# SOME THOUGHTS ON THE UPCOMING EXPERT SESSIONS

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## 1. The Appropriate Risk-Free Rate Within the Allowed Cost of Equity

In my 2021 report for the AER, I concluded that satisfying the NPV = 0 principle requires the regulator to use a risk-free rate matching the regulatory cycle rather than the asset life. To do so, I started with the simplest possible scenario, and then progressively made it more sophisticated. In each case, the conclusion stated above holds. If this conclusion is wrong, the analysis in each of these scenarios must be wrong and use of a risk-free rate matching the asset life must instead satisfy the NPV = 0 principle. I invite supporters of the latter position to focus on these examples, most particularly the second one as follows.

Suppose that regulated assets are purchased now for $100, with a life of two years. Suppose further that the regulatory cycle is one year, and revenues are set at the beginning of each year, and the resulting revenues are received at the end of that year. Suppose further that there is no debt, opex, capex, or taxes. Suppose further that regulatory depreciation is $50 per year. Suppose further that the only risk in this world is uncertainty about the future risk-free rates. Let the current one-year spot risk-free rate be 2%, and the current two-year spot rate be 3.3% (the usual upward sloping term structure) and the one-year risk-free rate in one year (which is not yet known) be designated $R$.

In one year's time, the residual life of the asset is one year, as is the regulatory cycle. So, proponents of the two different views would presumably agree that at this point the appropriate allowed rate is $R$, yielding revenues one year later of $50 + $50*$R$. In one year's time, these revenues to be received one year later are known for certain and therefore must be discounted using the prevailing one-year rate of $R$. Their value in one year is then

$$V_1 = \frac{\$50(1+R)}{1+R} = \$50$$

In respect of the first year's revenues, my view is that they should be set using the current one-year rate of 2%, leading to revenues of $50 + $100*0.02 = $52. Both these revenues received in one year and the value $V_1$ = $50 arising in one year are known now, and therefore both warrant discounting at the one-year risk-free rate of 2%. This yields a value now for the business of

$$V_0 = \frac{\$52}{1.02} + \frac{\$50}{1.02} = \$100$$

This matches the current RAB of $100, and therefore satisfies the NPV = 0 principle. If the regulator instead set the first year's revenues using an allowed rate equal to the current two-year rate of 3.3%, the resulting revenues in one year would be $50 + $100*0.033 = $53.30 rather than $52. These revenues are still known now and therefore still warrant discounting at the current one-year rate of 2%. So, the value now of the business will then be thus:

$$V_0 = \frac{\$53.30}{1.02} + \frac{\$50}{1.02} = \$101.30$$

This does not satisfy the NPV = 0 principle. The source of the problem is using the two-year risk-free rate now of 3.3% to determine the revenues to be received in one year, but these warrant discounting at the current one-year rate of 2% because they are known now.

If revenues for the first year are set using the current two-year rate of 3.3%, the only way in which the value now of the business would be $100 (and therefore satisfy the NPV = 0 test) would be to discount these revenues due in one year by the current two-year risk-free rate of 3.3%. However, this would be an error, because this discount rate applies only to certain cash flows or values arising in two years' time.

## 2. The Use of Multiple Estimators of the MRP

One of the issues to be discussed is the merits of using estimators of the MRP in addition to historical averaging of excess returns. This is a standard problem in statistics, and the usual recommendation is to weight estimators so as to minimise the Mean Squared Error (MSE) of the combined estimator.[1]

Letting $\hat{T}$ denote an estimator and $T$ the true value, the MSE of the estimator is as follows:

$$MSE = E\left[\hat{T} - T\right]^2$$

[1] The MSE is the average over the squared differences between estimated value and the true value.

$$= E\left[\hat{T} - E(\hat{T}) + E(\hat{T}) - T\right]^2$$

$$= E\left[\hat{T} - E(\hat{T})\right]^2 + \left[E(\hat{T}) - T\right]^2 \tag{1}$$

The first term in the last equation is the variance of the estimator and the second term is the square of the bias. Suppose that one estimator is biased down by 1% (as an estimator of the MRP for the next five years), and that its standard deviation is 2%. Suppose that a second estimator is unbiased, and that it also has a standard deviation of 2%. Using equation (1), the MSE of the first estimator is $.022^2$ and is therefore larger than that of the second estimator ($.02^2$). Consequently, if the choices were restricted to only one of these two estimators, the latter would be preferred. However, one could instead form a weighted-average of the two estimators with the weight on the first ($w$) chosen to minimise the MSE of the weighted-average, i.e., letting the two estimators be denoted 1 and 2, choose $w$ to minimise

$$MSE = E\left[w\hat{T}_1 + (1-w)\hat{T}_2 - T\right]^2$$

$$= E\left[w(\hat{T}_1 - T) + (1-w)(\hat{T}_2 - T)\right]^2$$

$$= w^2 E\left[\hat{T}_1 - T\right]^2 + (1-w)^2 E\left[\hat{T}_2 - T\right]^2 + 2w(1-w)Cov(\hat{T}_1, \hat{T}_2)$$

$$= w^2 MSE_1 + (1-w)^2 MSE_2 + 2w(1-w)Cov(\hat{T}_1, \hat{T}_2) \tag{2}$$

With $MSE_1$ and $MSE_2$ as given above and no correlation between the estimators, $MSE$ is minimised with

$$w = \frac{MSE_2}{MSE_1 + MSE_2} = \frac{0.02^2}{0.022^2 + 0.02^2} = 0.44$$

i.e., a 44% weight on the biased estimator and therefore a 56% weight on the unbiased estimator. Using equation (2), the MSE on the combined estimator is then $.015^2$. So, even if an estimator were materially biased, it might still warrant significant weight in a weighted-average estimator, and reduce the MSE of the combined estimator substantially below even the better of the two individual estimators.

An even better goal than choosing an estimator with minimal MSE for the MRP over the next regulatory cycle would be to choose an estimator with minimal MSE for the MRP over the *life* of the regulated assets. In this case, under or over estimation within a single regulatory cycle would be of no great consequence relative to aggregate errors over the entire life of the regulated asset. With such a long period, shorter term biases in an estimator will tend to wash out. This point may apply to historical averaging of excess returns.

## 3. The Use of Multiple Estimators of Beta

The AER uses Australian firms for estimating beta. Foreign firms might also be used. The latter may be more biased (even local firms are biased because some of their activities are unregulated), but are also likely to have a lower standard deviation on the estimator (because there are many more firms available). It is not then apparent which would have the lower MSE. Regardless, the MSE of a combined estimator may be lower.

For example, suppose the local firms yield an unbiased estimator (generous to them) with a standard deviation of 0.15, the foreign firms yield an estimator with a bias of 0.15 and a standard deviation of 0.1 (lower than when using only local firms because of the larger set of companies). The MSE of the local estimator is then $0.15^2$ whilst that of the foreign estimator is $0.18^2$. The optimal weight on the local estimator is then 60%, and hence 40% weight on the foreign estimator. The MSE of this optimal estimator is then $0.115^2$. This significantly outperforms exclusive use of local data.

Some of the problems with using foreign firms (which induce bias) can be corrected for. For example, the betas for the foreign firms are estimated relative to the market portfolio of the foreign country, which likely differs in its industry weights from Australia. The solution to this is to reconstruct the market portfolio for the foreign market using the same industry weights as Australia, and then reconstruct the time series of returns on that market portfolio, and then re-estimate the beta of the foreign firm against that new market portfolio. Another source of bias in using foreign beta estimates is that the leverage of the foreign market (aggregate debt to equity for the set of firms whose equity comprises the market portfolio) differs from Australia. Again, this can be corrected for.

## 4. How Much Historical Data to Use

Another issue is that of how much historical to use in estimating beta (and the MRP via historical averaging of excess returns). More recent data is likely to be less biased, but also likely to have a higher standard deviation on the estimator (because the number of observations is smaller). The trade-off is reflected in equation (1).

## 5. Geometric Versus Arithmetic Means

In using historical averages of excess returns to estimate the MRP, there is a choice of arithmetic and geometric means.

Consider the analysis in section 1, with a one-year regulatory cycle but with the cost of capital including a risk allowance. Let the allowed cost of capital be denoted *R* and the appropriate discount rate be denoted *k*. Following section 1, if *R* is a constant, the regulatory problem is then to choose *R* so that

$$V_0 = \frac{\$50 + \$100 * R}{1+k} + \frac{\$50}{1+k} = \$100$$

This implies *R* = *k*, i.e., the allowed rate equals the cost of capital. However, since *R* is obtained by historical averaging, *R* is then a random variable.[2] So too will the value now $V_0$. The regulatory problem is then to choose between arithmetic and geometric averaging so that E($V_0$) = \$100. Coupling this requirement with the preceding equation implies that

$$\$100 = \frac{\$50 + \$100E(R)}{1+k} + \frac{\$50}{1+k}$$

It follows that *E*(*R*) = *k*, i.e., the expected value of the estimate for the allowed cost of capital *R* must be equal to the true (but unknown) cost of capital *k*. The value *k* comes from the CAPM, and is by definition an expectation, i.e., an arithmetic mean over the population distribution of possible returns. The choice of *R* (arithmetic or geometric mean) must then be likewise, which requires an arithmetic rather than a geometric mean over past returns.

[2] Averaging applies to the MRP estimate rather than the entire return but this does not alter the fundamental issue here.

To illustrate this, suppose annual returns on the relevant asset are 0 or 20% with equal probability. The expected rate is 10%, which is $k$. Using past returns to estimate this, use of arithmetic averaging will yield $E(R) = 0.10$. Use of geometric averaging will not. For example, suppose two years of past returns are used for averaging. The possible arithmetic averages are then 0 (if both past years yield a return of 0), 0.10 (if one year yields a return of 0 and the other a return of 0.2), and 0.20 (if both years yield returns of 0.2), with respective probabilities of 0.25, 0.50, and 0.25. The value for $E(R)$ is then 0.10, which equals $k$. However, the possible geometric averages are 0, 0.095, and 0.20 with probabilities of 0.25, 0.50, and 0.25. The value for $E(R)$ is then 0.0975, which is not equal to $k$.