

**REVIEW OF FRONTIER REPORT ON GAMMA**

Dr Martin Lally  
Capital Financial Consultants Ltd

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## 1. Introduction

I have been asked by the AER to critically review Frontier Economics (2016), and to address a number of additional questions posed by the AER. I commence by reviewing the Frontier report.

## 2. Review of Frontier

Frontier (2016, section 1.3) presents an example to demonstrate the point that equilibrium prices are determined by all investors. This example was intended to rebut the claim by the ACT (2016) that, in respect of gamma, there is a choice between an average investor perspective and a marginal investor perspective. I fully concur with Frontier's example and the point being demonstrated.

Frontier (2016, sections 1.4, 1.5) describes marginal investors as the participants in transactions that constitute observed market prices but that these prices are still determined by all investors. This is a very unusual definition of "marginal investors"; the typical meaning, in a situation in which there is variation in personal tax rates across investors, is a group of investors subject to the same personal tax rates and whose tax rates are reflected in market prices, i.e., market prices are those that would prevail if all investors were of this type (for example, see Eades et al, 1984, page 18). This typical definition is a mere definition and therefore is innocuous. However, this definition sometimes leads to the comment that these "marginal investors" determine market prices (for example, see Ainsworth et al, 2015, pp. 13-15; Feuerherdt et al, 2010, page 375).<sup>1</sup> However, as noted by Michaely and Vila (1995, page 172) in the context of ex-dividend studies, an expected ex-dividend price drop-off (as a proportion of the dividend) equal to 1 in a market comprising investors with drop-off ratios of 1.25, 1 and 0.75 might be due to the actions of the first and third groups rather than the second group. Thus, it is possible that the first and third group rather than the second group 'determine' the expected drop-off ratio. Michaely and Vila (1993, pp. 180-181) go on to develop an equilibrium model of the expected drop-off ratio, and find that the ratio is determined by the personal tax rates of all investors rather than only a subset called marginal

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<sup>1</sup> Ironically, one of the authors of the latter paper (Stephen Gray) is also the author of the Frontier (2016) paper. So, Gray seems to hold conflicting views on the question of whether prices are determined by all investors or only the "marginal investors".

investors. This is akin to the value of theta in a CAPM framework being determined by all investors. Michaely and Vila (1993, page 180) refer (fairly) to marginal investor models of price determination as “nonequilibrium”, i.e., lacking a fundamental requirement in asset pricing models.

Frontier (2016, section 1.6) claims that the CAPM of Lally and van Zijl (2003) implies that the parameter called theta ( $\theta$ ) is a complex weighted average over the imputation credit utilization rates of all investors. Frontier (2016, section 1.6) also claims that, if this model were a perfect representation of the world, the observed market prices would accord with the models. I concur with both statements.<sup>2</sup> Frontier are hinting here at a relationship between theta and the market value of the credits, but are not sufficiently explicit. I therefore do so, as follows. The Officer (1994) model is a special case of Lally and van Zijl (2003), in which the personal tax rates on gross dividends and capital gains are equal. If the Officer (1994) model is correct, then the value of equity ( $S$ ) in a company delivering a pre company tax payoff ( $Y$ ) in only  $T$  years’ time (with associated company tax payments  $TAX$  and the resulting imputation credits  $IC$ ) would be as follows:

$$S = \frac{E(Y) - E(TAX) + E(IC)\theta}{(1 + k)^T}$$

where  $k$  is the discount rate specified in the Officer (1994) model. When  $T$  is very close to zero, it follows that

$$S = E(Y) - E(TAX) + E(IC)\theta$$

So, an extra \$1 of credits would raise equity value by  $\theta$  (theta). Accordingly, theta is the market value of \$1 of credits due imminently *if* the Officer (1994) model is correct. This provides support for estimating theta from market prices (as well as from other methods). Nevertheless, it must be stressed that  $\theta$  is still by definition a complex weighted average over the imputation credit utilization rates of all investors and this is not in any way changed by the fact that it will also be the market value of \$1 of credits due imminently *if* the Officer (1994) model is correct.

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<sup>2</sup> Frontier also refers to Monkhouse (1993), but I will focus upon the Lally and van Zijl (2003).

Frontier (2016, section 1.6) argues that models in which theta is a complex weighted average over the imputation credit utilization rates of all investors do not apply in the case where there are two markets (domestic and foreign) because those models derive the equilibrium price by equating demand and supply across the market. Prima facie, this is asserting that such models apply only in a world with one country and therefore are not relevant because the world comprises multiple countries. However, this is not correct. If the equity markets of countries are completely segregated, a CAPM of this type applies to each market (see Lally and van Zijl, 2003, page 191) and this is the Officer (1994) model used by Australian regulators. By contrast, if all countries' equity markets are completely integrated, there is only one market and a CAPM of this type applies to it but the market is now that for the entire world rather than for an individual country (see Lally, 1996).

Frontier (2016, section 1.6) argues that, since the CAPM is not a perfect representation of the world, only market evidence on the value for theta should be used where it is available rather than an estimate of what the parameter would have been if the model was accurate, i.e., only market evidence on the value for theta should be used where it is available rather than using the definition of theta expressed in terms of other parameters that arises from a theoretical model (being a complex weighted average over the utilization rates of investors).<sup>3</sup> A test of Frontier's adherence to the general principle here arises when estimating the cost of equity. To do so, all Australian regulators and most participants in these regulatory proceedings (including Frontier) adopt the Officer (1994) version of the CAPM followed by estimating its constituent parameters. An alternative approach would be to estimate the cost of equity for a regulated business without recourse to a theoretical model, and this is possible by using the DDM applied to an individual company.<sup>4</sup> Given Frontier's view about directly estimating theta only from market prices where such data is available, one might expect Frontier to adopt the same approach to estimating the cost of equity of a regulated business, and therefore favour exclusive use of a DDM approach applied to that business. However, Frontier does not do so. Estimating the cost of equity is difficult. Despite the opportunity to do so by using market prices rather than a theoretical model (which will accord with market prices only if the model is accurate), the standard Australian approach is to invoke a

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<sup>3</sup> In the initial stages of presenting this argument, Frontier implies that theta should be estimated from both its definition within the model and from the market value of the credits. However, in summarizing the argument, it is clear that they favour exclusive use of the estimate directly based upon market prices.

<sup>4</sup> Such an approach assumes that the current market value of the firm's equity is the present value of its dividends, which is uncontroversial.

theoretical model, to estimate each parameter within the model by a variety of methods, and to weight the results in accordance with their relative merits. Furthermore, in respect of these parameters, it may be possible to estimate some of them directly from market prices but it is typically not the only method and may not be the best one.

Frontier (2016, section 1.6) also argues that this direct approach to the estimation of theta (using market prices without recourse to a theoretical model) is consistent with the way in which every other WACC parameter is estimated. However, this is not the case. For example, in respect of the MRP, there is no method for estimating it directly using market prices. Instead, one starts with the definition for this parameter within the Officer model (the expected market return net of the risk free rate plus the product of theta and the expected imputation credit as a proportion of equity value):

$$MRP = E(R_m) - R_f + \frac{\theta E(IC_m)}{S_m} \quad (1)$$

Estimates of the individual terms are then determined, in accordance with their definitions, and then inserted into equation (1). This process parallels the estimation of theta, commencing with its definition as a weighted average over the utilization rates of investors. So, Frontier's claim that estimating theta directly from market values (and disregarding estimates that invoke a theoretical model) is consistent with the way in which every other WACC parameter is estimated is false.

Frontier (2016, section 1.6) argues that the CAPM assumes that there are no taxes or transactions costs but the MRP is estimated from market prices that do reflect investors' consideration of those things rather than estimated as if the theoretical assumptions of the CAPM did hold. This is intended to contrast the process for estimating the MRP from that of theta when its definition in the CAPM is invoked. However, the same point applies to estimating theta by commencing with its definition in the CAPM as a weighted average over investors' utilization rates. This definition leads to estimating theta as the proportion of Australian equities held by those investors who are able to use the credits (Lally, 2016a, pp. 16-17). This proportion is empirically determined and the empirical proportion will reflect the true nature of the world rather than the CAPM's assumptions. So, in respect of both theta and the MRP, recourse to the CAPM provides definitions for both parameters expressed in

terms of other parameters that can be empirically estimated, and empirical estimates of these parameters are adopted even if the assumptions of the CAPM (if valid) would give rise to different values for these parameters to those actually observed.

Frontier (2016, section 1.6) also argues that the AER's estimate of theta using the equity ownership approach or the tax statistics approach is not the complex weighted average as specified in the definition of theta but a simple average. The claim that the AER's approach involves a simple average is false; their equity ownership approach weights the utilization rates of investors by their holdings of risky assets, whilst their tax statistics approach weights the redemption rates of investors by the level of redemptions. For example, if investors A and B have utilization rates of 1 and zero and hold 70% and 30% of the relevant risky assets respectively, the AER's estimate would be 0.7 (the value-weighted average) rather than 0.5 (the simple average). However it is correct that the AER does not incorporate other terms specified in the definition of theta because they do not readily lend themselves to estimation and instead assumes that these additional terms are equal across investors (see Lally, 2016a, page 16). Lally (2016b, pp. 15-17) analyses this issue and concludes that this assumption leads to theta being overestimated by about 0.06.

Frontier (2016, section 2.2) argues that theta in equation (1) must be the market value of the credits because the other terms in the MRP (the expected dividend yield and the expected rate of capital gain) are market values. To assess this claim, I decompose the term  $E(R_m)$  in equation (1) into its two parts and therefore the MRP in the Officer model is as follows:

$$MRP = \frac{E(DIV)}{S_m} + \frac{E(Cap Gain)}{S_m} - R_f + \frac{\theta E(IC_m)}{S_m}$$

In this equation, the treatment of imputation credits is identical to that of the dividends. In both cases, the numerator terms are not by definition market values. In respect of the dividends, these are the payments before personal taxes and transactions costs whilst their market value will be affected by the personal tax rate on dividends relative to capital gains, transactions costs and time delays in receiving them. Similarly, the imputation credit terms in the numerator are the payments before personal taxes and transactions costs, scaled down by theta to reflect the proportion of investors who cannot use them, whilst their market value will be affected by the personal tax rate on dividends relative to capital gains, transactions

costs and time delays in receiving them. It is the denominator terms in the last equation (the market value of equity) that produces an MRP that is a market rate. However, whilst Frontier's analogy is wrong, theta will be the market value of \$1 of credits due imminently if the Officer (1994) model in which theta appears is valid (as explained earlier). Similarly, the dividend term *DIV* in the last equation will be the market value of the dividends due imminently if the Officer (1994) model is valid.

### **3. AER Questions**

The AER has also raised a number of questions with me. The first of these is whether the AER's approach to estimating gamma post company tax (but before personal costs and personal taxes) based upon its 'utilisation value' conceptualization of distributed imputation credits is consistent with the way in which the AER estimates the cost of equity using a domestic version of the CAPM. The AER estimates the cost of equity using the Officer (1994) model, gamma is a parameter within that model, and therefore the AER must estimate gamma as defined within the Officer model. A rigorous derivation of the Officer model (Lally and van Zijl, 2003) reveals that gamma is the product of two parameters: the distribution rate for credits (the proportion of company taxes paid to the ATO that are attached to dividends as credits) and theta (a utilization rate for credits constituting a weighted-average over investors' utilization rates for the credits). So, the AER's focus upon the 'utilisation value' of the distributed credits (rather than the market value) is entirely appropriate.

The second of the AER's questions is whether the AER's current gamma estimate of 0.4 remains appropriate to ensure that regulated firms receive a post company tax cost of equity inclusive of imputation credits that is at least sufficient to contribute to a rate of return that would be expected to meet the allowed rate of return objective (ARORO). As discussed in Lally (2016a), I favour an estimate for theta of 1. In addition, conditional upon recognizing the existence of foreign investors when estimating theta as the AER does, I favour an estimate of theta of 0.60. Coupled with an estimate for the distribution rate of at least 0.83, my estimate for gamma is therefore at least 0.50. Accordingly, I consider the AER's estimate of 0.40 to be too low. Since lower values for gamma are beneficial to the regulated businesses, it follows that the AER's estimate of 0.40 is more than sufficient to contribute to a rate of return that meets the ARORO.

The third of these questions is whether theta for the benchmark efficient entity should or must be estimated by dividend drop-off (DDO) studies or some other form of study using market prices given the AER's use of the Officer (1994) model. As stated in the first paragraph of this section, within the Officer model, theta is by definition a complex weighted average over the imputation credit utilization rates of individual investors. Nothing in this definition implies that this parameter should or must be estimated using some form of market study. Furthermore, as discussed in the previous section, theta is the market value of \$1 of credits due imminently *if* the Officer (1994) model is correct. This provides support for estimating theta from market prices but it does not preclude recourse to other methods. A range of methods are available. As discussed in Lally (2013), I consider that studies using market prices (including DDO studies) warrant very limited weight for an extensive range of reasons.

#### **4. Conclusions**

This report has reviewed a recent report from Frontier Economics, and also addressed a number of questions posed by the AER. The principal conclusions are as follows.

In respect of Frontier, I agree with two of the points raised by them, as follows. Firstly, Frontier demonstrates that equilibrium prices are determined by all investors. Secondly, Frontier argues that, within CAPMs in which the parameter called theta is a complex weighted average over the imputation credit utilization rates of all investors, the observed market prices would accord with the models if these models were a perfect representation of the world.

In respect of the AER, the first question raised by them is whether the AER's approach to estimating gamma post company tax (but before personal costs and personal taxes) based upon its 'utilisation value' conceptualization of distributed imputation credits is consistent with the way in which the AER estimates the cost of equity using a domestic version of the CAPM. The AER estimates the cost of equity using the Officer model, gamma is a parameter within that model, and therefore the AER must estimate gamma as defined within the Officer model. A rigorous derivation of the Officer model reveals that gamma is the product of two parameters: the distribution rate for credits (the proportion of company taxes paid to the ATO that are attached to dividends as credits) and theta (a utilization rate for credits constituting a



weighted-average over investors' utilization rates for the credits). So, the AER's focus upon the 'utilisation value' of the distributed credits (rather than the market value) is entirely appropriate.

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The third of the AER's questions is whether theta for the benchmark efficient entity should or must be estimated by dividend drop-off (DDO) studies or some other form of study using market prices given the AER's use of the Officer model. Within the Officer model, theta is by definition a complex weighted average over the imputation credit utilization rates of individual investors, and nothing in this definition implies that this parameter should or must be estimated using some form of market study. Theta is the market value of \$1 of credits due imminently *if* the Officer model is correct, and this provides support for estimating theta from market prices but it does not preclude recourse to other methods. A range of methods are available, and I consider that studies using market prices (including DDO studies) warrant very limited weight for a range of reasons.

## REFERENCES

ACT, 2016. *Application by SA Power Networks [2016] ACompT 11* ([www.competitiontribunal.gov.au](http://www.competitiontribunal.gov.au)).

Ainsworth, A., Partington, G., and Warren, G., 2015. “Do Franking Credits Matter? Exploring the Financial Implications of Dividend Imputation” ([www.cifr.edu.au](http://www.cifr.edu.au)).

Eades, K., Hess, P., and Kim, E., 1984. “On Interpreting Security Returns during the Ex-Dividend Period”, *Journal of Financial Economics*, vol. 13, pp. 3-34.

Feuerherdt, C., Gray, S., and Hall, J., 2010. “The Value of Imputation Tax Credits on Australian Hybrid Securities”, *International Journal of Finance*, vol. 10, pp. 365-401.

Frontier Economics, 2016. *Perspectives for the Estimation of Gamma*, report prepared for AGN, Multinet Gas, and Ausnet Services ([www.aer.gov.au](http://www.aer.gov.au)).

Lally, M., 1996, “The CAPM Under Dividend Imputation and International Portfolio Selection”, *Pacific Accounting Review*, Vol. 8 (1), pp. 48-65.

\_\_\_\_\_ 2013, *The Estimation of Gamma*, report prepared for the AER ([www.aer.gov.au](http://www.aer.gov.au)).

\_\_\_\_\_ 2016a. *Gamma and the ACT Decision*, report prepared for the AER ([www.aer.gov.au](http://www.aer.gov.au)).

\_\_\_\_\_ 2016b. *Review of the ACT’s Gamma Decision*, report prepared for the QCA ([www.qca.org.au](http://www.qca.org.au)).

\_\_\_\_\_ and van Zijl, T., 2003, ‘Capital Gains Tax and the Capital Asset Pricing Model’, *Accounting and Finance*, vol. 43, pp. 187-210.

Michaely, R., and Vila, J., 1995. “Investors’ Heterogeneity, Prices, and Volume around the Ex-Dividend Date”, *Journal of Financial and Quantitative Analysis*, vol. 30, pp. 171-198.

Monkhouse, P., 1993, ‘The Cost of Equity under the Australian Dividend Imputation System’, *Accounting and Finance*, vol. 33 (2), pp. 1-18.

Officer, R., 1994, ‘The Cost of Capital of a Company under an Imputation Tax System’, *Accounting and Finance*, vol. 34, pp. 1-17.