Attachment 7.22
Grey et. al., Comparison of OLS and LAD regression techniques for estimating beta
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Comparison of OLS and LAD regression techniques for estimating beta

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1. Preparation of this report

This report was prepared by Professor Stephen Gray, Dr Jason Hall, Professor Robert Brooks and Dr Neil Diamond. Professor Gray, Dr Hall, Professor Brooks and Dr Diamond acknowledge that they have read, understood and complied with the Federal Court of Australia’s Practice Note CM 7, Expert Witnesses in Proceedings in the Federal Court of Australia. Professor Gray, Dr Hall, Professor Brooks and Dr Diamond provide advice on cost of capital issues for a number of entities but have no current or future potential conflicts.
2. Executive summary

In the Capital Asset Pricing Model (CAPM), the cost of equity capital depends upon systematic risk, the extent to which equity returns are expected to move with overall market movements, commonly referred to as beta. This association can be estimated with an ordinary least squares (OLS) regression using data from firms in the same industry as the firm of interest. But these OLS beta estimates are well-known to have large standard errors and can be influenced by a small number of extreme returns (see, for example, Gray, Hall, Klease and McCrystal, 2009). This imprecision in beta estimates from OLS regression means that the cost of capital estimate could be materially mis-stated. This has motivated academics, practitioners, and commercial data providers to consider refinements, alternatives and adjustments to OLS regression when estimating beta. For example, the Australian Energy Regulator (AER) has considered least absolute deviation (LAD) regression (AER, 2009). This report examines the performance of LAD against OLS regression in the context of beta estimation.

LAD regression places relatively less weight on observations with extreme stock and market returns. It is not generally used in beta estimation (in academic research or commercial practice). Conceptually, this could be because (a) LAD provides better estimates than OLS and practitioners are unaware of this benefit, or (b) LAD provides even worse estimates than OLS and practitioners recognise this already. We have been asked by the Energy Networks Association to assess whether the OLS and LAD regression techniques are appropriate for the purpose of beta estimation. We conducted a literature search and were unable to find other papers that have documented the actual difference in beta estimates computed using OLS and LAD regression in a large sample – probably due to the fact that LAD estimation is not generally used to estimate CAPM betas.

We compiled beta estimates for each month using returns information from all prior months. We then document the distribution of beta estimates from two samples: (a) a sample of 1,103 stocks for which at least 10 years of returns are available for beta estimation, and (b) a sample of 2,585 stocks for which at least 36 four-weekly returns are used in beta estimation (2.75 years of returns).

The first thing to note is that LAD estimates are systematically lower than OLS estimates. From the sample in which at least 10 years of returns data is used in estimation, the average OLS estimate is 0.89, the average LAD estimate is 0.76 and 75% of observations have lower LAD estimates than OLS estimates. Furthermore, across all ten industry groups we constructed, the average LAD estimate is lower than the average OLS estimate. Finally, LAD beta estimates are generally lower than OLS estimates even for the lower end of the distribution of beta estimates. For example, the 5th percentile of OLS beta estimates is 0.16, compared to 0.05 for LAD estimates. We observe these same three

1 Our returns are constructed over four weeks rather than every month, so each return is over the same time period, but we use the term monthly returns for convenience. There are 13.04 four-weekly returns per year, that is, 365.25 days ÷ 28 days = 13.04 four-weekly periods.
results when we consider the sample in which at least 36 months of returns are used in beta estimation.

Having observed that LAD beta estimates are systematically lower than OLS estimates, we tested whether this was due to an inherent bias in either (or both) techniques when applied to beta estimation. The market capitalisation weighted average of beta estimates must equal one, by definition, if the market is comprised entirely of the stocks being evaluated. If the market capitalisation weighted average of beta estimates is generally below one, it means that the manner in which the estimates are computed leads to a downward bias. So we constructed a market index from sample firms which had returns continuously available over the most recent 10 years and documented the beta estimates resulting from this in-sample market index. We summarised the beta estimates compiled from 10, five and three years of returns.

The value weighted LAD estimates are systematically below one, and (for all but the very large stocks) this difference is material. For beta estimates computed using 10 years of returns, the value weighted LAD estimate is 0.98, compared to 1.00 for OLS estimates. Considering estimates over five years and three years, the value weighted LAD estimates are 0.96 and 0.99, respectively, while the OLS estimates are 1.00. This tells us that the nature of stock return data generates LAD estimates with a downward bias and unbiased OLS estimates.

At a high level, this difference may appear to be small, but the actual impact is much larger. We split the sample into the top 20 stocks by market capitalisation and the remaining 237 stocks. For the top 20 stocks, the OLS and LAD estimates both have a value weighted average of 1.02. Outside of the top 20 stocks, the value-weighted OLS estimate is 0.94, compared to 0.85 for the LAD estimates. So while the bias in total weighed average LAD estimates appears small, the bias is in fact significant in all but the very largest of stocks.

To understand further the impact on cost of capital estimation we should consider the equal weighted average estimates. The value weighted estimates allow us to conclude that the LAD estimates are biased downwards because we know that the value-weighted beta estimate should equal 1.00. But in estimating the cost of capital, what is most commonly done is to construct an equal-weighted average beta estimate from a sample of comparable firms. The motivation behind equal weighting is to not have the outcome influenced too heavily by the estimates from a small number of large stocks. For example, if comparable firm analysis was performed on the basis of value weighting for the telecommunications industry, the beta estimate would basically be the single estimate for Telstra.

On an equal weighted basis the LAD estimates are lower than the OLS estimates by 0.16, when estimated using 10 years of returns. Recall that across the full sample the difference in average beta estimates was 0.13 (that is, 0.89 versus 0.76) and the average difference across 10 industries was 0.16. This is the magnitude of the bias which would actually be incorporated into comparable firm analysis under LAD estimation. In short, if an analyst compiles an equal weighted average of LAD beta

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estimates, which we know have a downward bias, that average is generally 0.15 lower than the estimates from OLS estimation.

The implication is that, in the context of beta estimation, the LAD technique does not mitigate against outliers in the manner intuition suggests. An outlier is an observation that occurs with greater frequency in a sample than we would expect, and which has a disproportionate impact on the results. Outliers should lead to some beta estimates being too high relative to the true systematic risk, and some beta estimates being too low. In producing beta estimates with a downward bias, LAD estimation reduces the weight on observations which lead to high beta estimates, but there is not the same reduction in weight on observations which lead to low beta estimates.

We conclude that, due to the bias in LAD beta estimates, the LAD technique should not be used for the purposes of beta estimation. The OLS technique shows no bias for beta estimation and is widely used for beta estimation. Therefore the OLS technique is a more appropriate technique for beta estimation.

3. Issue and evaluation approach

When estimating systematic risk using stock returns, beta is computed as the slope of the line of best fit between excess market returns (the independent variable) and excess stock returns (the dependent variable).2 We have been asked by the Energy Networks Association to assess whether the OLS and LAD regression techniques are appropriate for the purpose of beta estimation.

The OLS and LAD estimation techniques employ different criteria to determine which line best fits the data. The line of best fit then provides the predicted stock return for each possible value of the market return. The difference between the predicted stock return and actual stock return is known as the error term or residual. The criteria used in OLS regression to set this line is to minimise the sum of squared error terms. In LAD regression the criteria used is to minimise the sum of absolute error terms. This means that LAD regression will place relatively less weight on observations with extreme stock and market returns. LAD regression is not generally used in beta estimation (in academic research or commercial practice), but it is not clear whether this is because (a) LAD provides better estimates than OLS and practitioners are unaware of this benefit, or (b) LAD provides even worse estimates than OLS and practitioners recognise this already. We were unable to find other papers that have documented the actual difference in beta estimates computed using OLS and LAD regression in a large sample.

To be appropriate for cost of capital estimation, a beta estimate should be unbiased, meaning that it does not systematically over- or under-state systematic risk. Hence, the particular question we address in this paper is whether OLS and LAD estimation generate unbiased beta estimates.3 Of

2 The term “excess returns” refers to returns relative to the risk free rate.
3 There may well be research questions best addressed by LAD regression or OLS regression. But we are only concerned with one particular topic, namely estimation of systematic risk from historical stock returns.
course, unbiasedness is a necessary but not sufficient condition for a good estimate. An unbiased estimate will be of limited use if it is imprecise and/or unreliable.

To address this question, we first compute OLS beta estimates according to the following equation. Every four weeks we compile beta estimates using returns information from all prior periods. In OLS regression the intercept (α) and coefficient on excess market returns (β) is determined in order to minimize the sum of squared errors.

\[ r_{i,t} - r_{f,t} = \alpha_i + \beta_i (r_{m,t} - r_{f,t}) + \varepsilon_{i,t} \]

where:
- \( r_{i,t} \), \( r_{m,t} \), and \( r_{f,t} \) = return on stock \( i \), the return on the equity market and the risk free rate, respectively in period \( t \);
- and
- \( \varepsilon_{i,t} \) = an error term for stock \( i \) during period \( t \).

LAD estimates are constructed using the same equation, but the coefficients (α and β) are estimated using a different decision rule as to what is optimal. In the above equation, the error term (\( \varepsilon \)) represents the difference between the actual excess stock return and the predicted excess return. In OLS regression the coefficients are set in order to minimise the sum of squared errors. In LAD regression the coefficients are set in order to minimise the sum of absolute errors.

In the first part of the paper we summarise the distribution of beta estimates in the entire sample and across ten different industry groups. From this description it is clear that LAD estimates are systematically lower than OLS estimates, not just on average, but across the entire returns distribution. In other words, even stocks with very low OLS estimates have even lower LAD estimates. This phenomenon is consistent across industries and we note that other papers using samples from entirely different markets and firm types report materially lower beta estimates using this type of technique.

The question is whether the systematic difference between OLS and LAD beta estimates is due to random chance in our sample, or due to a systematic bias. To test this, we reconstruct the market index using the sub-sample of firms which have returns information continuously available for the last 10 years of our sample. If an estimation technique is unbiased then the market capitalisation weighted average estimate would be equal to one. The results show that the weighted average OLS estimate is one and the weighted average LAD estimate is below one, a result which is repeated for beta estimates constructed using five and three years of returns. Outside of the largest 20 stocks, the difference between OLS and LAD estimates is substantial, and averages about 0.15.

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4 Our returns are constructed over four weeks rather than every month, so each return is computed over the same time period.
4. Data

4.1. Sample construction

We compiled beta estimates for 2,585 Australian-listed stocks using returns computed from 2 January 1976 to 4 May 2012. The returns interval is four weeks, computed using Friday closing prices, so there are 474 four-week periods. The market index is the All Ordinaries Index from 1 May 1992 and the Datastream Australia Total Market Index prior to this date. The estimate of the risk free rate is the yield to maturity on 10 year Australian government bonds as reported by the Reserve Bank of Australia, converted to a 28 day yield. We excluded firms with less than 36 four-weekly returns observations and individual observations with a four-weekly return of more than 200%.\(^5\)

The beta estimates at each date comprise all available returns information prior to that date. In our test of the relationship between expected returns and realised returns, we perform the analysis using two samples. Sample A requires at least 131 returns observations (10 years) to be used in beta estimation, and the Sample B requires at least 36 returns (2.75 years) observations to be used. There are two reasons we evaluate the results over two time periods. First, we want to document whether the results differ if shorter and longer time periods are used in beta estimation. Second, in practice beta estimates are often compiled over different time periods, either because there is limited data actually available, or because the analyst determines that a particular time period will provide more relevant information. We want to ensure that our results have broad applicability.

To ensure that our tests are performed over the same period of time, for both samples the first beta estimates are compiled from 17 January 1986 to 6 April 2012, which allows at least 131 returns observations. As a consequence, the first four-weekly period of realised returns ends on 14 February 1986 and the last period ends on 4 May 2012.

Sample A has 93,101 beta estimates from 1,103 firms. On average, each beta estimate is compiled using 206 returns periods in estimation, equivalent to 15.8 years of returns. Sample B comprises 247,652 beta estimates from 2,585 firms. On average, each beta estimate is compiled using 125 returns periods in estimation, equivalent to 9.6 years of returns.

4.2. Beta estimates

In Table 1 we summarise the distribution of beta estimates. For Sample A, the mean OLS beta estimate is 0.89 with a standard deviation of 0.53. Despite the long estimation period there is substantial dispersion of OLS beta estimates. Half of the OLS beta estimates lie outside the range of 0.49 to 1.24, and 10 per cent of estimates are either above 1.87 or below 0.16.

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\(^5\) Application of this filter results in the exclusion of less than 0.5% of observations. Some of these observations will represent returns of stocks which happen to be volatile. Other cases will represent data errors and the assumptions used by Datastream in accounting for changes in capitalisation. The most extreme stock return prior to the application of this filter was 15,133% for Equatorial Resources on 3 May 2002.
Table 1. Beta estimates

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std Dev</th>
<th>5th</th>
<th>25th</th>
<th>50th</th>
<th>75th</th>
<th>95th</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Sample A - At least 131 periods used in beta estimation (N = 93,101; 1,103 firms)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OLS estimate</td>
<td>0.89</td>
<td>0.53</td>
<td>0.16</td>
<td>0.49</td>
<td>0.82</td>
<td>1.24</td>
<td>1.87</td>
</tr>
<tr>
<td>LAD estimate</td>
<td>0.76</td>
<td>0.47</td>
<td>0.05</td>
<td>0.41</td>
<td>0.72</td>
<td>1.09</td>
<td>1.54</td>
</tr>
<tr>
<td><strong>Panel B: Sample B - At least 36 periods used in beta estimation (N = 247,652; 2,585 firms)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OLS estimate</td>
<td>0.96</td>
<td>0.77</td>
<td>–0.01</td>
<td>0.45</td>
<td>0.85</td>
<td>1.37</td>
<td>2.35</td>
</tr>
<tr>
<td>LAD estimate</td>
<td>0.81</td>
<td>0.56</td>
<td>–0.01</td>
<td>0.35</td>
<td>0.74</td>
<td>1.19</td>
<td>1.98</td>
</tr>
</tbody>
</table>

The sample comprises four-weekly return observations from 2,585 stocks listed on the Australian Stock Exchange from 2 January 1976 to 4 May 2012 for which at least 36 returns observations are available. From 1 May 1992 the market index is the All Ordinaries Index. Prior to this date the market index is the Datastream Australia Total Market Index. The risk free rate is the yield to maturity on 10 year Australian government bonds, converted to a a four-weekly rate. The compilation of the estimates is described Section 4.

Consider now the LAD estimates, which have a mean estimate of 0.76 and a standard deviation of 0.47. While the LAD estimates exhibit less dispersion than the OLS estimate we observe that the LAD estimates are systematically lower than the OLS estimates. Across all of the percentiles in the table, the LAD estimates are lower than the OLS estimates. For example, the 5th percentile of OLS estimates is 0.16 implying that 5% of OLS estimates are less than 0.16. For LAD estimation, 5% of estimates are less than 0.05. A quarter of OLS estimates are less than 0.49 and a quarter of LAD estimates are less than 0.41.

So we have the first indication that LAD regression, when applied to beta estimation, seems to reduce beta estimates rather than adjust the distribution symmetrically. Observations which would lead to high beta estimates appear to be considered outliers by LAD estimation, so are given less weight in beta estimation compared with OLS. But observations which would lead to low beta estimates do not have their weighting reduced relative to OLS by the same degree.

The implication is the same if we refer to returns for Sample B. In this case the mean OLS estimate is 0.96, which declines to 0.81 for LAD estimation. The 5th percentile of the distributions is the same at –0.01, about 5% of OLS and LAD estimates are negative. In contrast, under OLS estimation, 5% of estimates are more than 2.35, but for LAD estimation 5% of estimates are above 1.98.

To examine further the difference between LAD and OLS estimates we looked cross-sectionally at the industry level. In Table 2, we present the mean beta estimates for Sample A and B split into ten industry groups formed on the basis of FTSE Industry Classification Benchmark (ICB) codes. The table shows lower mean beta estimates across all ten industry groups when the estimates are performed using LAD estimation rather than OLS. The magnitude of the difference between the OLS and LAD estimate is reasonably consistent across industries. For the sample of observations in which at least 10 years of returns are available for analysis, in eight of ten industry groups the average difference is within the range of 0.08 to 0.16. For the larger sample the corresponding average difference for these groups is within the range of 0.10 to 0.18.
Comparison of OLS and LAD regression techniques for estimating beta

Table 2. Industry beta estimates

<table>
<thead>
<tr>
<th>Industry</th>
<th>At least 131 periods used in estimation (Sample A)</th>
<th>At least 36 periods used in estimation (Sample B)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N</td>
<td>OLS</td>
</tr>
<tr>
<td>Oil &amp; Gas</td>
<td>7,207</td>
<td>1.19</td>
</tr>
<tr>
<td>Basic Materials</td>
<td>29,140</td>
<td>1.15</td>
</tr>
<tr>
<td>Industrials</td>
<td>14,831</td>
<td>0.70</td>
</tr>
<tr>
<td>Consumer goods</td>
<td>8,485</td>
<td>0.54</td>
</tr>
<tr>
<td>Health care</td>
<td>4,412</td>
<td>0.95</td>
</tr>
<tr>
<td>Consumer services</td>
<td>7,288</td>
<td>0.72</td>
</tr>
<tr>
<td>Telecommunications</td>
<td>685</td>
<td>1.46</td>
</tr>
<tr>
<td>Utilities</td>
<td>1,549</td>
<td>0.82</td>
</tr>
<tr>
<td>Financials</td>
<td>16,188</td>
<td>0.66</td>
</tr>
<tr>
<td>Technology</td>
<td>3,316</td>
<td>1.10</td>
</tr>
<tr>
<td>Full sample</td>
<td>93,101</td>
<td>0.89</td>
</tr>
</tbody>
</table>

The table comprises mean beta estimates across 10 industry super-sectors formed according to the International Classification Benchmark of FTSE.

To recap, observe that, on average across the sample, LAD estimates are lower than OLS estimates by 0.13 when estimated using 10 years of data and 0.15 when estimated using 36 months of data. Further, even at the very low end of the distribution of estimates, the LAD estimates are lower than OLS estimates. And this difference is systematic across industries. Across eight of ten industries the average difference between OLS and LAD estimates lies within the range of 0.08 to 0.18. For the remaining two industries the difference is even higher.

The observation that LAD estimates are consistently lower than OLS estimates is not unique to our sample. We observe the same phenomenon in two other empirical papers from different markets relying upon different sample types. For example, Mills, Coutts and Roberts (1996) compiled beta estimates for 65 companies listed in the United Kingdom following the announcement of a management buyout. The mean OLS estimate was 0.82 and the mean LAD estimate was 0.34.

Chan and Lakonishok (1992) examined a sample of 661 initial public offerings in the United States. For the first 10 trading days they estimated the systematic risk of this sample by computing OLS estimates and trimmed quantile estimates. Trimmed quantile regression is an alternative technique to LAD estimation which also places less weight on returns further from the mean, and which generates similar coefficient estimates to LAD estimation. In this case, the beta estimate is made not by observing returns in time series for individual stocks, but by observing the returns on the sample of stocks each trading day. Over the 10 trading days examined the average OLS beta estimate was 1.66, the average trimmed quantile estimate was 1.08 and the trimmed quantile estimate is lower than the OLS estimate for every trading day.

Consider the beta estimates from these three papers together. We have samples from three markets (Australia, the United Kingdom and the United States), with different composition (all listed firms, buyout firms, IPOs), and in all three instances the outlier-resistant estimates are materially lower than OLS beta estimates. The justification for LAD estimation is that it leads to lower standard errors in cases in which the variables are not normally distributed. In the particular context of beta
estimation, the LAD technique consistently reduces high OLS estimates by far more than it increases low OLS estimates. The question is whether this occurs due to attributes of our sample, or whether one of the techniques results in biased beta estimates. We address this question in the next section.

5. Test for bias

In this section, we directly test whether there is something pervasive about LAD estimation which generates a downward bias in beta estimates. We constructed a market index from the 257 stocks which had all returns information available for 10 years (that is, 131 four-weekly returns) ending on 4 May 2012. We computed their starting market capitalisation weights and held these constant over the following ten years. Under these assumptions the market value weighted beta should be exactly 1.00, if the estimation technique is unbiased. We compiled OLS and LAD beta estimates, and repeated the exercise using exactly the same sample, using the most recent five and three years of returns. Results are presented in Table 3.

In the left section of the table we present the value-weighted average beta estimates. The OLS and LAD estimates would only be unbiased if the estimates were also exactly equal to 1.00. Value-weighted OLS estimates are exactly equal to 1.00 in all three cases, which means that the OLS estimate is unbiased. However, the value-weighted LAD estimates are all below 1.00, at 0.98 when betas are estimated using 10 years of returns information, 0.96 when five years of returns are used and 0.99 when three years of returns are used. The application of LAD estimation results in downwardly biased beta estimates.

This extent of this bias is highlighted upon further analysis of the results. We separately analyse the largest 20 stocks in terms of market capitalisation. These stocks comprise 74% of the weight in the 10 year returns sub-sample, 67% of the weight in the five year returns sub-sample and 76% of the weight in the three year returns sub-sample.

Across these top 20 stocks, the value-weighted OLS and LAD estimates are approximately the same. But across the remainder of the sample the value-weighted LAD estimates are materially lower than the OLS estimates. Outside the top 20 stocks, the value-weighted LAD estimates are lower than the OLS estimates by 0.09, 0.08 and 0.07, respectively.

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6 Of course, unbiasedness is a necessary but not sufficient condition for a good estimate. An unbiased estimate will be of limited use if it is imprecise and/or unreliable.
Table 3. Beta estimates constructed from in-sample market index

<table>
<thead>
<tr>
<th></th>
<th>Value-weighted average</th>
<th></th>
<th>Equal-weighted average</th>
<th></th>
<th>% of estimates below OLS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All        Top 20  Ex top 20</td>
<td>All        Top 20  Ex top 20</td>
<td>All        Top 20  Ex top 20</td>
<td>All        Top 20  Ex top 20</td>
<td></td>
</tr>
<tr>
<td><strong>Panel A: 10 years of returns</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OLS</td>
<td>1.00       1.02     0.94</td>
<td>1.15     1.08     1.16</td>
<td>76%        40%      79%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LAD</td>
<td>0.98       1.02     0.85</td>
<td>0.99     1.05     0.98</td>
<td>76%        40%      79%</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Panel B: 5 years of returns</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OLS</td>
<td>1.00       0.99     1.02</td>
<td>1.12     0.98     1.13</td>
<td>69%        60%      70%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LAD</td>
<td>0.96       0.97     0.94</td>
<td>0.99     0.94     0.99</td>
<td>69%        60%      70%</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Panel C: 3 years of returns</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OLS</td>
<td>1.00       1.04     0.88</td>
<td>1.02     0.97     1.03</td>
<td>62%        45%      64%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LAD</td>
<td>0.99       1.05     0.81</td>
<td>0.91     1.00     0.91</td>
<td>62%        45%      64%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

What this means is that the LAD technique itself leads to a downward bias in beta estimates. The value weighted average LAD estimates are consistently below one which cannot occur if the technique generates unbiased estimates of systematic risk. The actual bias that will be reflected in beta estimates for the cost of capital will approximate the difference in equal-weighted averages. In the actual process of cost of capital estimation, an analyst will compile beta estimates for a set of comparable firms generally formed on the basis of industry. In most instances these estimates are given equal weight in the analysis, rather than market capitalisation weight, in order for the output to not be unduly influenced by a small number of the largest stocks. For example, in compiling a beta estimate for telecommunications, if a value-weighted estimate was compiled the estimate would basically be the estimate for Telstra.

This means that the conclusions that will actually be drawn from beta estimates will be equal weighted averages, so any actual bias incorporated into cost of capital estimates will reflect differences in equal weighted averages. If we consider equal weighted beta estimates, across the whole sample the average LAD estimates are lower than the OLS estimates by 0.16, 0.13 and 0.11 across the three estimation periods. The magnitude of these average differences is approximately the same as we reported in the descriptive statistics – 0.13 for the sample in which at least 10 years of returns are available for analysis and 0.15 for the sample in which at least 2.75 years of returns are available for analysis. So the direct implication of using the biased LAD technique over OLS is that the average beta estimate is reduced by 0.15.

This impact is not due to a small number of extreme beta estimates. To document this, we report the percentage of instances in which the LAD was less than the OLS estimate. For LAD estimation, this percentage is 76%, 69% and 62% across the three estimation periods. So in other words, in roughly two-thirds to three quarters of cases the LAD estimate will be less than the OLS estimate.
6. Conclusion

OLS regression estimates of systematic risk are sensitive to extreme stock returns. This has led researchers to investigate whether alternative regression techniques, which mitigate the influence of outliers, have merit in estimating systematic risk. One technique recently adopted by the AER is LAD estimation.

We conclude that LAD beta estimates exhibit a downward bias. The first indication that a bias might exist is that LAD estimates are systematically lower than OLS estimates – on average for the full sample, on average for all ten industries considered, and for a substantial majority of individual firm estimates. The tendency for LAD estimates to be consistently lower than OLS estimates is consistent with other relevant evidence from IPOs in the United States and management buyout companies in the United Kingdom.

To directly test whether LAD estimates are biased we constructed a market index from available firms, and documented that the market capitalisation weighted average LAD estimates are consistently below one while the market capitalisation weighted average OLS estimate is one. Outside of the top 20 stocks the LAD estimates are consistently lower than OLS estimates, at around 0.10 on a value-weighted basis and 0.15 on an equally-weighted basis.

The implication of this test is that LAD estimates have a material downward bias. This makes its use in beta estimation inappropriate.
7. References


8. Terms of reference and qualifications

This report was prepared by Professor Stephen Gray, Dr Jason Hall, Professor Robert Brooks and Dr Neil Diamond. Professor Gray, Dr Hall, Professor Brooks and Dr Diamond have made all they enquiries that they believe are desirable and appropriate and that no matters of significance that they regard as relevant have, to their knowledge, been withheld.

Professor Gray, Dr Hall, Professor Brooks and Dr Diamond have been provided with a copy of the Federal Court of Australia’s “Guidelines for Expert Witnesses in Proceeding in the Federal Court of Australia.” The Report has been prepared in accordance with those Guidelines, which appear in the terms of reference.