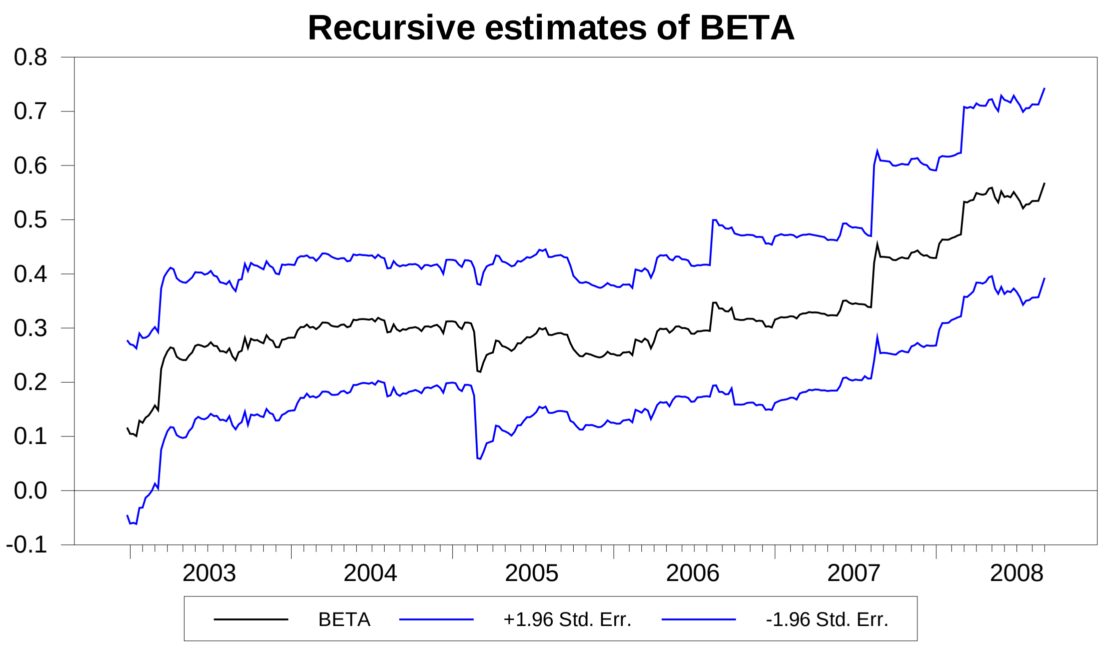

Figure A4: P1: 2002:01:01 – 2008:09:01, value weights

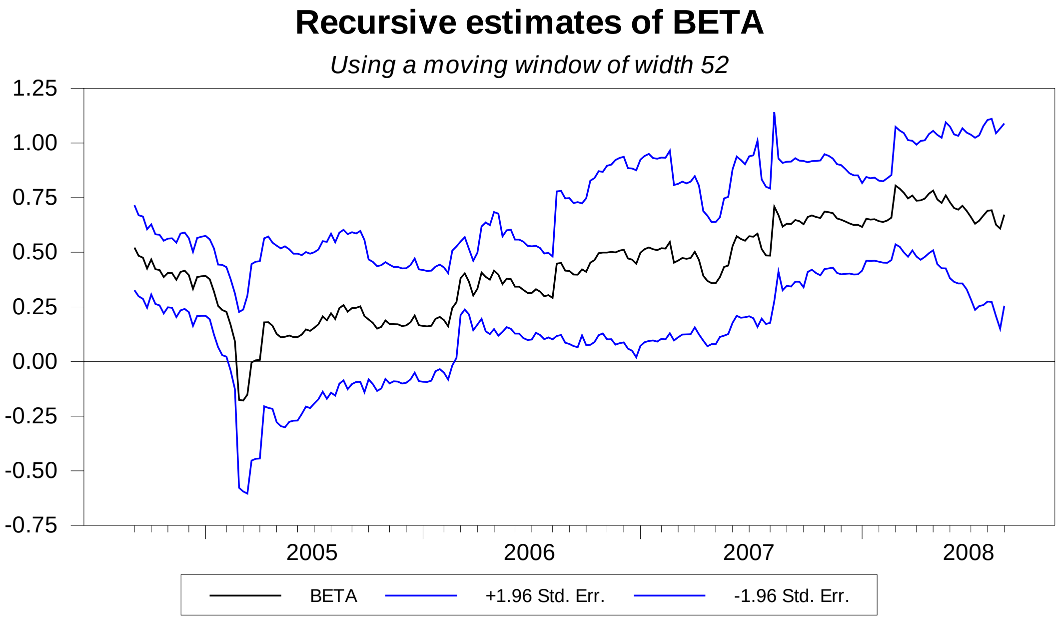

Figure A5: P1, 2003:09:01 - 2008:09:01, equal weights

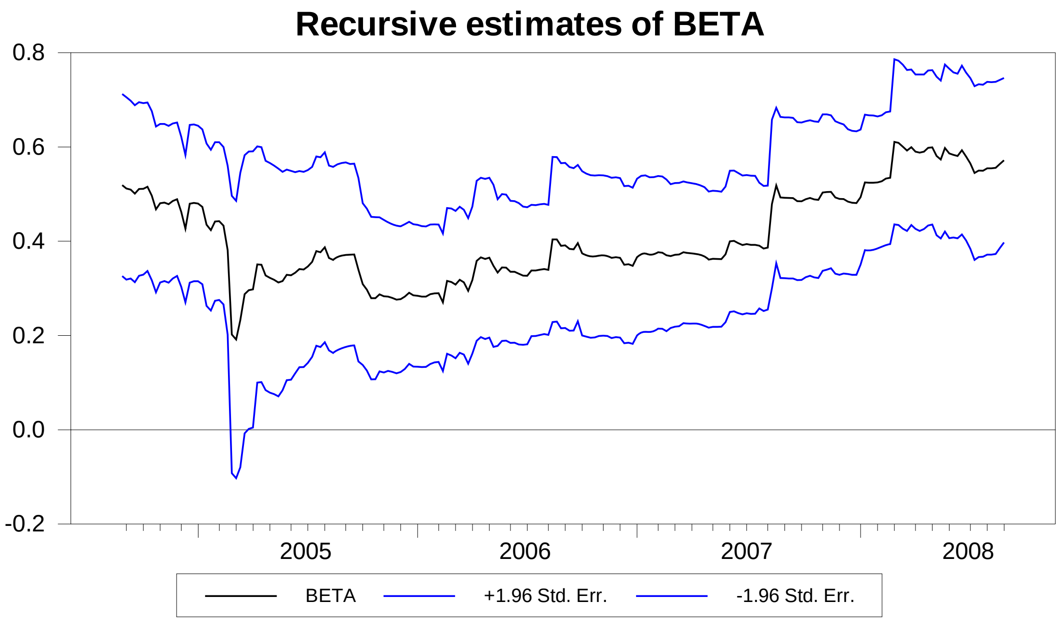

Figure A6: P1, 2003:09:01 - 2008:09:01, equal weights

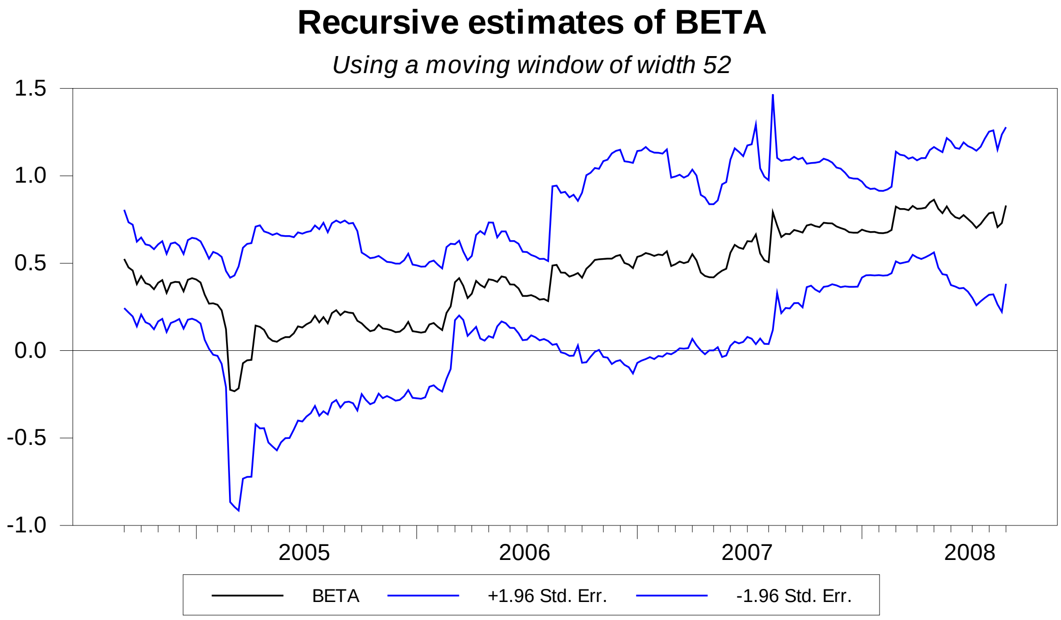

Figure A7: P1, 2003:09:01 - 2008:09:01, value weights

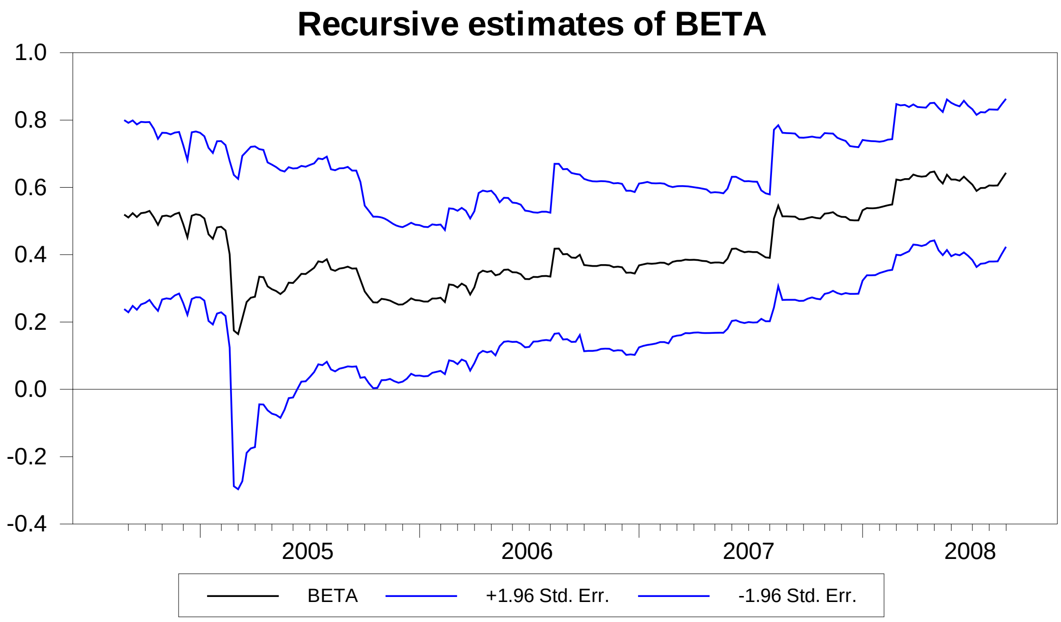

Figure A8: P1, 2003:09:01 - 2008:09:01, value weights

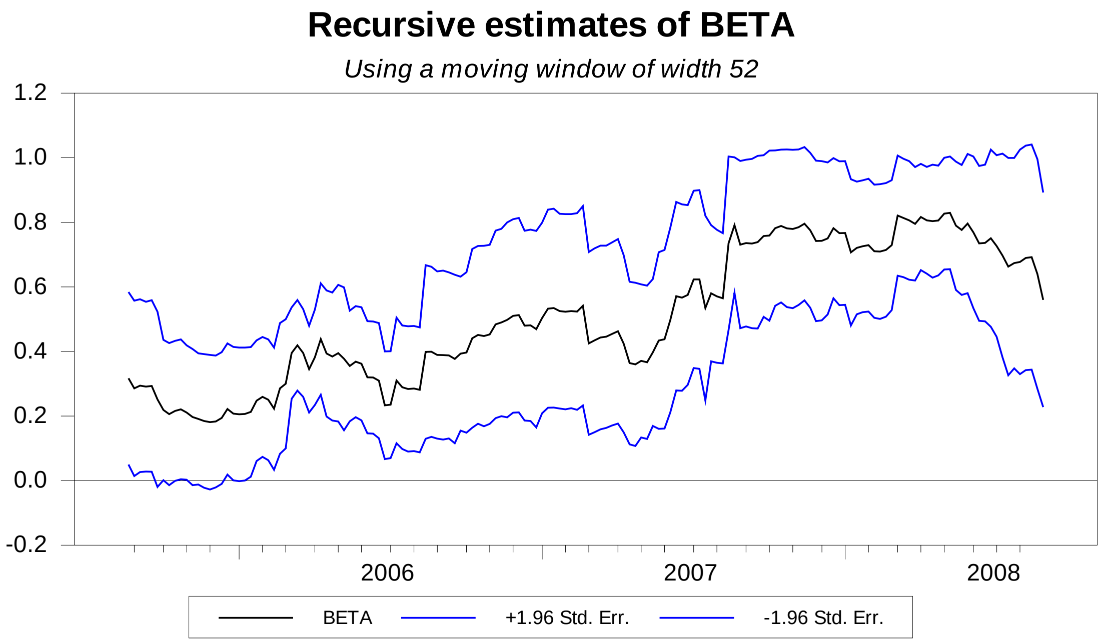

Figure A9: P2: 2004:08:20 – 2008:09:01 Equal Weights

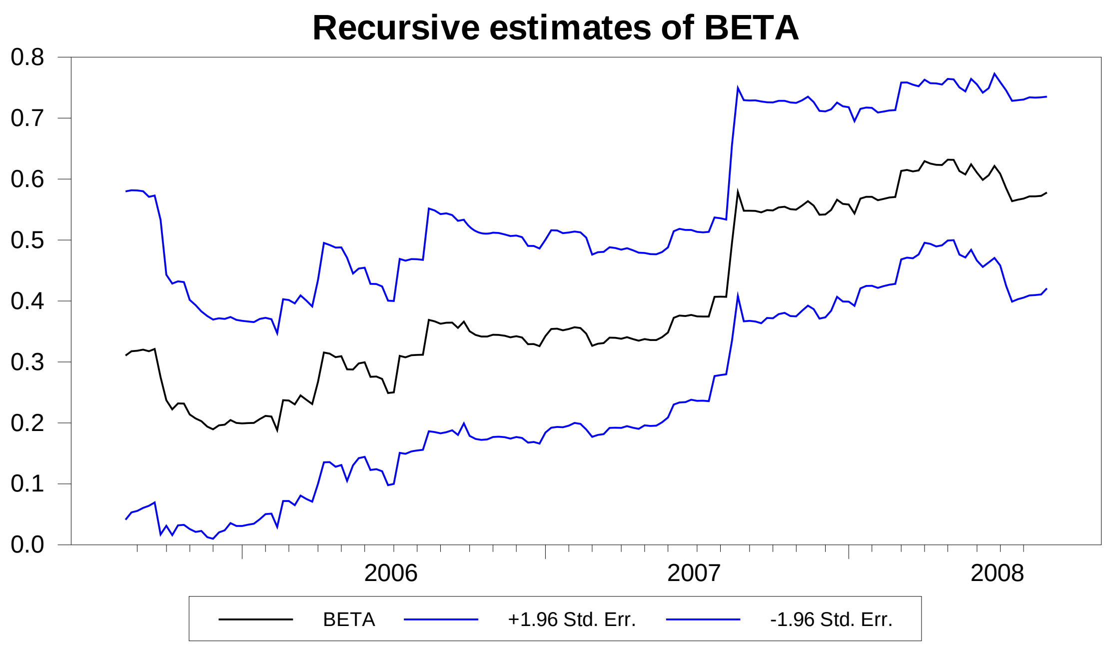

Figure A10: P2: 2004:08:20 – 2008:09:01 Equal Weights

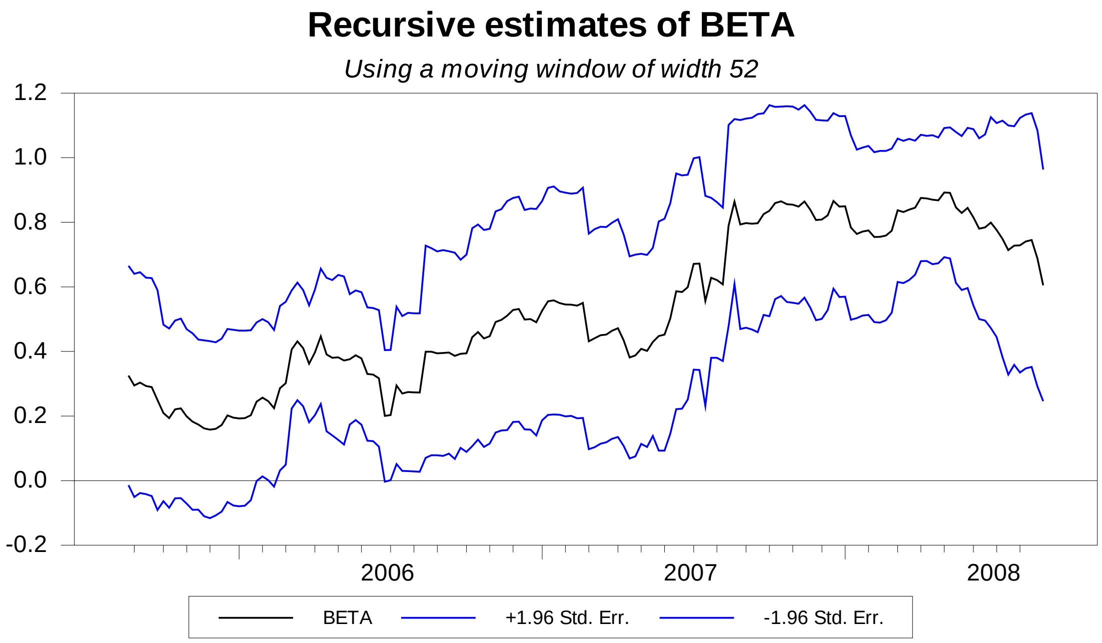

Figure A11: P2: 2004:08:20 – 2008:09:01 Value Weights

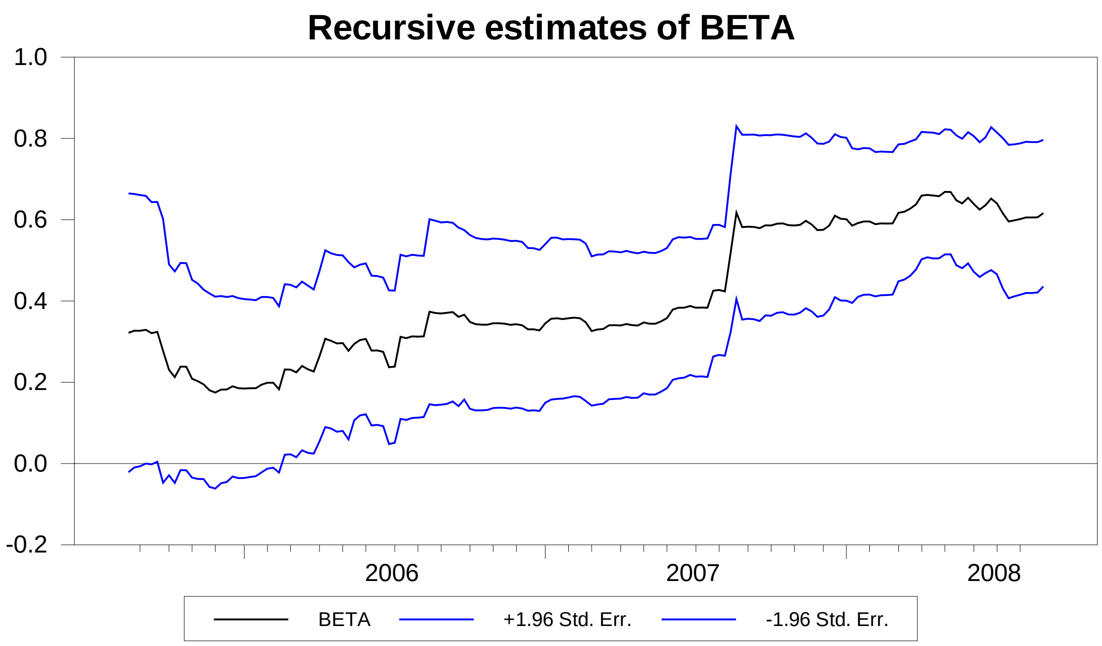

Figure A12: P2: 2004:08:20 – 2008:09:01 Value Weights

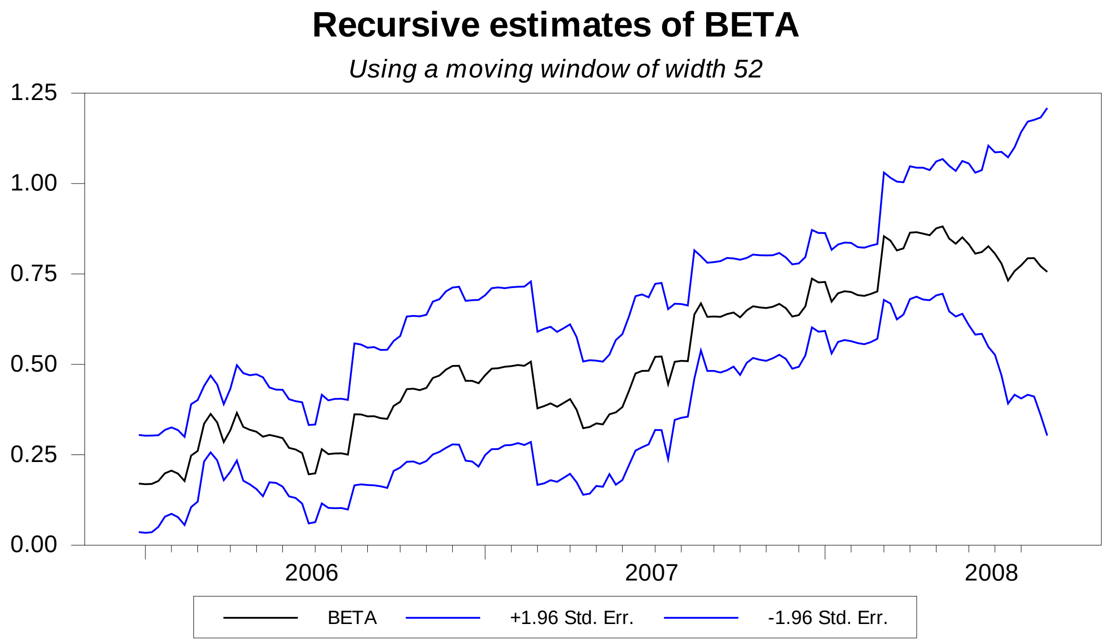

Figure A13: P3: 2004:12:24 – 2008:09:01 Equal Weights

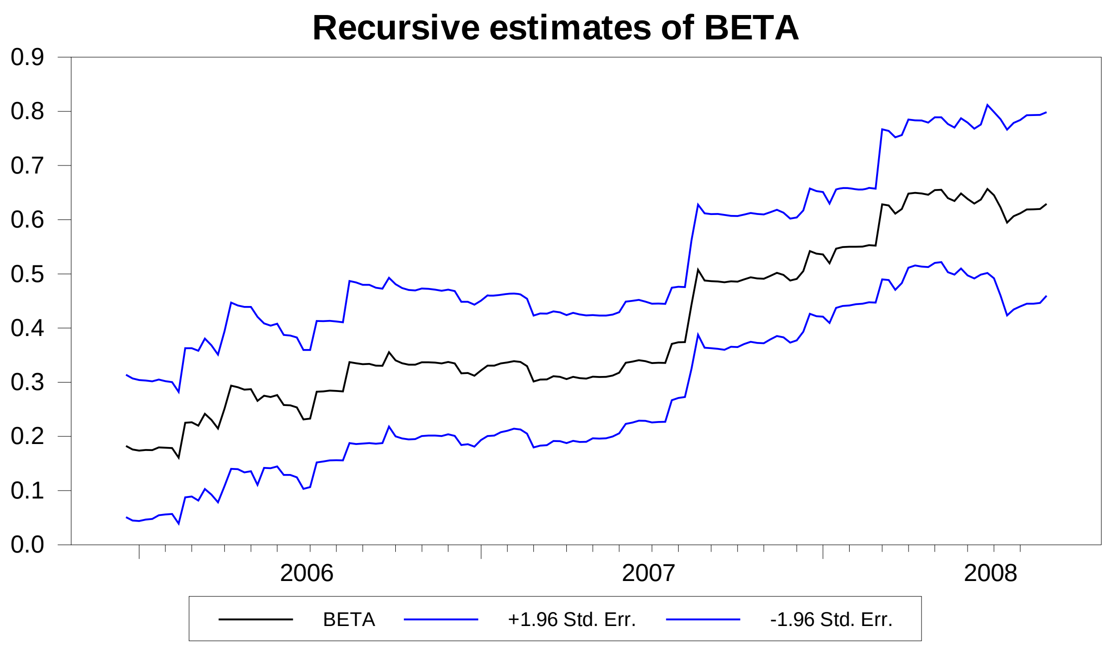

Figure A14: P3: 2004:12:24 – 2008:09:01 Equal Weights

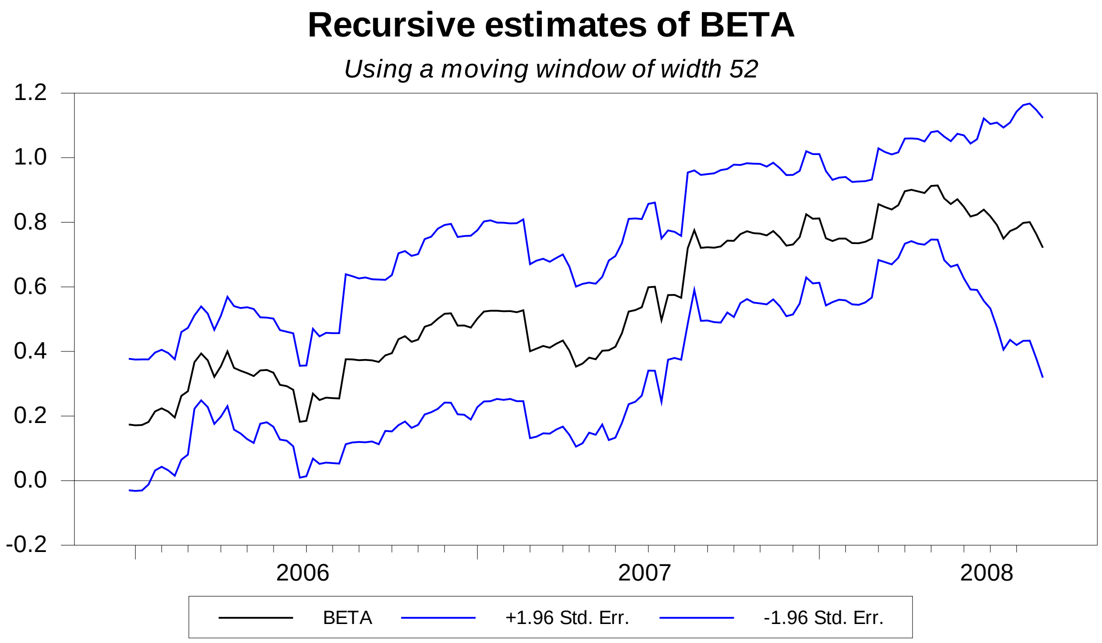

Figure A15: P3: 2004:12:24 – 2008:09:01 Value Weights

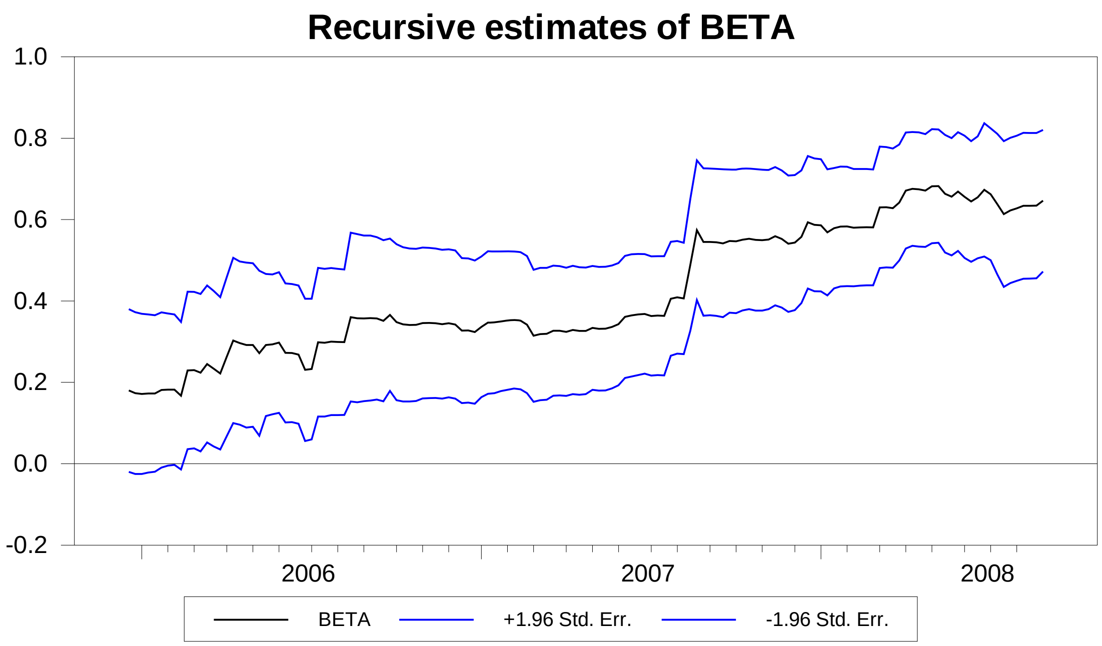

Figure A16: P3: 2004:12:24 – 2008:09:01 Value Weights

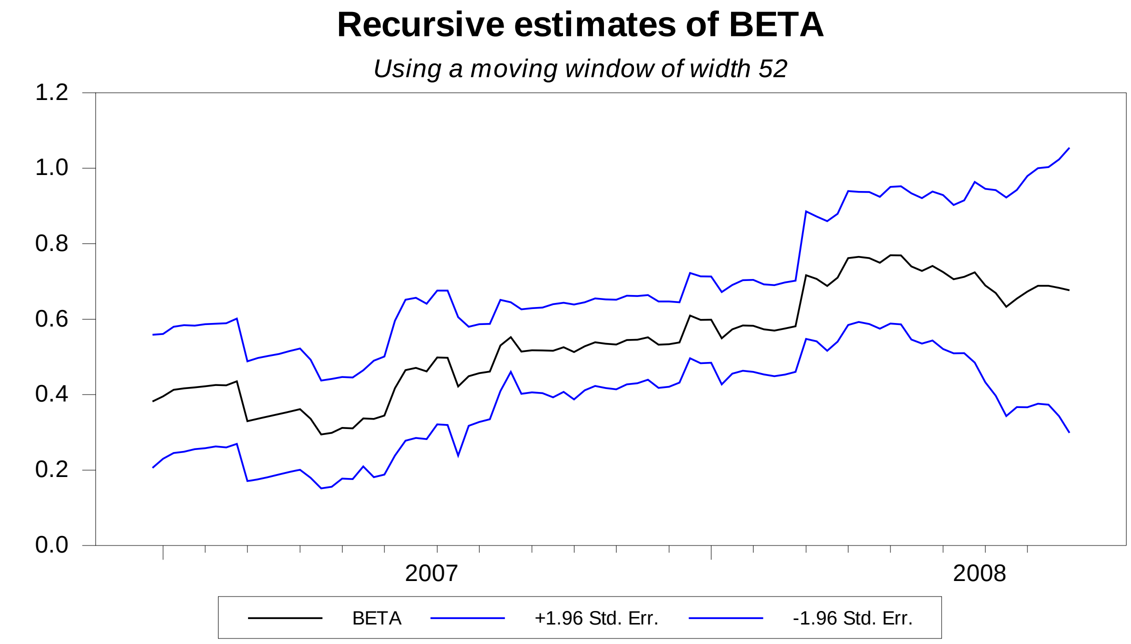

Figure A17: P4: 2005:12:23 – 2008:09:01 Equal Weights

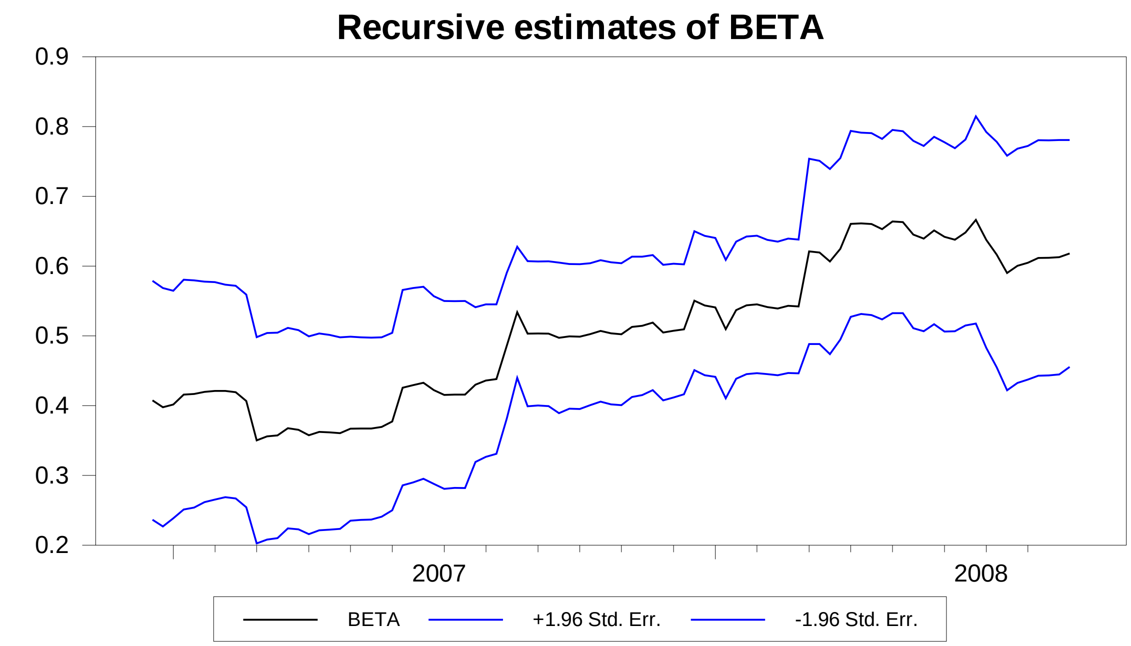

Figure A18: P4: 2005:12:23 – 2008:09:01 Equal Weights

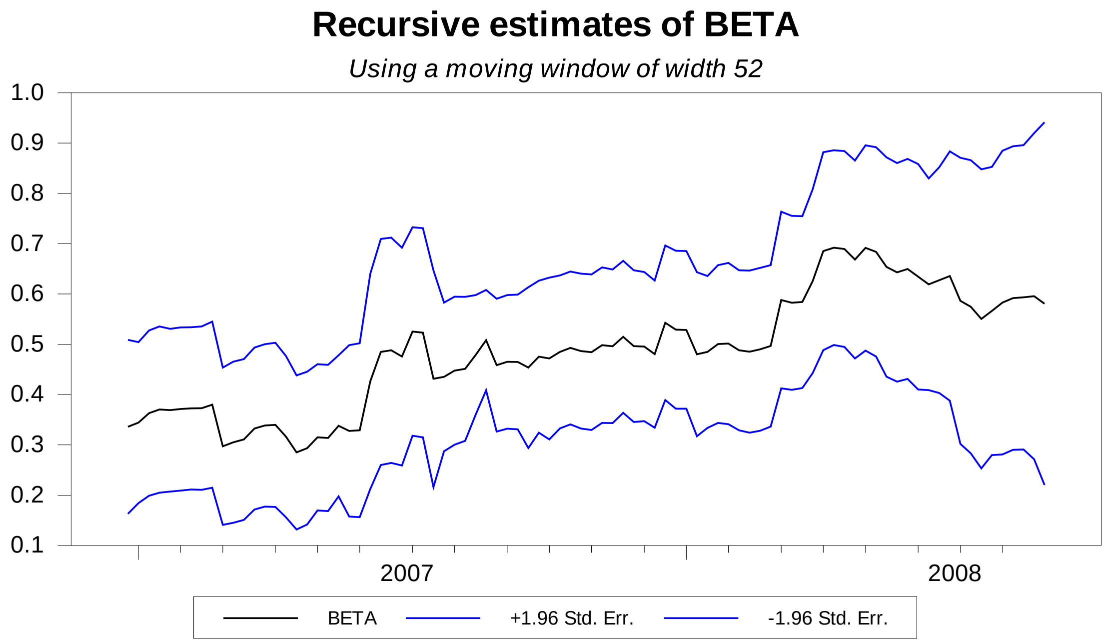

Figure A19: P4: 2005:12:23 – 2008:09:01 Value Weights

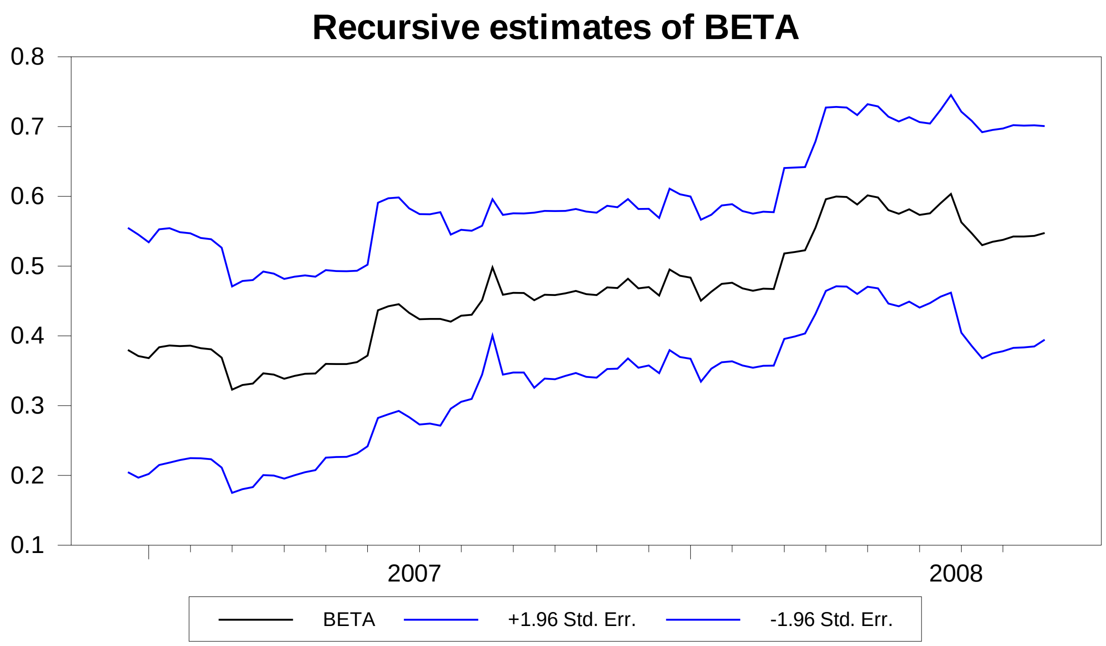

Figure A20: P4: 2005:12:23 – 2008:09:01 Value Weights

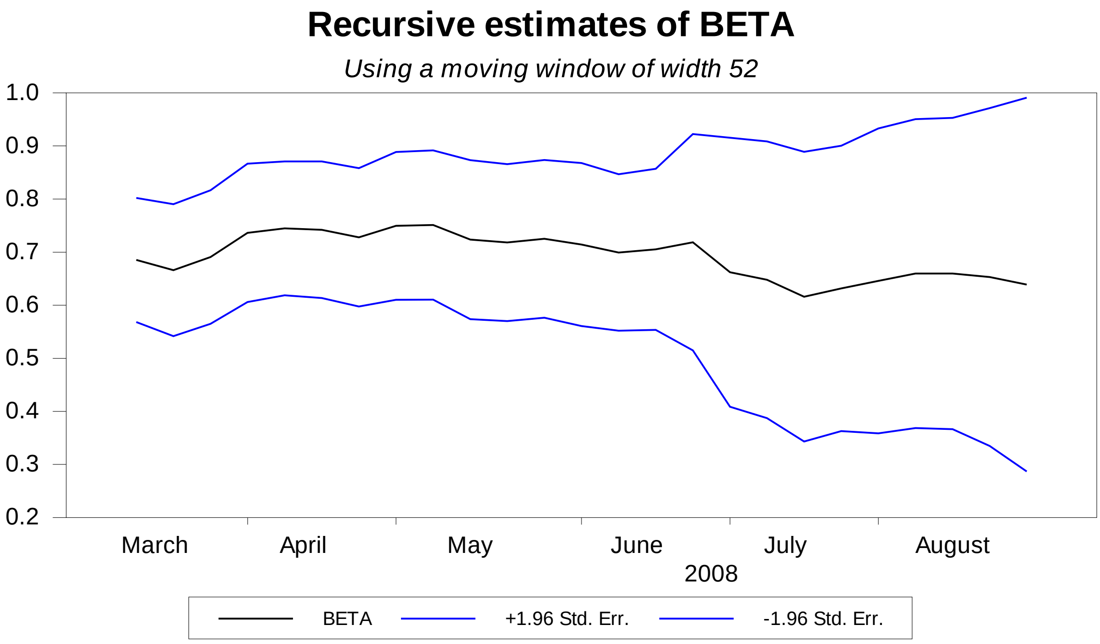

Figure A21: P5: 2007:03:09 – 2008:09:01 Equal Weights

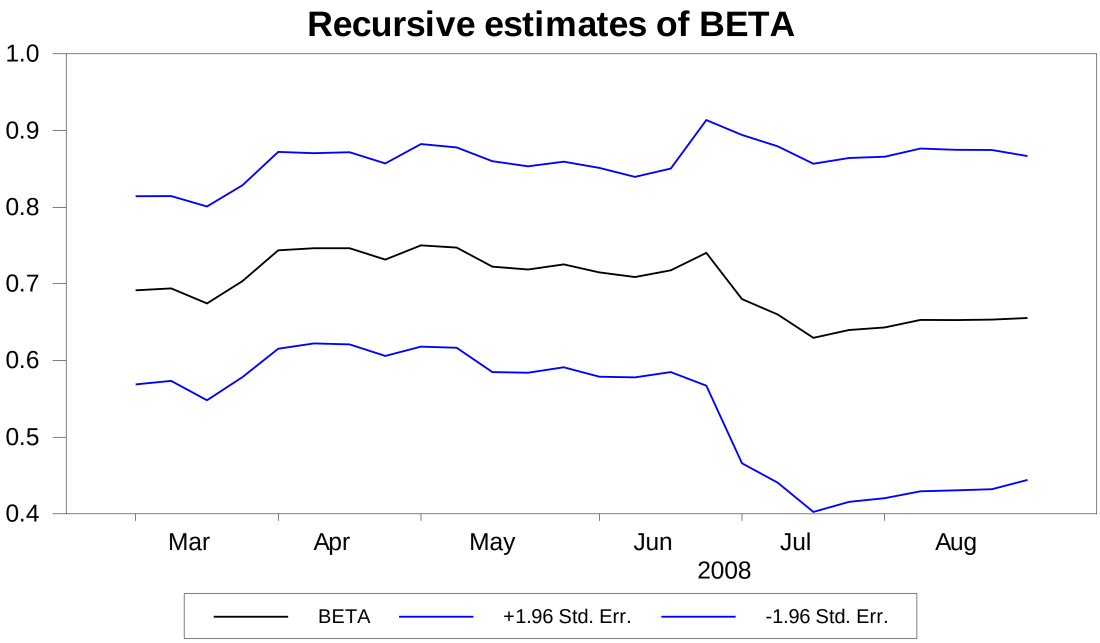

Figure A22: P5: 2007:03:09 – 2008:09:01 Equal Weights

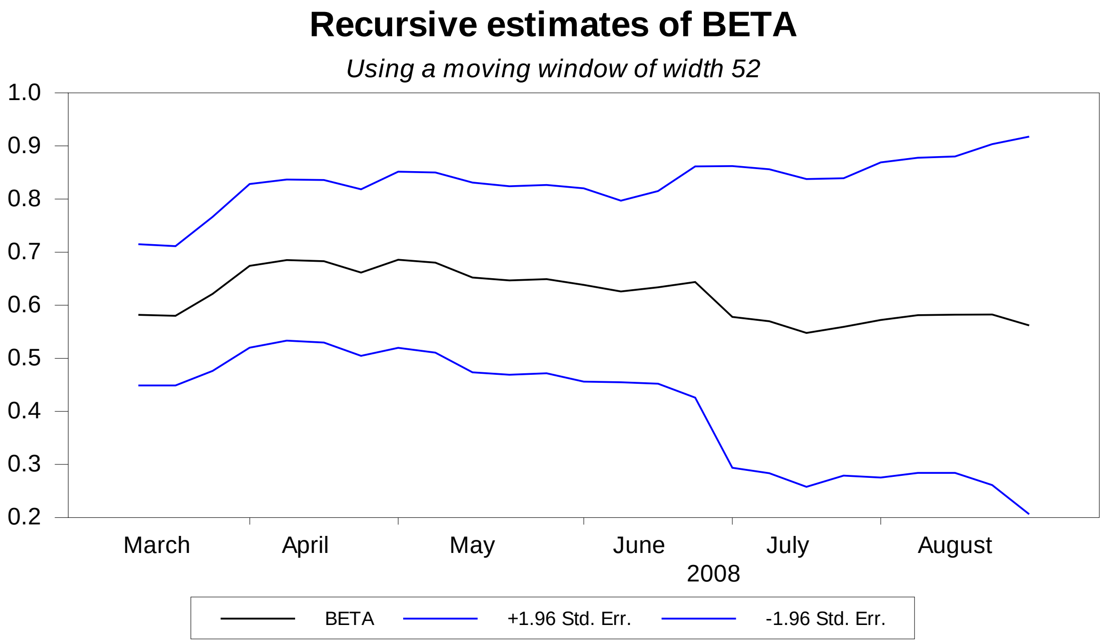

Figure A23: P5: 2007:03:09 – 2008:09:01 Value Weights

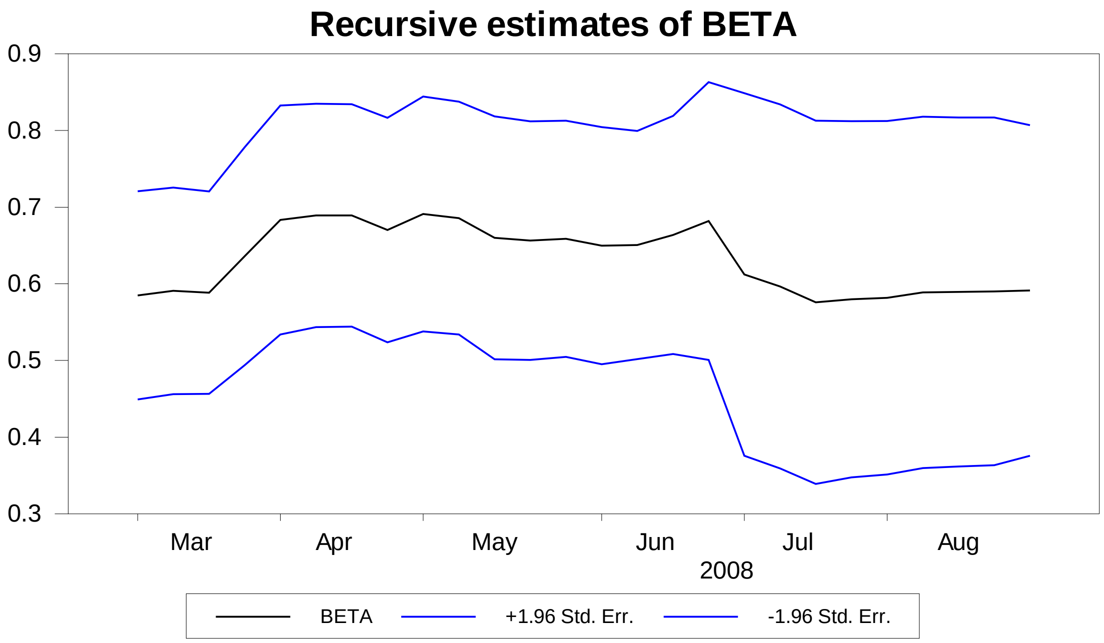

Figure A24: P5: 2007:03:09 – 2008:09:01 Value Weights

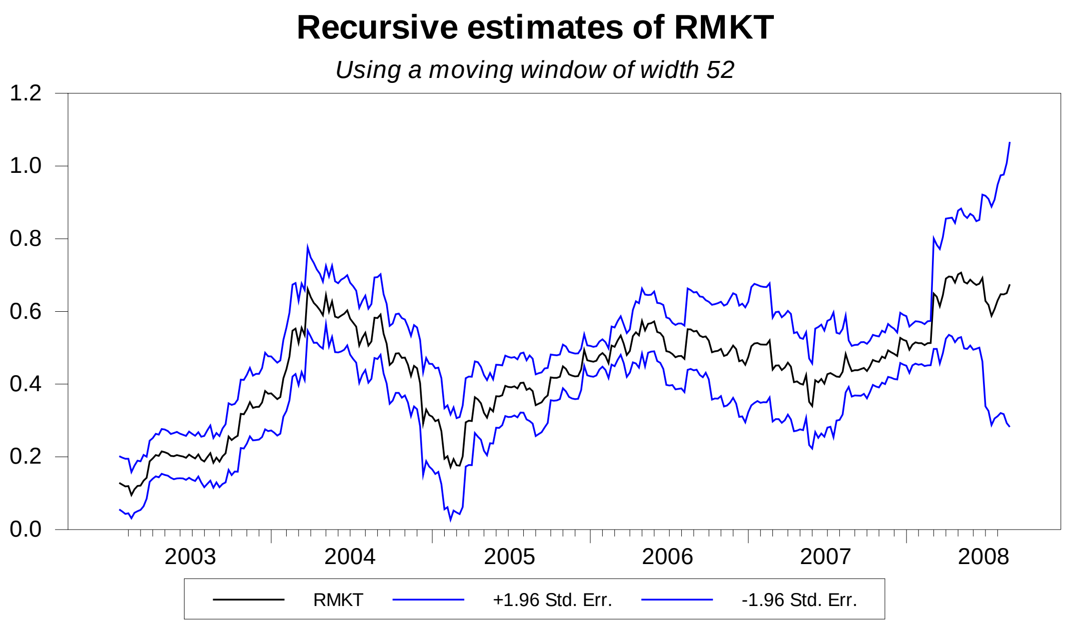

Figure A25: Average Portfolio 2002:01:01 - 2008:09:01

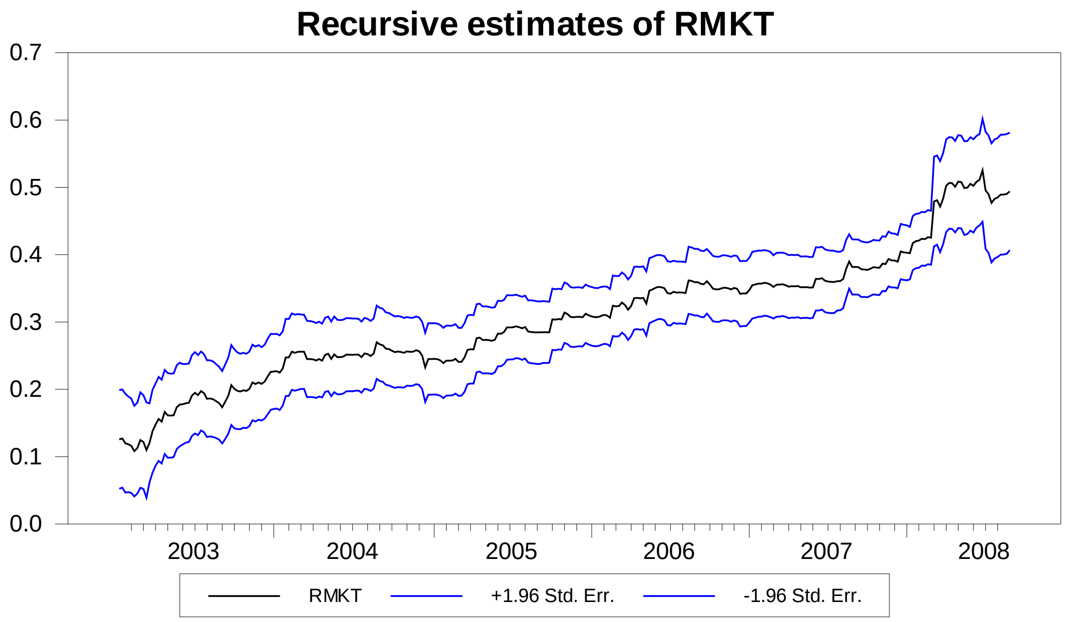

Figure A26: Average Portfolio 2002:01:01 - 2008:09:01

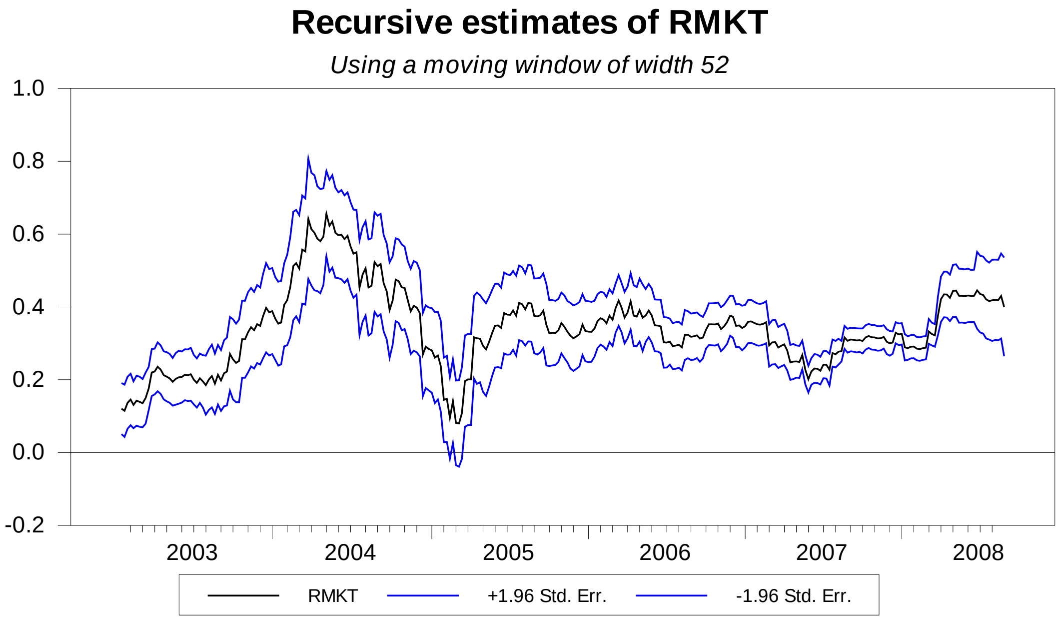

Figure A27: Median Portfolio 2002:01:01 - 2008:09:01

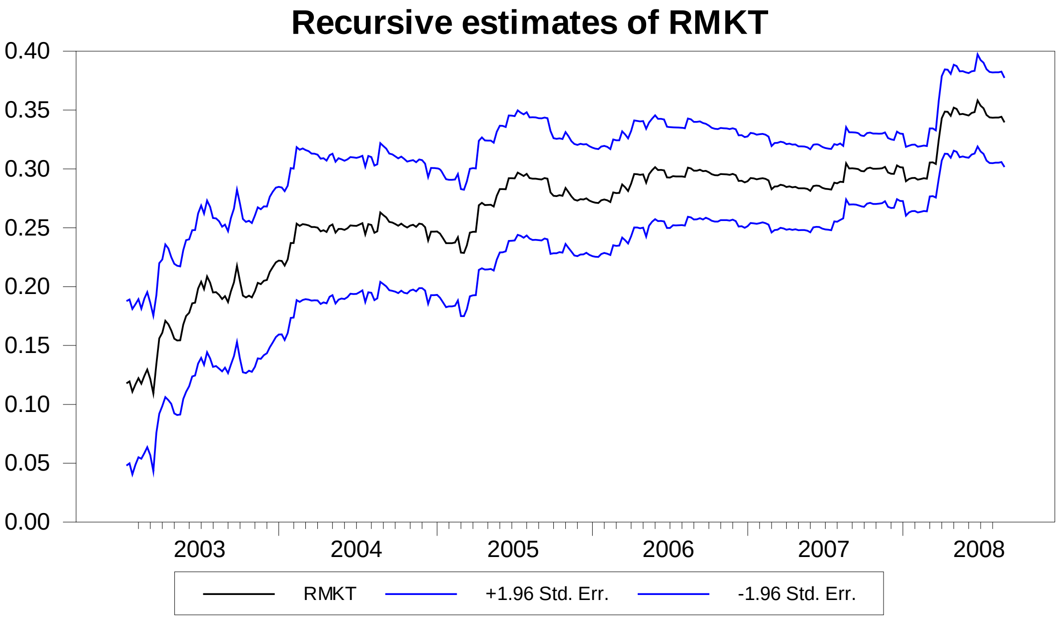

Figure A28: Median Portfolio 2002:01:01 - 2008:09:01

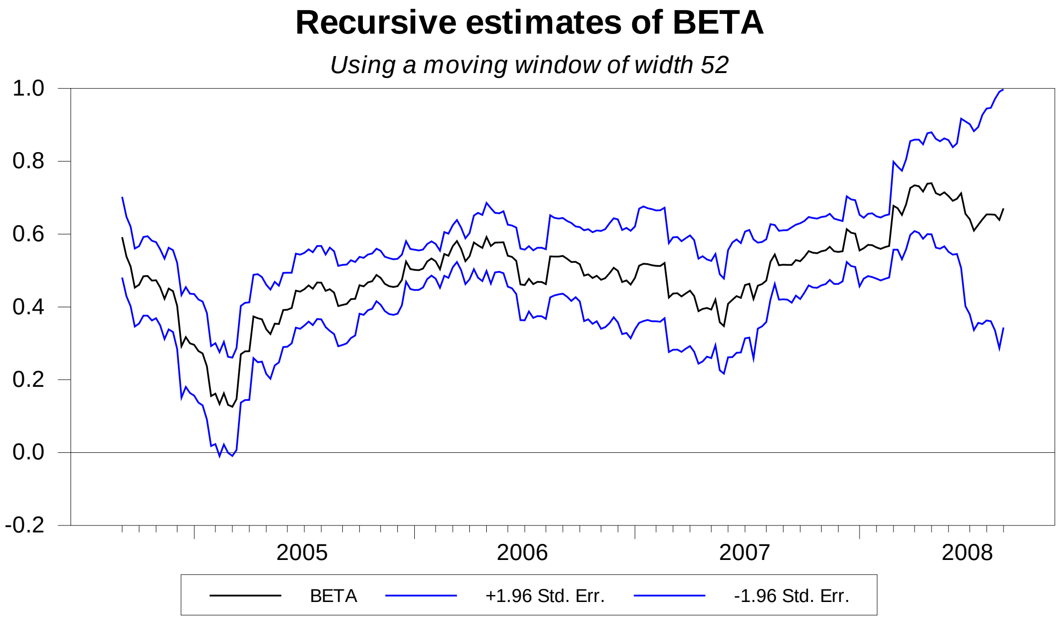

Figure A29: Average Portfolio 2003:09:01 - 2008:09:01

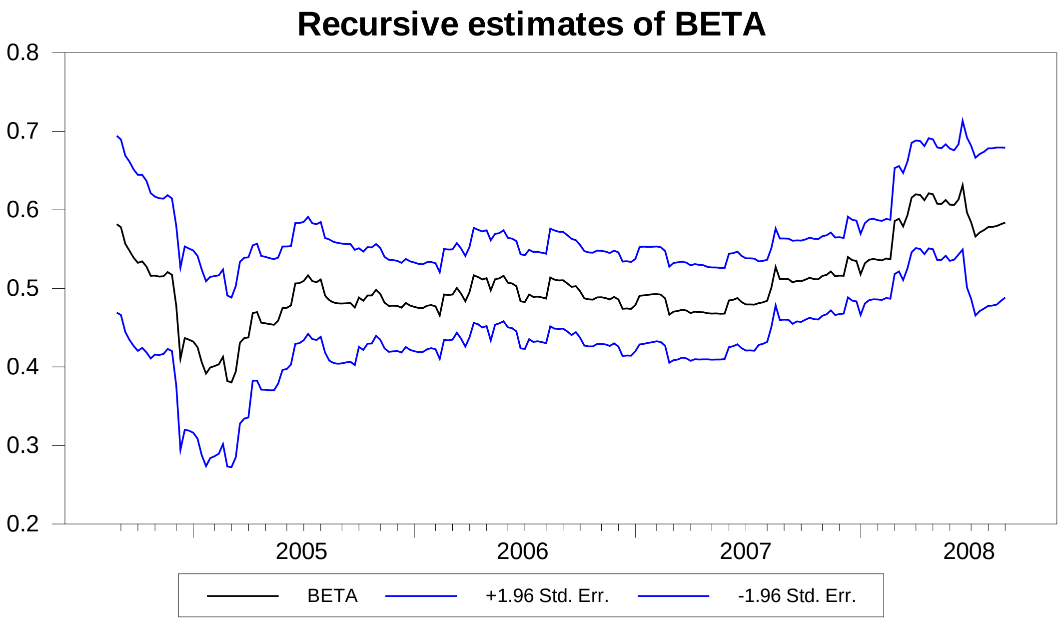

Figure A30: Average Portfolio 2003:09:01 - 2008:09:01

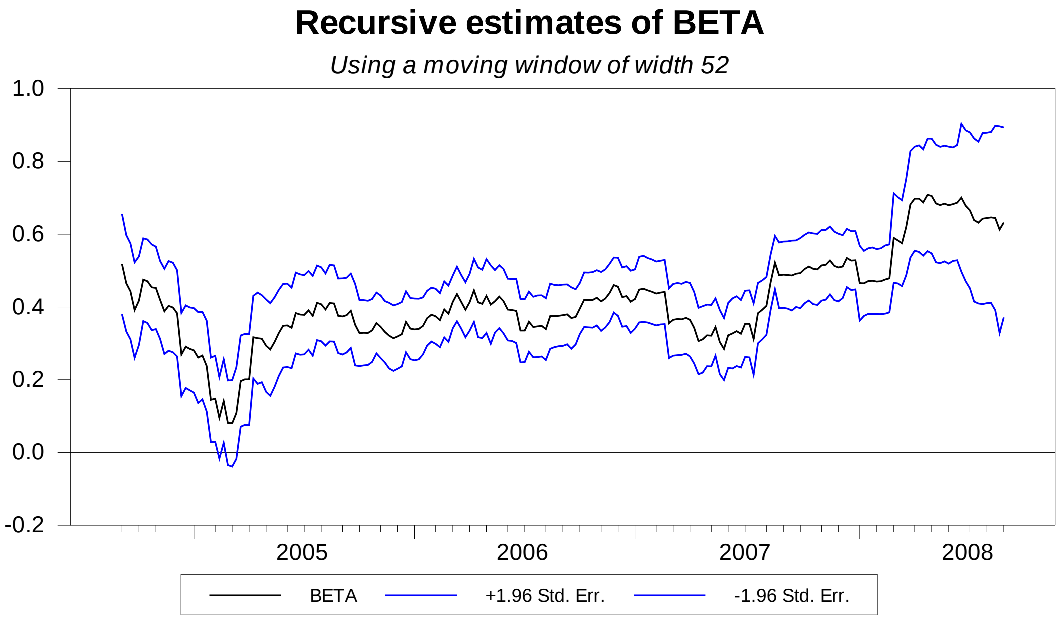

Figure A31: Median Portfolio 2003:09:01 - 2008:09:01

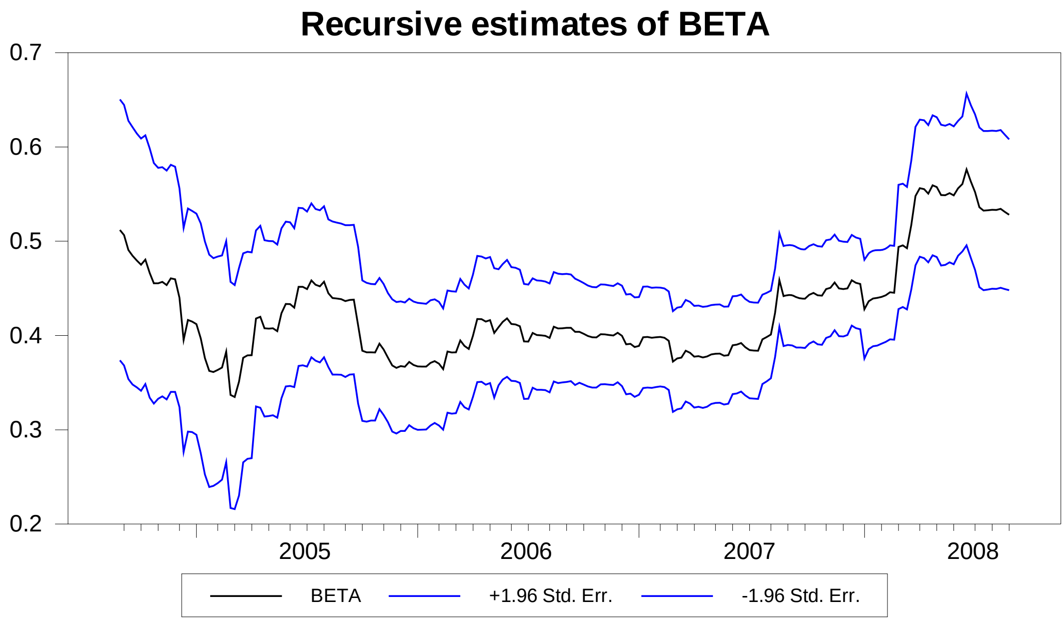

Figure A32: Median Portfolio 2003:09:01 - 2008:09:01

Recursive Least Squares Estimates:

Monthly Data

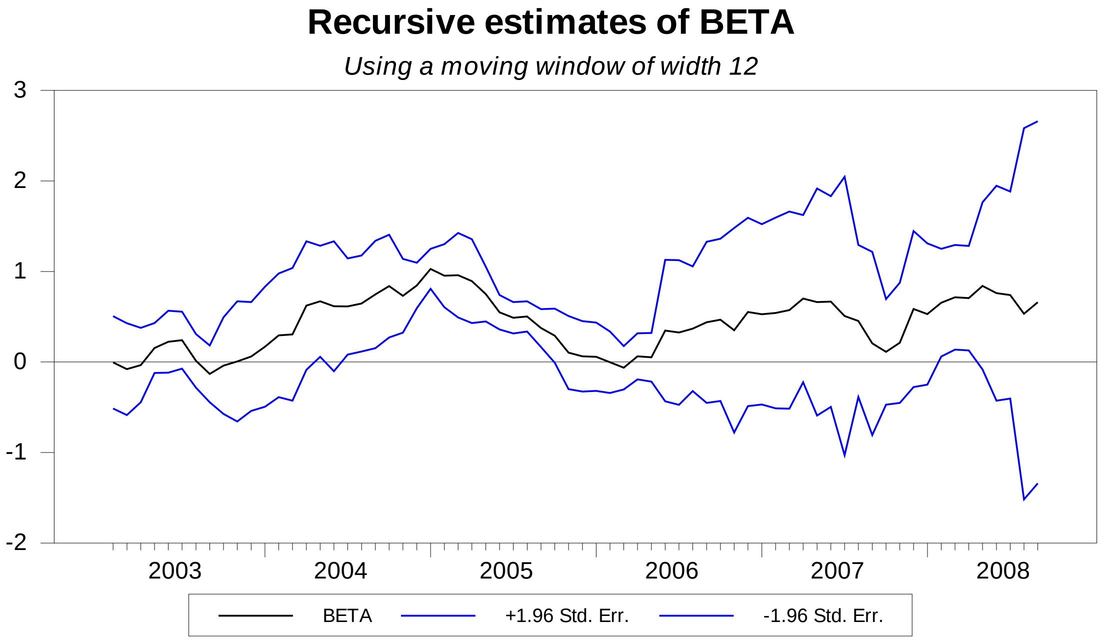

Figure A33: P1: 2002:01 – 2008:09, equal weights

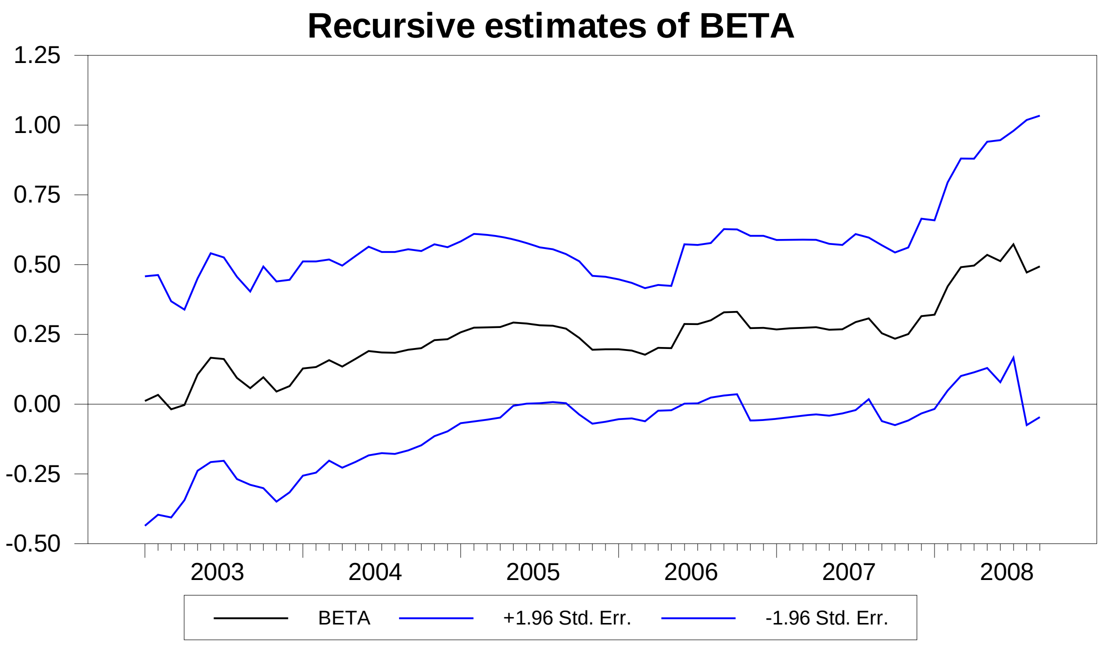

Figure A34: P1: 2002:01:01 – 2008:09:01, equal weights

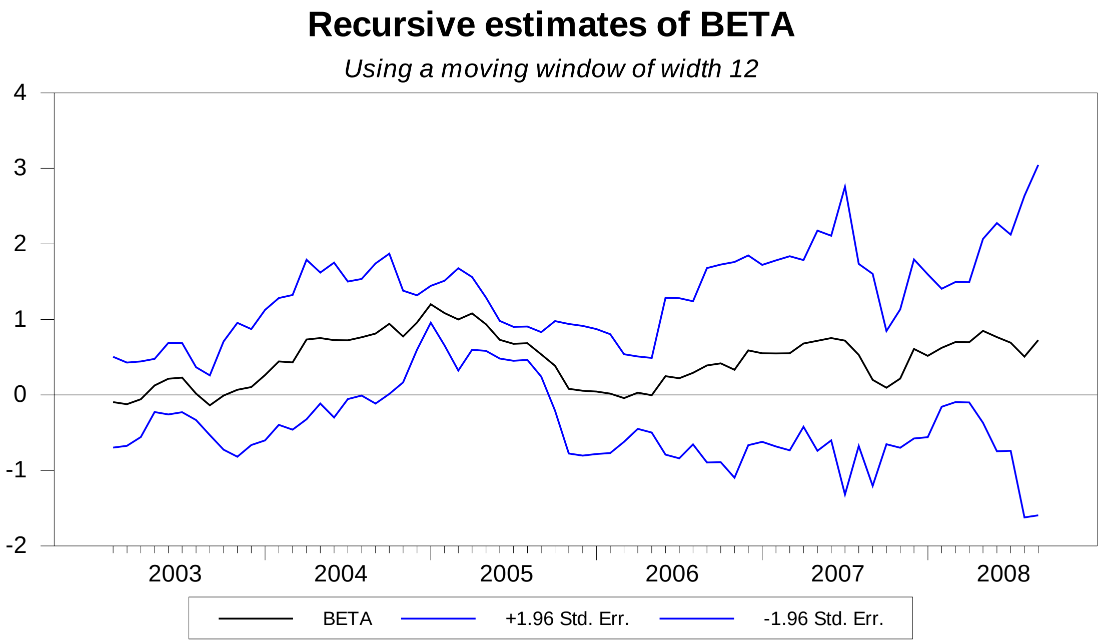

Figure A35: P1: 2002:01 – 2008:09, value weights

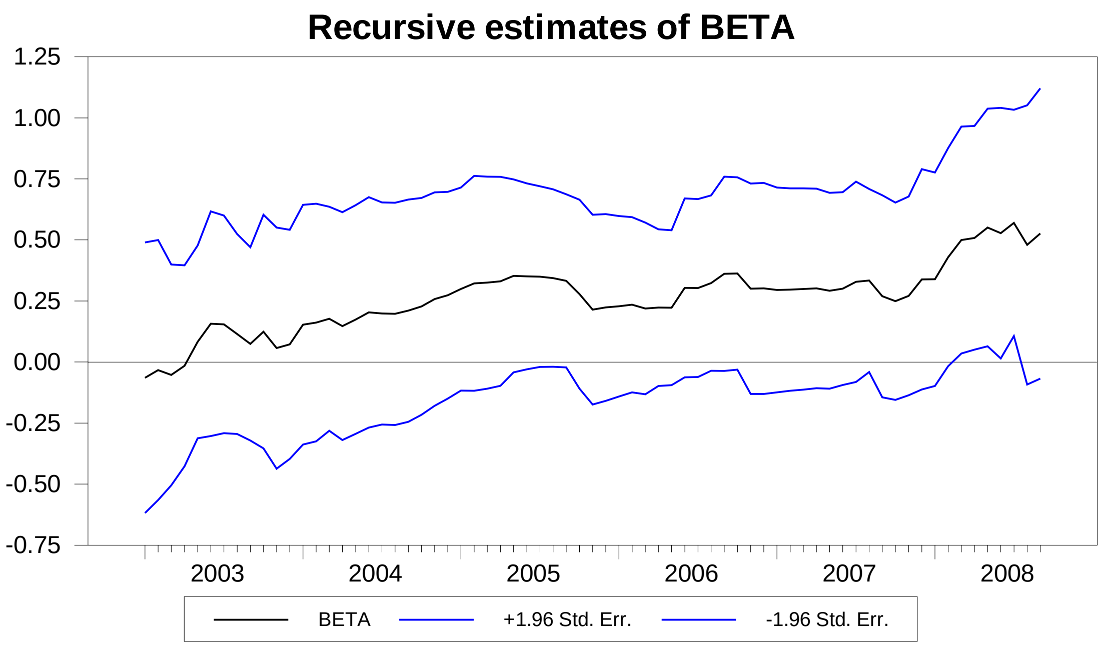

Figure A36: P1: 2002:01 – 2008:09, value weights

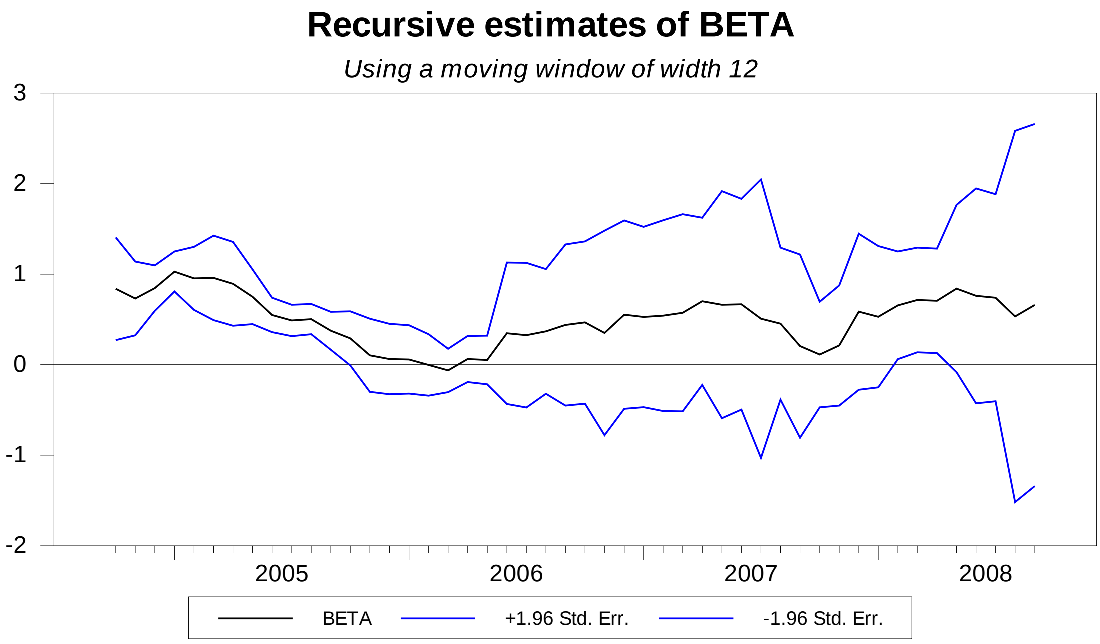

Figure A37: P1, 2003:09 - 2008:09, equal weights

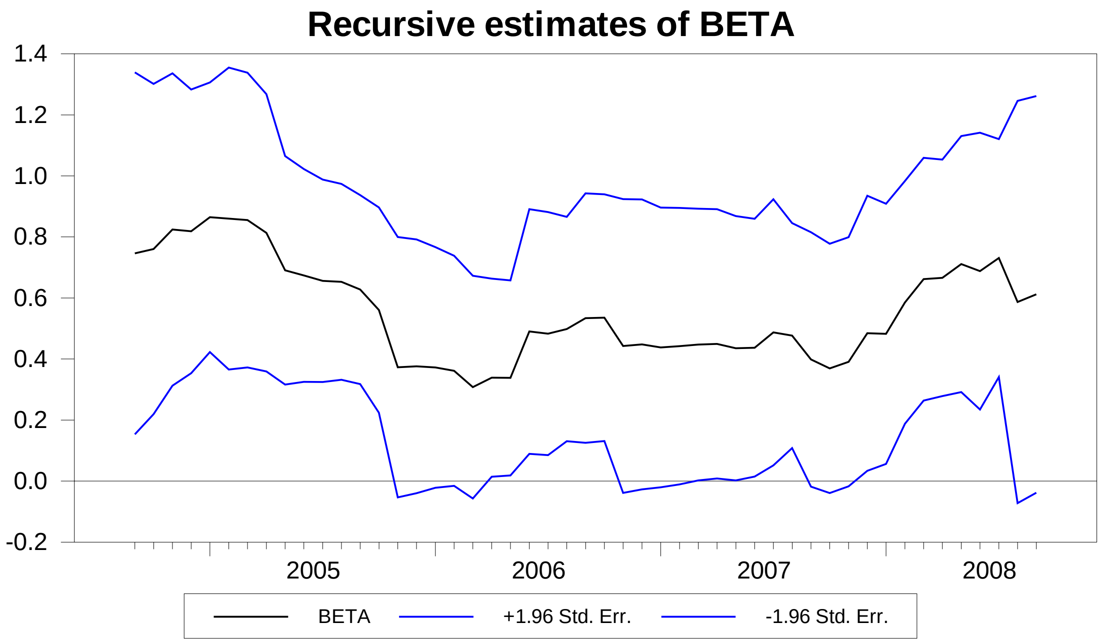

Figure A38: P1, 2003:09 - 2008:09, equal weights

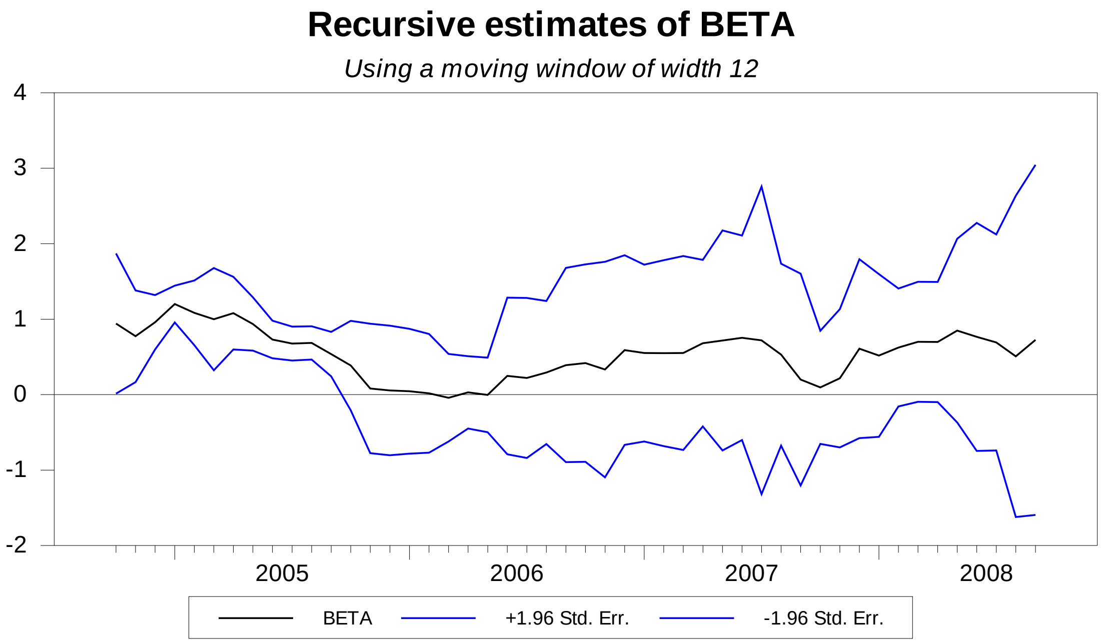

Figure A39: P1, 2003:09 - 2008:09, value weights

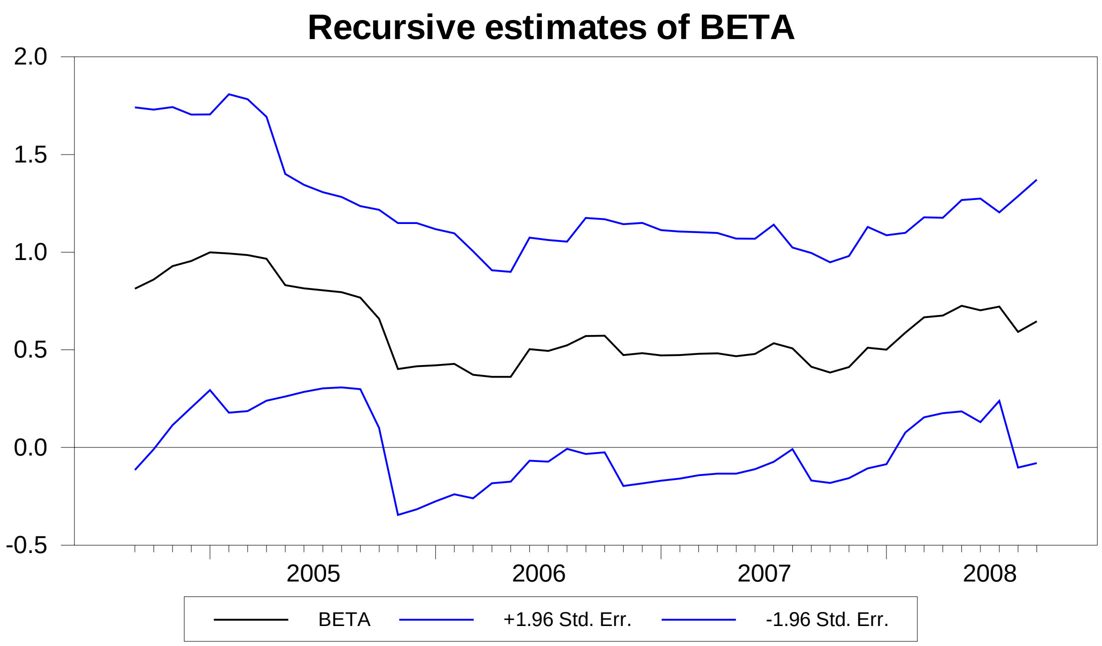

Figure A40: P1, 2003:09 - 2008:09, value weights

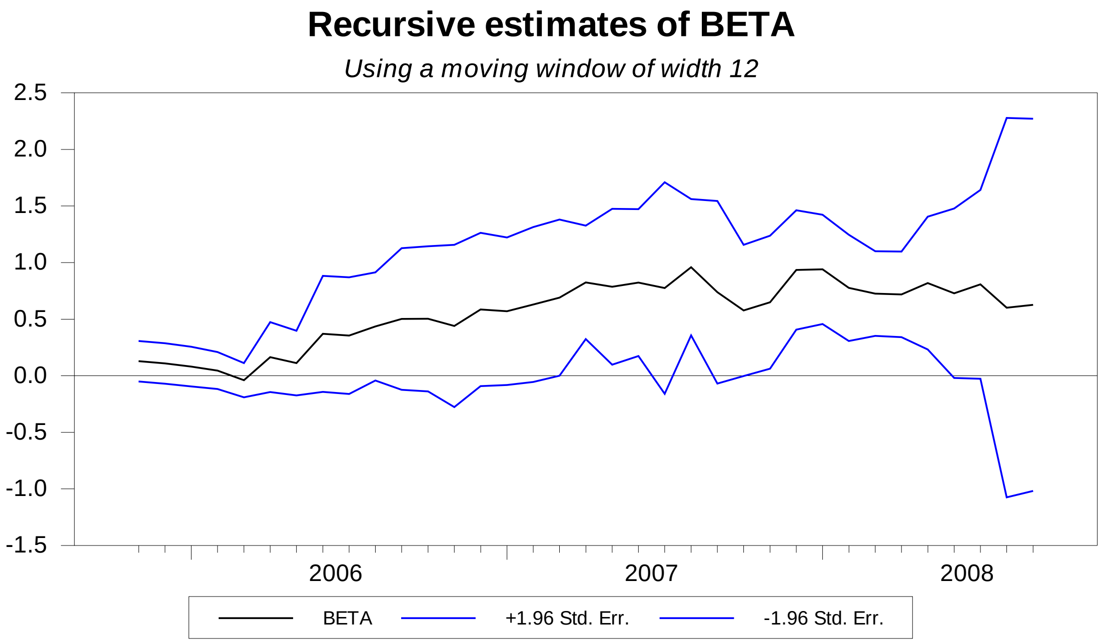

Figure A41: P2: 2004:08 – 2008:09 Equal Weights

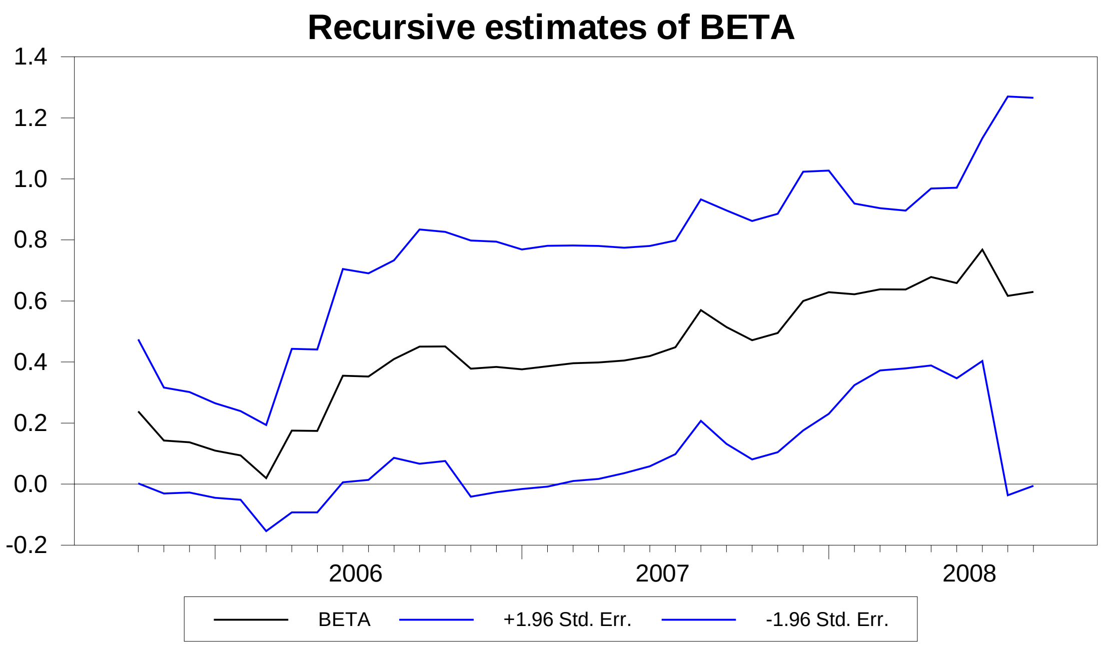

Figure A42: P2: 2004:08 – 2008:09 Equal Weights

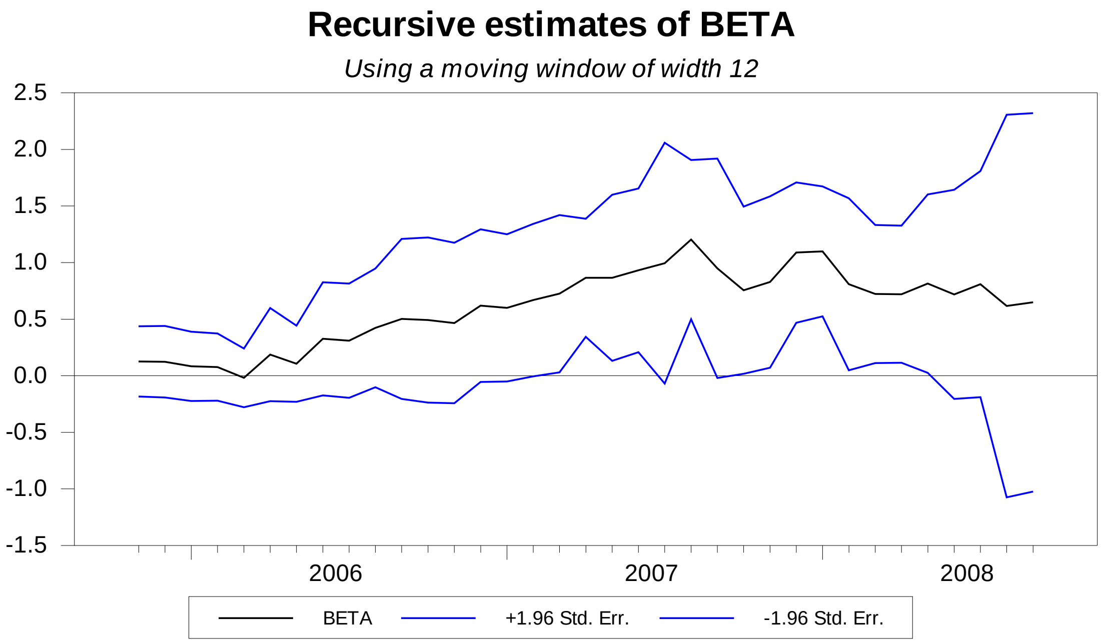

Figure A43: P2: 2004:08 – 2008:09 Value Weights

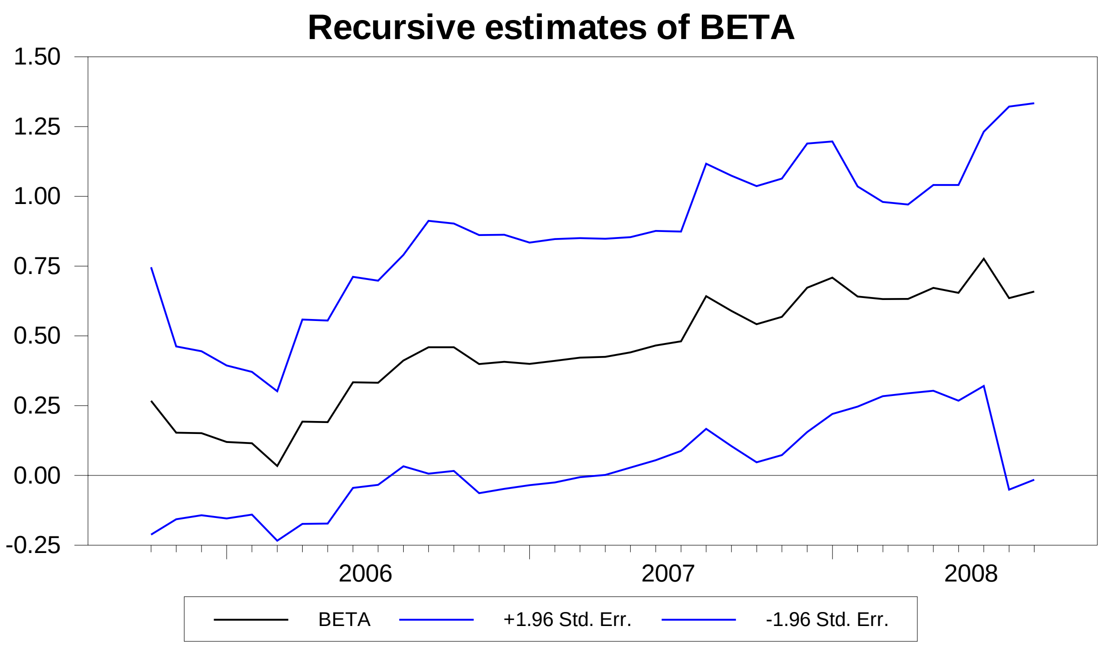

Figure A44: P2: 2004:08 – 2008:09 Value Weights

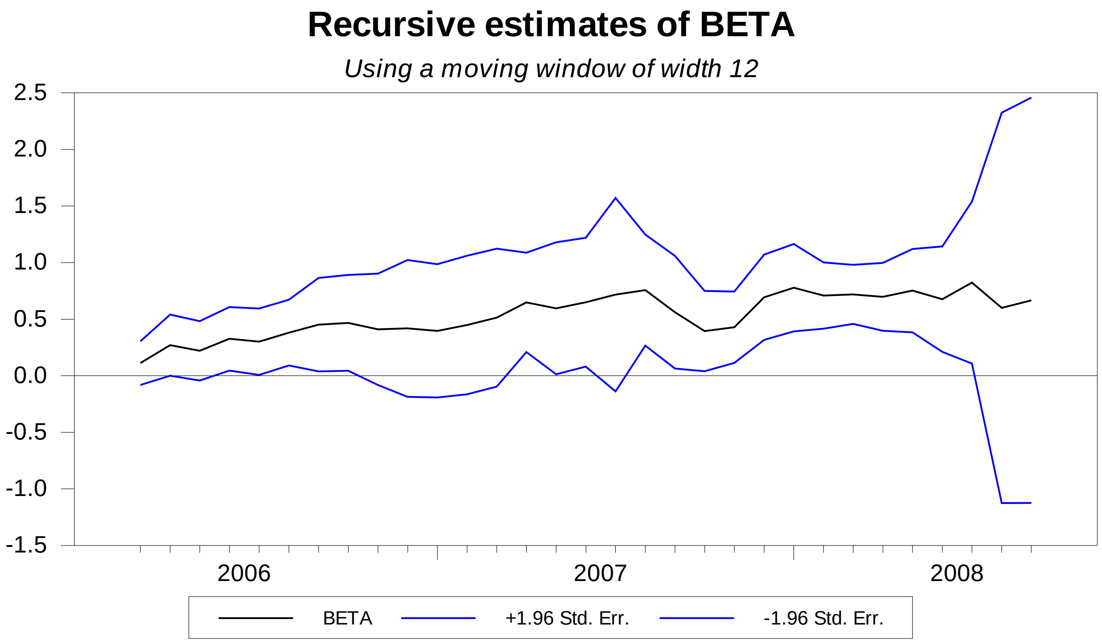

Figure A45: P3: 2004:12– 2008:09 Equal Weights

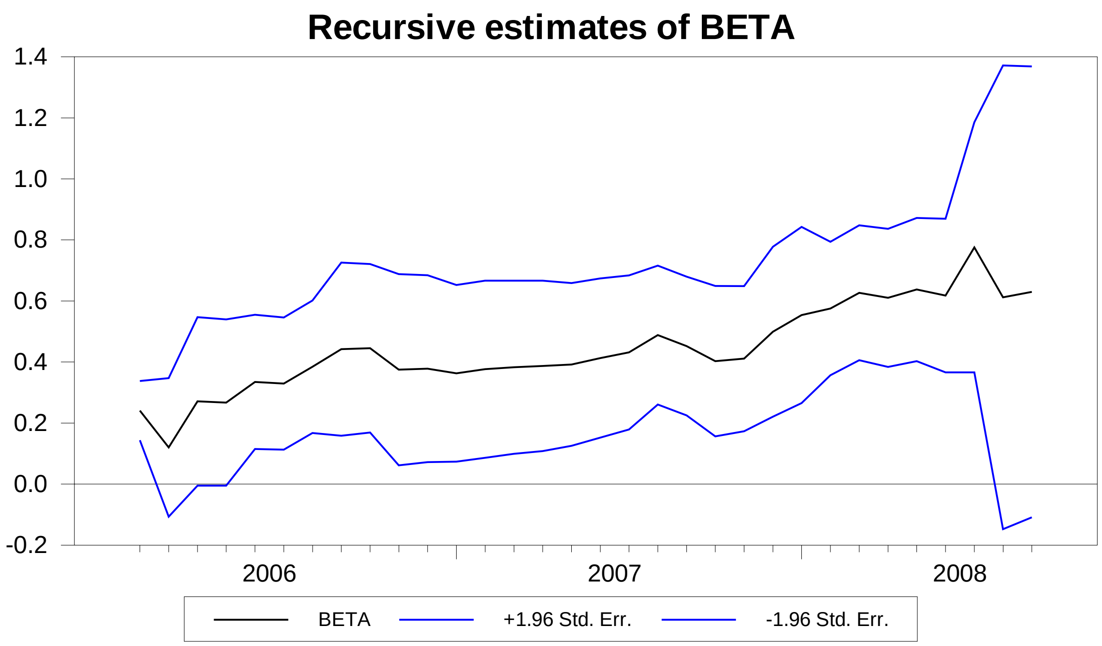

Figure A46: P3: 2004:12 – 2008:09 Equal Weights

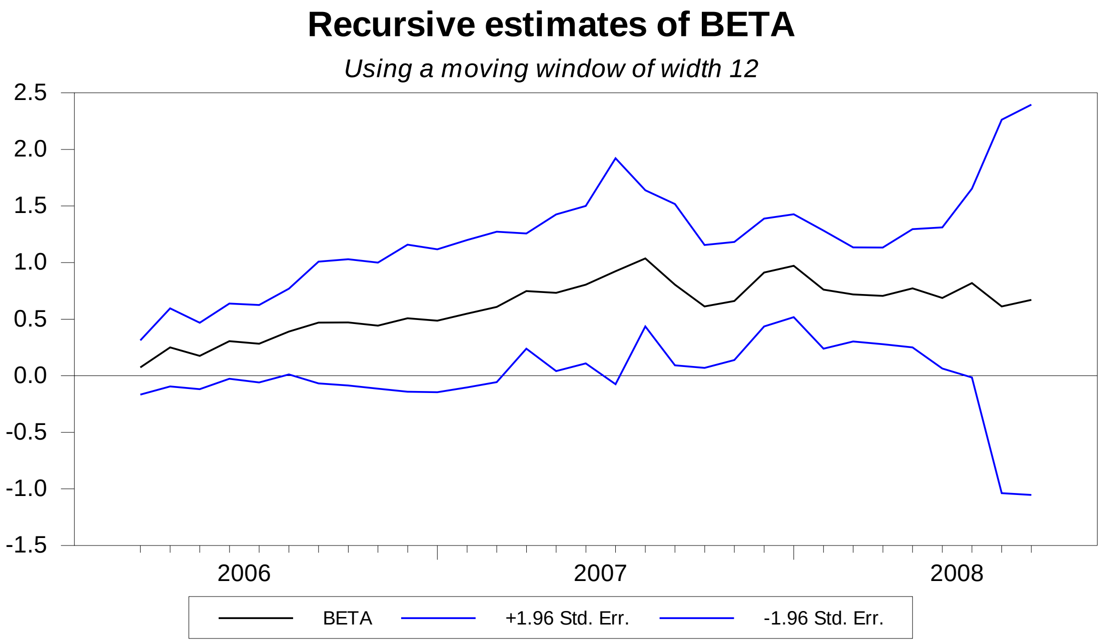

Figure A47: P3: 2004:12 – 2008:09 Value Weights

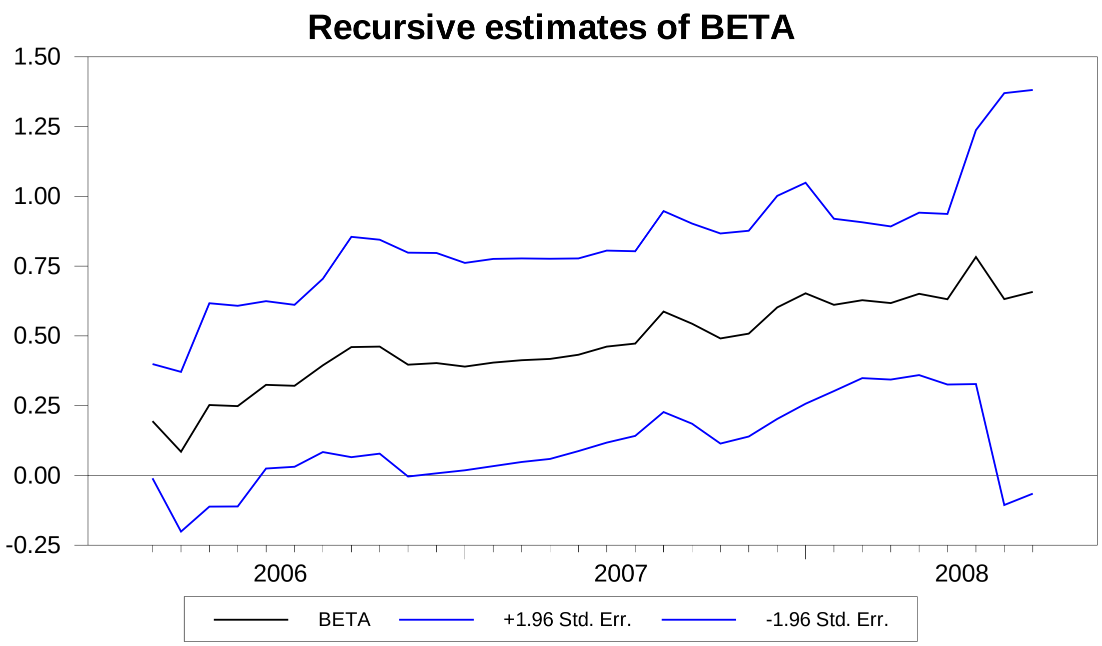

Figure A48: P3: 2004:12 – 2008:09 Value Weights

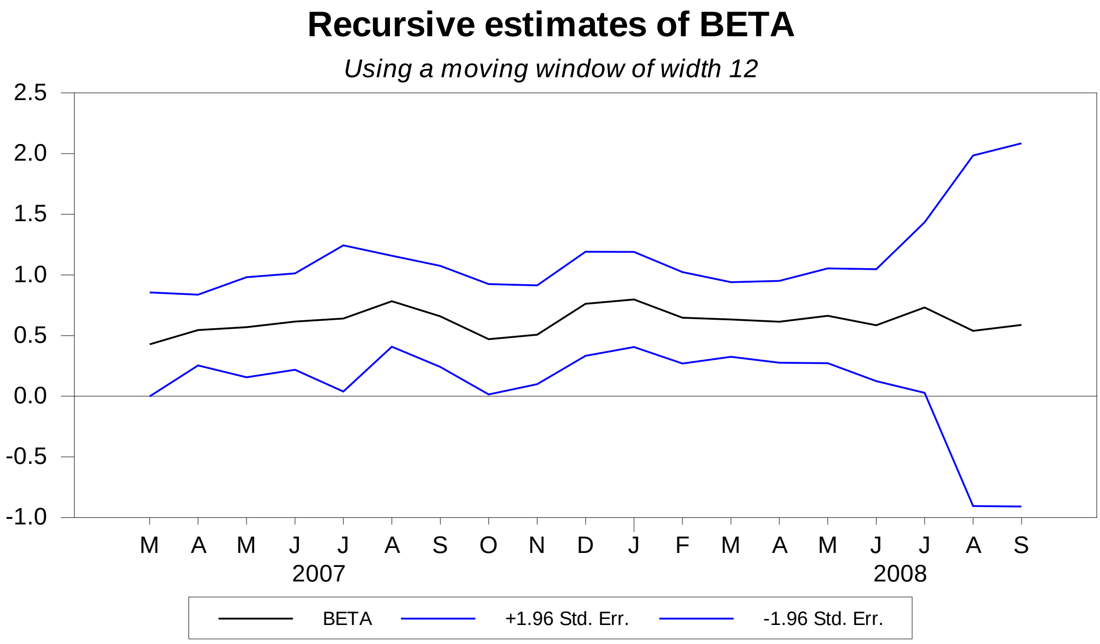

Figure A49: P4: 2006:01 – 2008:09 Equal Weights

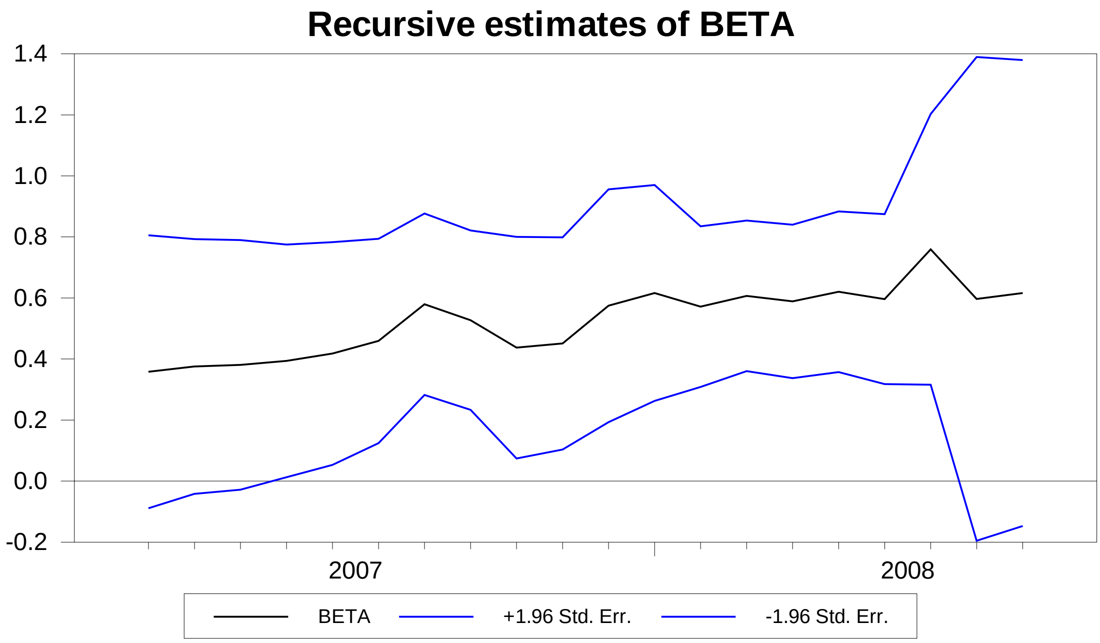

Figure A50: P4: 2006:01 – 2008:09 Equal Weights

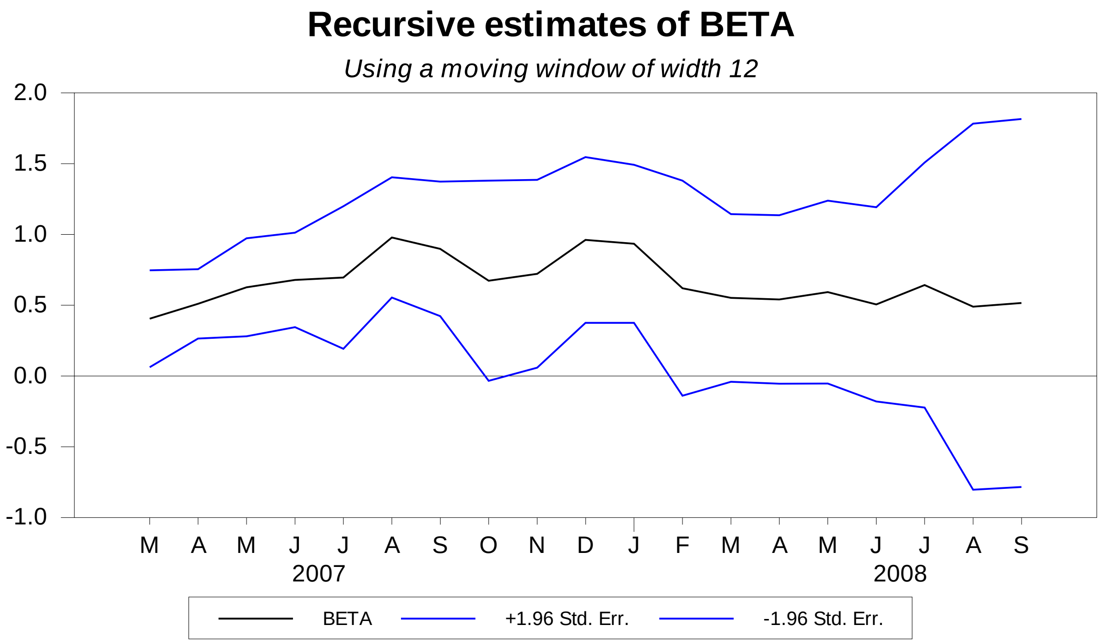

Figure A51: P4: 2006:1 – 2008:09 Value Weights

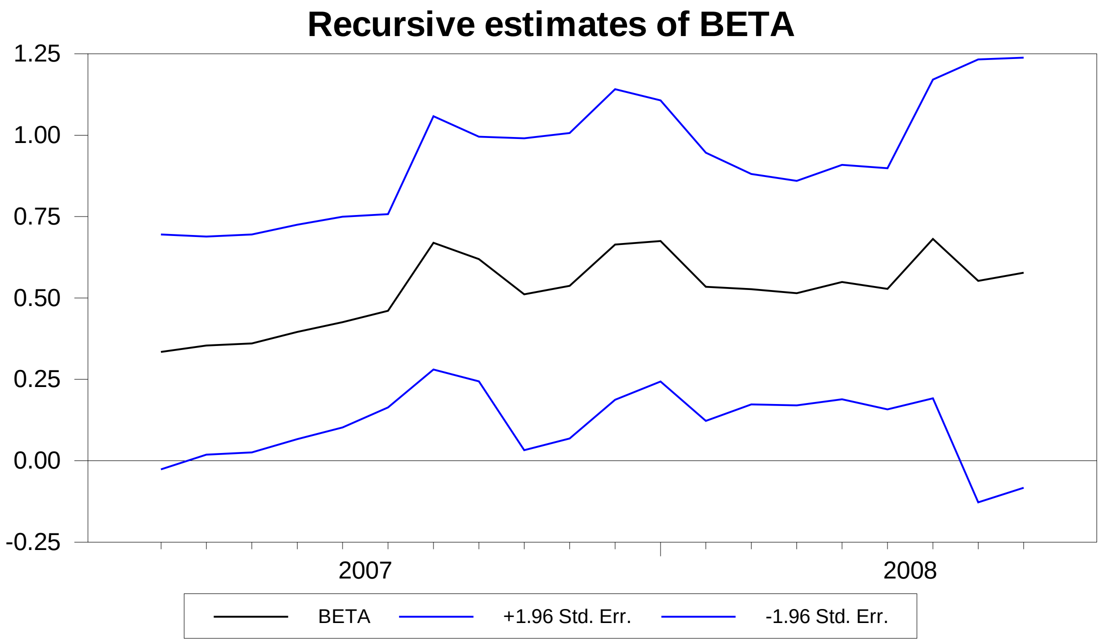

Figure A52: P4: 2006:01 – 2008:09 Value Weights

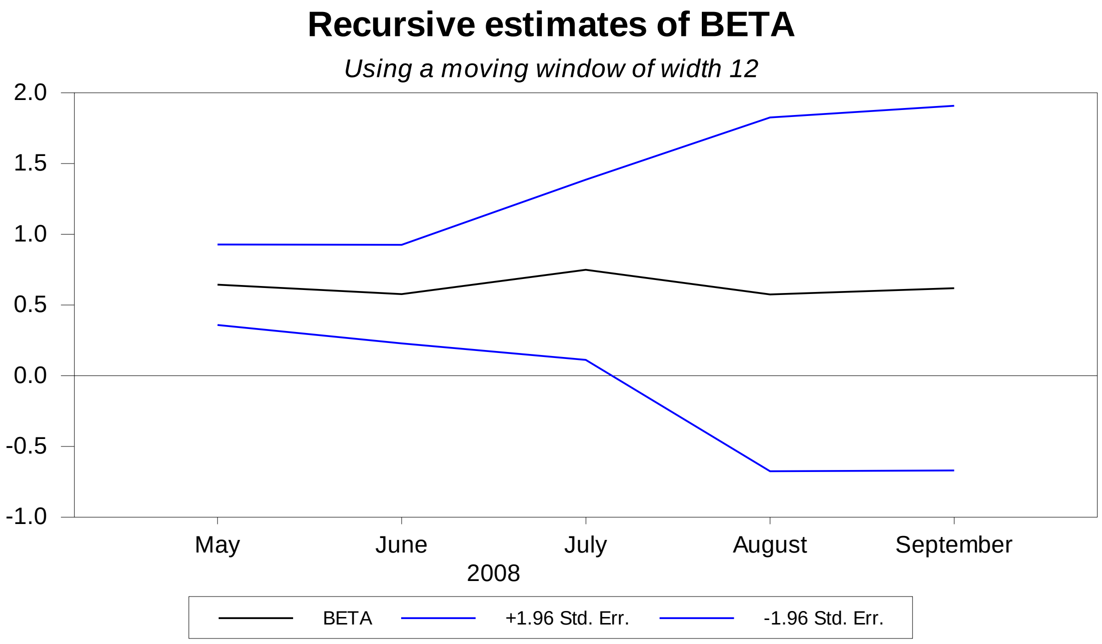

Figure A53: P5: 2007:03 – 2008:09 Equal Weights

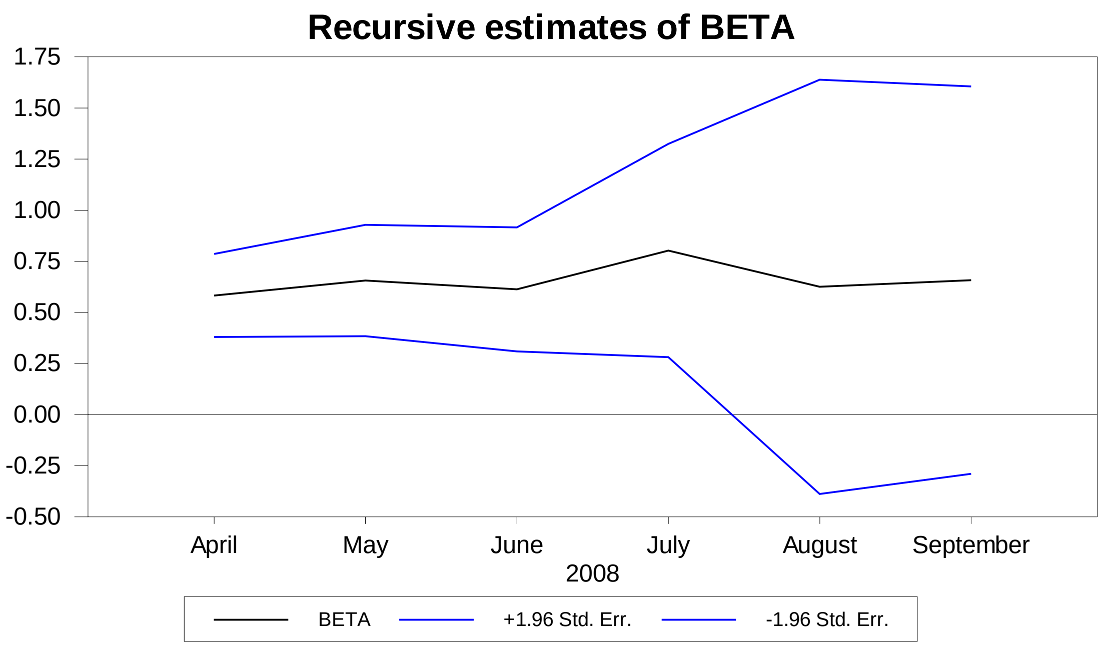

Figure A54: P5: 2007:03 – 2008:09 Equal Weights

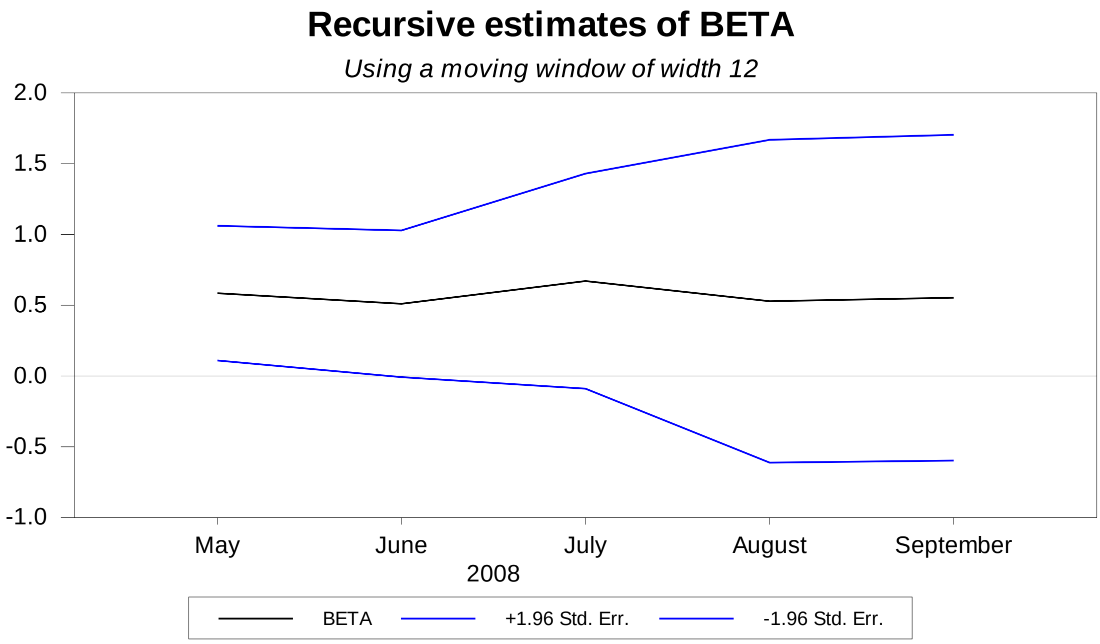

Figure A55: P5: 2007:03 – 2008:09 Value Weights

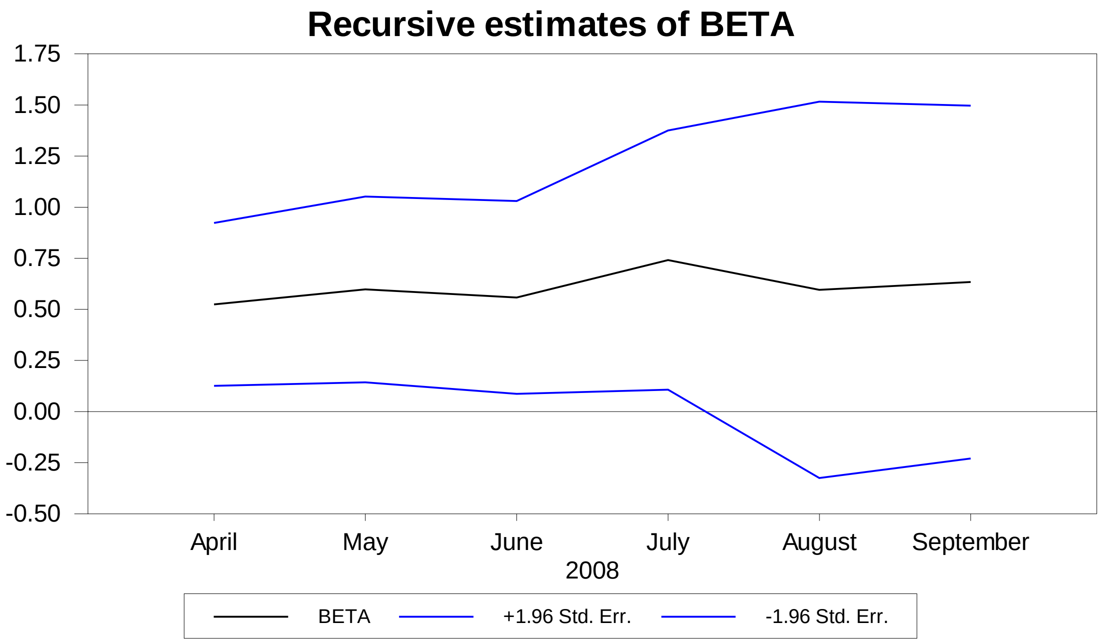

Figure A56: P5: 2007:03 – 2008:09 Value Weights

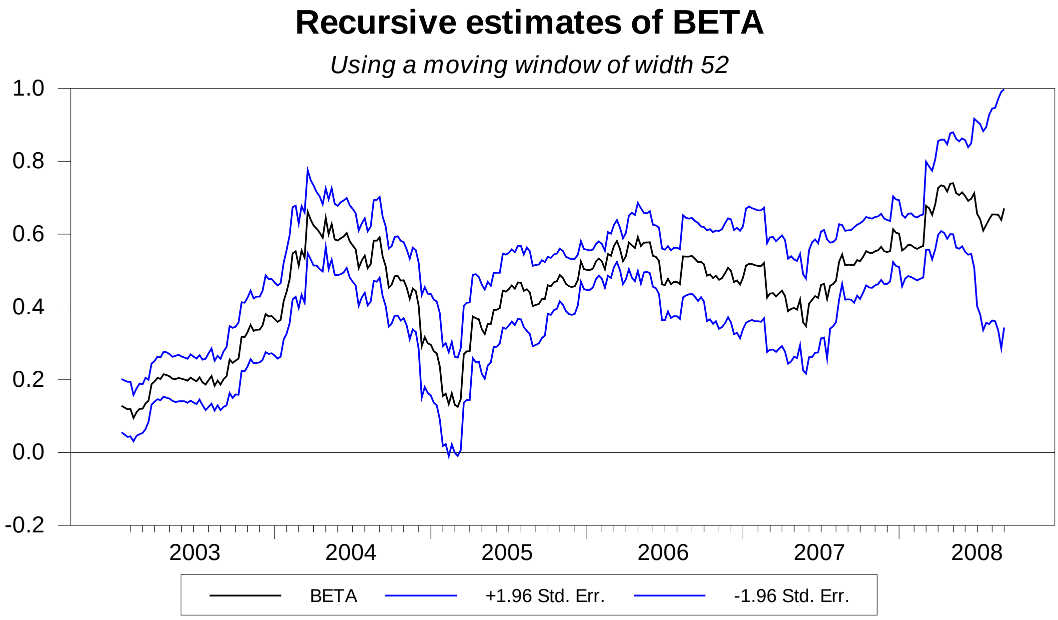

Figure A57: Average Portfolio 2002:01 - 2008:09

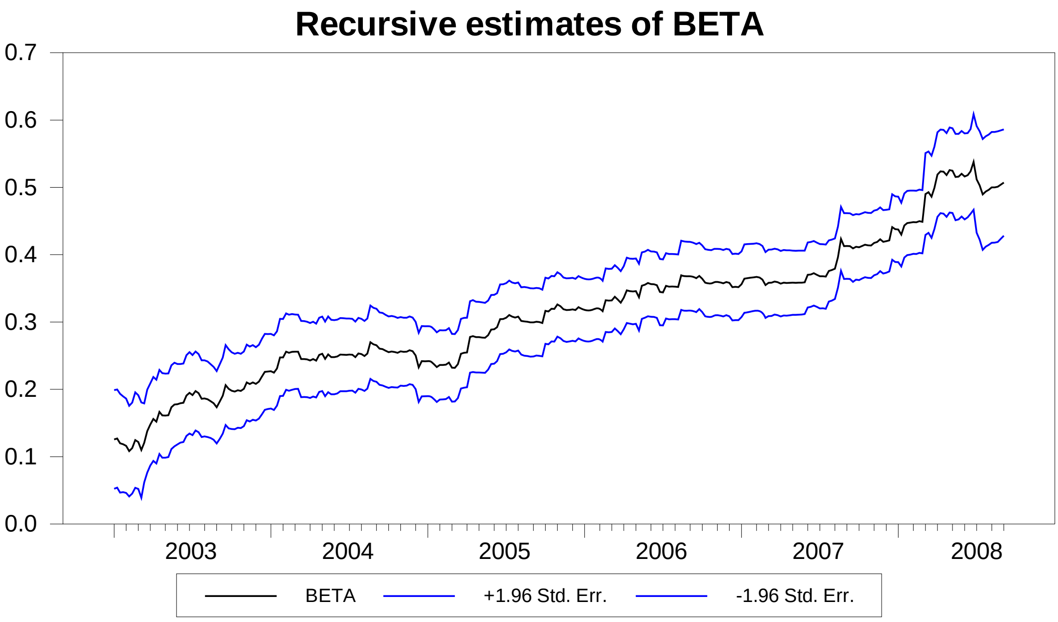

Figure A58: Average Portfolio 2002:01 - 2008:09

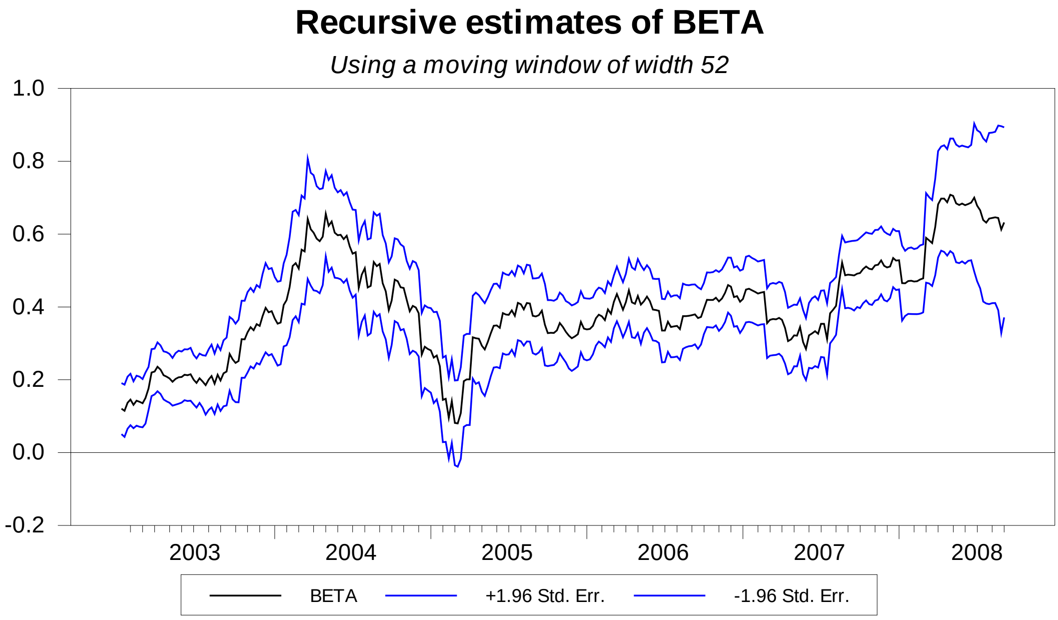

Figure A59: Median Portfolio 2002:01 - 2008:09

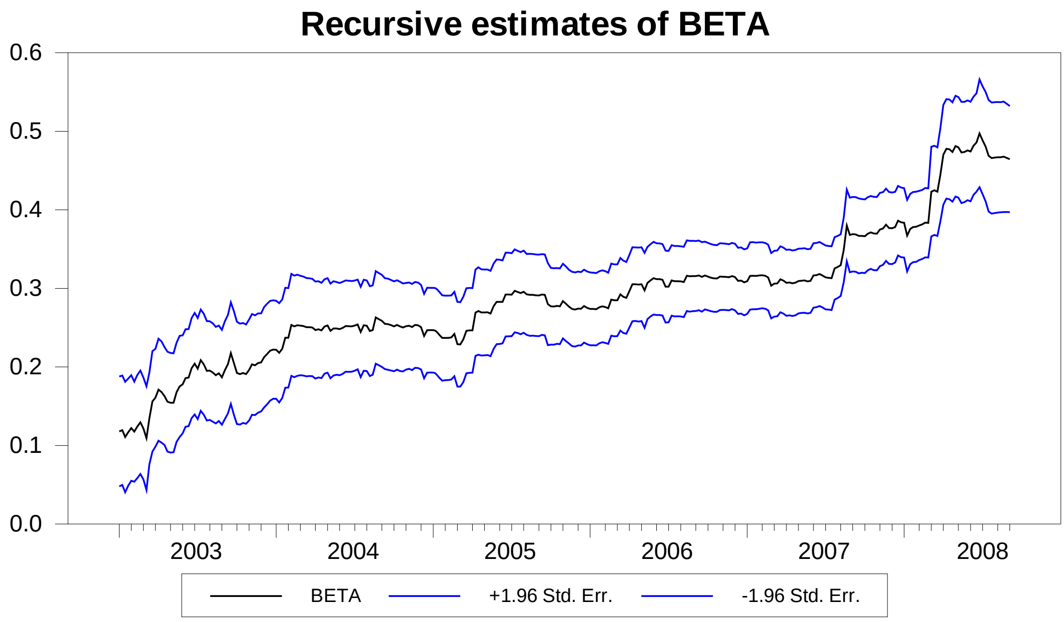

Figure A60: Median Portfolio 2002:01 - 2008:09

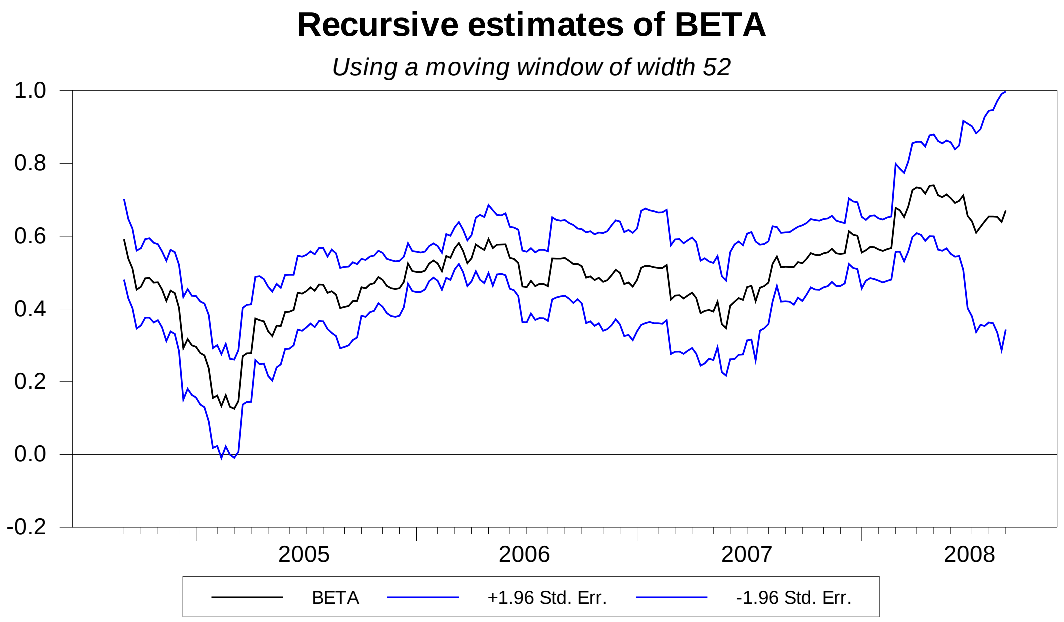

Figure A61: Average Portfolio 2003:09 - 2008:09

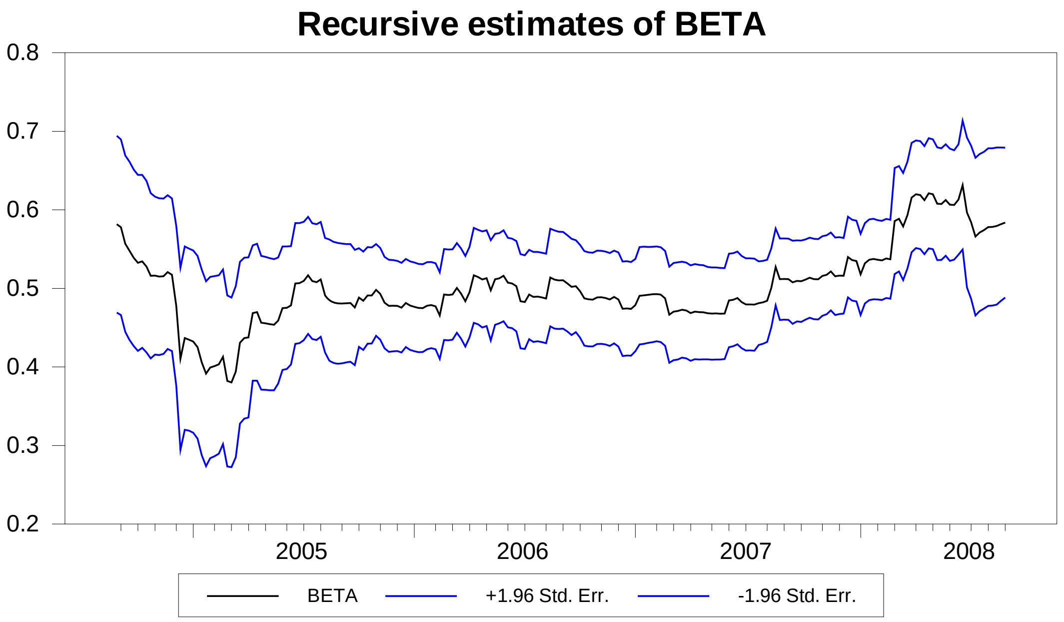

Figure A62: Average Portfolio 2003:09 - 2008:09

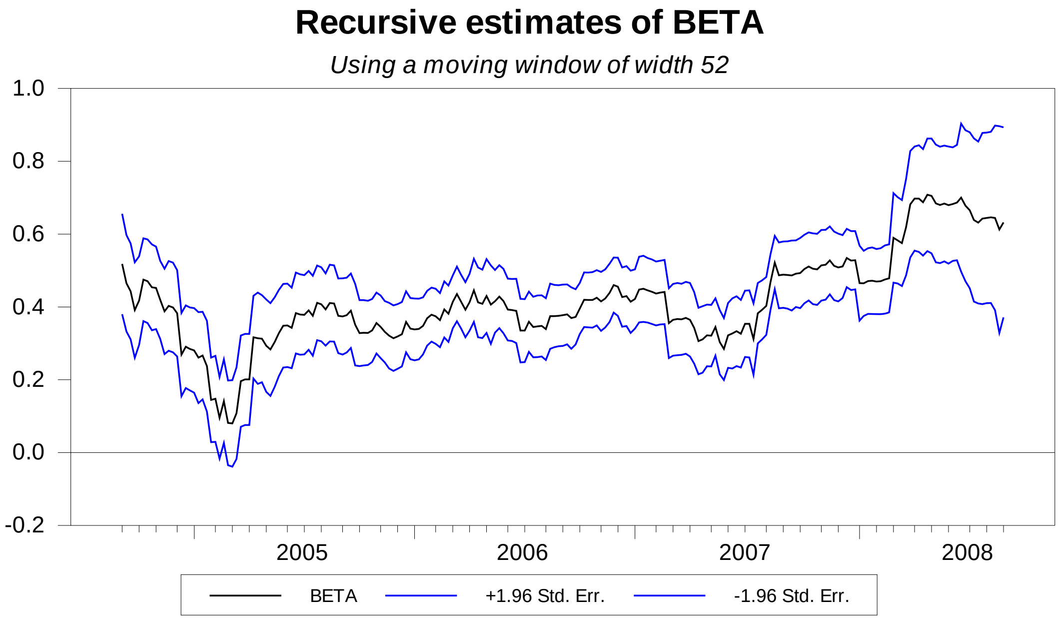

Figure A63: Median Portfolio 2003:09 - 2008:09

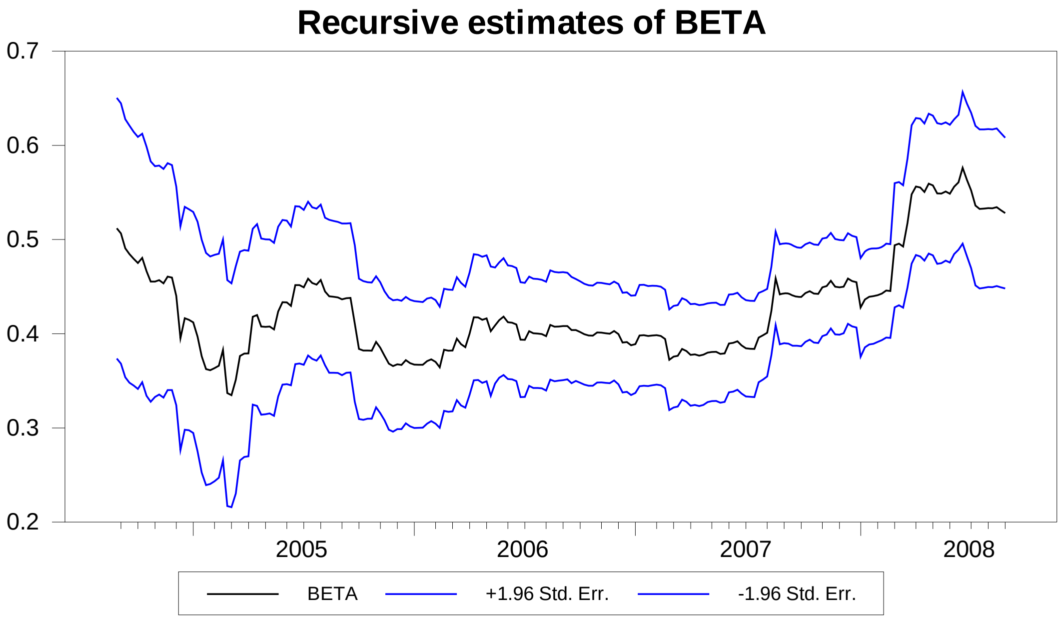

Figure A64: Median Portfolio 2003:09 - 2008:09