



Perspectives for the estimation of gamma

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1 The complex weighted-average investor and the marginal investor

1.1 Overview

1 In October 2016, the Australian Competition Tribunal (Tribunal) handed down its *SAPN* decision.¹ When considering the AER’s approach to estimating gamma, the Tribunal characterised the issue as a choice between an “average investor” perspective and a “marginal investor” perspective. However, there are not two theoretical perspectives or frameworks.

2 We explain below that, under certain theoretical asset pricing models, the value of imputation credits that is reflected in stock prices will be a complex weighted average (by investor wealth and risk aversion) of the ability of each investor to utilise imputation credits. Under the assumptions of the theoretical representative investor models, there would be an equivalence between the complex weighted-average and the observed market price.

3 However, in practice estimates of the market value differ from the AER’s estimates of the average utilisation rate. We explain below that this is because (a) the assumptions of the theoretical model do not hold in practice, and (b) in any event, the AER estimates a simple average of utilisation rates rather than the complex weighted average that is required by those models.

4 Thus, there is not a choice between theoretical “average investor” and “marginal investor” perspectives. Rather, the choice is between:

- a. An estimate of what the value of credits *would have been* if the assumptions of the theoretical model *did* hold in the real world, and if the simple average *was* the same as the complex weighted average; or
- b. An estimate of the market value of credits, which reflects the outworking of the process by which a market-clearing price is obtained, even where that process is too complex to be captured by a simple economic model.

1.2 Representative investor asset pricing models

5 Throughout its Guideline materials, the AER refers to a “complex weighted-average investor” or “representative investor.” These terms are drawn from the academic literature on representative investor asset pricing models, one example

¹ *Application by SA Power Networks* [2016] ACompT11.

of which is the Sharpe-Lintner Capital Asset Pricing Model. Under these models, all investors are endowed with some initial wealth and they trade with each other until an equilibrium is reached. The equilibrium price of each asset in the market will reflect the demand for that asset by each investor. How much of an asset an investor might demand will be a function of that investor's wealth and risk aversion. Other things equal, wealthier investors will have a higher demand for all assets and less risk-averse investors will have a higher demand for risky assets.

- 6 Under these models, the equilibrium price of each asset will reflect a weighted average over all investors – where the weights reflect investor wealth and risk aversion. The AER makes this point in its Guideline materials:

The representative investor is a weighted average of investors in the defined market. Specifically, investors are weighted by their value weight (equity ownership) [wealth] and their risk aversion.²

- 7 In the remainder of this section, we set out a simple stylised example to illustrate how the equilibrium market price is set collectively by all investors in the market and how the equilibrium price is ultimately determined by the complex weighted average or “representative” investor, and we consider the application to the estimation of the value of imputation credits in the regulatory process.

1.3 A simple numerical example

- 8 Consider a simple economy in which there are two investors (A and B) who are endowed with some initial wealth which they allocate between two risky assets (1 and 2). Suppose that:
- a. Both companies have 100 shares outstanding (which is the supply side of the equilibrium); and
 - b. Investors A and B have \$100 of wealth and \$50 of wealth, respectively.
- 9 Now suppose that a price is announced for each asset and each investor indicates how many shares they would like to buy at that price. The prices for assets 1 and 2 are announced at \$1.00 and \$0.50, respectively, and the demands from each investor are as follows:

² AER Rate of Return Guideline, Explanatory Statement, pp. 119-120.

	Asset A	Asset B	Total Cost
Announced price per share	\$1.00	\$0.50	
Demand from investor 1	93	14	\$100
Demand from investor 2	20	60	\$50
Total demand	113	74	\$150

10 Note that each investor divides their total wealth between the two assets. For example, investor 1 plans to buy 93 shares of A at \$1.00 each and 14 shares of B at \$0.50 each, spending his total wealth of \$100. Similarly for investor 2.

11 This is not an equilibrium – there are 100 shares of each asset to be sold, so there is excess demand for asset A and not enough demand for asset B. Therefore the price of A must increase (to make it slightly less attractive) and the price of B must decrease, resulting in revised demand from each investor as follows:

	Asset A	Asset B	Total Cost
Announced price per share	\$1.10	\$0.40	
Demand from Investor 1	83	22	\$100
Demand from Investor 2	17	78	\$50
Total demand	100	100	\$150

12 At these new prices, the market *is* in equilibrium. The aggregate demand matches the total supply of 100 shares for each asset. This shows that both investors have influenced the equilibrium price.

13 Note that in the first (disequilibrium) case, investor 1 wanted to invest 93% of his wealth into asset A (\$93 out of \$100) and investor 2 wanted to invest 40% of his wealth into asset A (\$20 out of \$50). Note also that investor 1 has 67% of total wealth (\$100) and investor 2 has 33% of total wealth. Thus, the weighted average investor wants to invest 75% of his wealth into asset A ($0.67 \times 93\% + 0.33 \times 40\%$). Since total wealth is \$150, 75% amounts to \$113, which is 113 shares if the price is set to \$1.00 – so the market is not in equilibrium, because the total supply is only 100.

14 In the equilibrium case, the weighted average investor wants to invest 73% of his wealth into asset A ($0.67 \times 91\% + 0.33 \times 37\%$). Since total wealth is \$150, 73% amounts to \$110 – which is 100 shares if the price is set to \$1.10 – the market is in equilibrium because the weighted-average investor demands exactly the right

number of shares. The weighted average investor seeks to invest \$110 into asset A, which equates to 100 shares at \$1.10 each.

- 15 Note that this example simplifies things by weighting only on wealth. Risk aversion is relevant in determining why, when the price went up from \$1.00 to \$1.10, did Investor 1 change demand from 93 to 83 (rather than, say, 82 or 84). This change depends on that investor's risk aversion via a mathematical equation known as the investor's "utility function". Incorporating utility functions into the above example would add considerable complexity, however the intuition for how risk aversion affects the analysis can be conveyed with the following simple extension.
- 16 Suppose that, in addition to the two risky assets (A and B) there is also a risk-free asset. Suppose investor A has \$200 wealth, invests \$100 in the risk-free asset and the remaining \$100 as set out in the table above. Suppose that investor B has \$50 of wealth and invests none of it in the risk-free asset, just mimicking the investments in the table above. That is, investor A is more risk averse than investor B, so invests more into the risk-free asset. This situation is summarised in the table below.

	Asset A	Asset B	Risk-free asset	Total Cost
Announced price per share	\$1.10	\$0.40		
Demand from Investor 1	83	22	\$100	\$200
Demand from Investor 2	17	78	\$0	\$50
Total demand	100	100	\$100	\$250

- 17 In this case, the weighted-average is taken by weighting by total wealth *and* the proportion of that wealth that the investor elects to invest in the risky assets (the second component being a measure of risk aversion). Thus, the weighted-average in the equilibrium case above is that the weighted average investor wants to invest 44% of his total wealth into asset A, computed as:

$$0.80 \times 0.5 \times 91\% + 0.20 \times 1.0 \times 37\% = 44\%.$$

- 18 Investor 1 has 80% of the total wealth (\$200 vs \$50) and places 50% of it into the risky assets with 91% of the investment in risky assets going to asset A.
- 19 Investor 2 has 20% of total wealth, invests all of it into risky assets with 37% of it going to asset A.
- 20 In this example, total wealth is \$250, so the weighted average investor (weighted by wealth and a measure of risk aversion) seeks to invest \$110 (44%) into asset A.

21 This example shows that both investors have influenced the equilibrium price according to their wealth and risk aversion. If either investor had a different level of wealth or a different degree of risk aversion, the equilibrium outcome would have been different. Ultimately, the equilibrium price of the shares was set so that the aggregate demand for shares (by investors in the market) was equal to the supply of shares in the market.

1.4 The marginal investor perspective

22 The concept of a “marginal investor” becomes relevant when trade occurs. Suppose that investor 1 seeks to increase his holding in Asset A by five shares (e.g., because he has received a pay rise), and that investor 2 agrees to sell him those shares for \$1.12 each. Both investors consider that this trade makes them better off, so the trade occurs and the market price is recorded as \$1.12, this being the new equilibrium value of the asset.

23 What we know from this is that investor 1 values those shares at *at least* \$1.12 and investor 2 values them at *at most* \$1.12. Thus, there are two marginal investors (a buyer and a seller) who may assign different values to the shares.

24 The observed market price is not some ‘marginal investor’ theoretical construct, it is simply the observed market price. It reflects the equilibrium value of an asset. If the market valued the asset higher, trading would continue and the price would rise, and vice versa. The observed price at a point in time reflects the market equilibrium valuation at that point in time.

1.5 Two different perspectives?

25 Whenever a market price is observed, it can be said to have been produced by a trade between two ‘marginal investors’ – a buyer and a seller. Thus, the ‘marginal investor’ perspective is nothing more than the use of observed market prices.

26 Under the assumptions of the theoretical representative investor models, the observed equilibrium price of an asset in the market would be set by the complex weighted-average investor. There would be an equivalence between the complex weighted-average and the observed market price.

1.6 Application to the regulatory estimate of gamma

27 The same complex weighted-average approach can be used to model the equilibrium ‘price’ or ‘value’ of imputation credits. Theoretical papers such as Monkhouse (1993) and Lally and van Zijl (2003) show that, under the assumptions of those models, the value of imputation credits that will be reflected in the equilibrium stock price is the complex weighted-average (by

- wealth and risk aversion) of the extent to which each investor is able to utilise/redeem the credits that they receive.
- 28 In a world that complied precisely with the assumptions that underpin the derivation of the complex weighted average (or “representative”) investor, the observed market price (in equilibrium) would be the same as the outcome of the model. This would only occur if all of the assumptions held in the real world. If that were the case, the market price would be the same as the price paid by the marginal investor which would be the same as the price paid by the complex weighted-average investor.
- 29 However, the models that suggest that theta can be conceptualised as a complex weighted average over investors do not apply in the case where there are TWO markets – a domestic market with some domestic investors and some domestic assets and a foreign market with foreign investors and foreign assets. Those models derive the equilibrium price by equating demand and supply across THE market, as in the example above.
- 30 Those models are, however, useful in identifying that the required return on equity must be adjusted by the value of imputation credits, gamma, but they do not imply that the available real-world estimate of the market value of credits should be discarded in favour of a theoretical conceptualisation.
- 31 Similarly, the CAPM identifies that the required return on equity is a function of the beta and MRP parameters. Having identified the relevant parameters and their role in determining the required return, market prices are then used to estimate them. For example:
- a. The CAPM assumes that there are no taxes or transactions costs, but the MRP is estimated from market prices that do reflect investors’ consideration of those things – the MRP is not estimated as it would have been if the theoretical assumptions actually did hold in the real world; and
 - b. The Black CAPM evidence suggests that the real-world relationship between beta and returns is somewhat different from the theoretical relationship under the CAPM. The AER’s approach is to adjust its beta estimate to accommodate this evidence from market prices – not to impose the theoretical relationship.
- 32 That is, where real world market evidence is available it is used – it is not supplanted by estimates of what the parameter *would have been* if the theoretical assumptions actually did hold in the real world.
- 33 In any event, the “complex weighted-average investor” is not the same as the “equity ownership approach”, and is not the same as the “tax statistics” approach. These two estimation approaches take a simple average of utilisation

rates. This is not a derivation of how the complex weighted average investor would value the asset in question. The simple average is not the same as the complex weighted average because:

- a. it ignores risk aversion entirely;
- b. it ignores all investor wealth outside Australian shares; and
- c. it ignores the fact that the weighted average investor can only be derived in the case of a single market.

34 Thus, there should be no surprise that there is a difference between the estimated market value of imputation credits and the simple average utilisation rate estimates that the AER has adopted. Differences will arise between those two estimates because:

- a. The complex weighted-average would only equal the market value under the theoretical assumptions of the model that derives that complex weighted-average. The key assumption that there is a single market where an equilibrium can be derived by equating demand and supply within the market does not hold in the real world; and
- b. In any event, the AER is unable to estimate the complex weighted-average, so it estimates the simple average instead.

35 Consequently, the regulator is left with two estimates – an estimate of the value of imputation credits in the market and an estimate of the simple average utilisation rate. Thus, the regulator must choose between:

- a. An estimate of what the value of credits *would have been* if the assumptions of the theoretical model *did* hold in the real world, and if the simple average *was* the same as the complex weighted average; or
- b. An estimate of the market value of credits, which reflects the outworking of the process by which a market-clearing price is obtained, even where that process is too complex to be captured by a simple economic model.

In our view, the market value estimate should be used, reflecting the complex process by which the market-clearing price is determined. This is the same approach that is used to estimate every other WACC parameter.

This conclusion would apply even if the AER proceeds on the basis that prices are ultimately set by a representative investor. This is because the market price embodies the complex weighted-average valuation, irrespective of how complex that might be, whereas the AER's equity ownership and tax statistics estimates do not.

2 The role of gamma in the regulatory process

2.1 Overview

36 The Australian regulatory framework, as reflected in the AER’s Post-tax Regulatory Model (PTRM) requires an estimate of gamma in two steps of the process:

- a. The first step is to produce an estimate of the total required return on equity, including the benefits of imputation credits. We refer to this as the “with-imputation” required return on equity.
- b. The second step is to remove the assumed value of imputation credits to produce an estimate of the “ex-imputation” required return on equity. This figure then flows into the revenue allowance.

37 In our view it is clear that, in the context of these calculations, gamma must reflect the value of credits – the worth of credits to investors. The reason for this conclusion is two-fold, as set out below.

2.2 Grossing-up must reflect market value

38 In the first step above, the AER estimates the total required return on equity using the SL-CAPM. The AER’s primary estimate of the MRP is the mean of historical excess returns over various long historical periods beginning in 1883. These estimates take the return on a broad stock market index each year and subtract the risk-free rate that was available to investors in that year.

39 Prior to the introduction of imputation in 1987, the observed stock market return already reflected the total return.³ However, post-imputation the observed market return is not the total return to equity holders – since it reflects only dividends and capital gains, the estimated value of imputation credits must be added via a process that the AER calls “grossing-up.” In our view, this grossing-up must reflect the market value of credits. The stock market index reflects the market value of dividends and capital gains, so the market value of imputation credits must be added to it. Adding anything other than the market value of credits would result in apples being added to oranges, producing a mish-mash that has no economic meaning.

³ That is, prior to 1987, shareholders received returns in the form of dividends and capital gains, both of which are reflected in the observed market index.

40 For example, suppose that, prior to imputation, investors required a total return on equity of 8%. We estimate this by observing stock prices in the market, so it reflects the market value of credits.⁴

41 Now suppose imputation begins and the face amount of credits is 2% but their value to investors is 1% (because of personal costs or other value impacts associated with credits).

42 In this case, stock prices would adjust so that the observed return from dividends and capital gains only would be 7%, as investors receive the other 1% of value that they require from the credits. This 7% represents the return that investors would require from dividends and capital gains, conditional on receiving credits with a value of 1%. It is a market value figure for dividends and capital gains that reflects all factors that affect the market value of those things.

43 In order to obtain an average of the total return on equity over the whole period, we would take the pre-1987 figures unadjusted (as they already reflect the total return) and the post-1987 figures would have to be grossed-up by adding 1% to reflect the value that investors obtain from imputation credits. Thus, all of the figures that are being averaged are comparable – they all reflect the total market value return.

44 Clearly, it makes no sense to gross-up the post-1987 data to reflect the face amount of credits (2%). This would involve adding a face amount to a market value which is inconsistent in itself. To then average the resulting figure with the pre-1987 market returns adds a further inconsistency. Rather, market values must be used throughout this step for it to have any economic meaning at all.

2.3 The deduction for the value of imputation credits

45 In the second step above, the PTRM removes the estimated value of imputation credits to produce an estimate of the ex-imputation required return on equity, which then flows into the revenue allowance. This step must also be done on a market value basis. To see why, consider the simple numerical example above where investors require a total return on equity of 8%. If the AER were to deduct the face amount of credits (2%) it would then allow the firm to obtain revenues that were sufficient to pay a 6% return to shareholders. If those shareholders only value the imputation credits they receive at 1%, they will be left under-compensated.

46 The reduction in the allowed return to equity holders must reflect the market value to equity holders of the imputation credits that are the reason for that reduction.

⁴ That is, market prices reflect all considerations that investors make when determining what the share is worth to the.

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