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The Low Beta Anomaly and Interest Rates

The reasons for outperformance in smart beta portfolios remains a mystery. We extend previous literature on the link between portfolio performance and macroeconomic factors by exploring the response of a low beta portfolio to interest rate movements. The implications for fund managers heavily invested in low-risk strategies where the immediate risk lies in the future rise in interest rates are worth considering. In particular, low beta funds appear to go up when interest rates fall more than when interest rates rise. We focus on the case of US equity investment based on the capital asset pricing model (CAPM). We find that the anomaly is partially explained by interest sign changes due to macroeconomic events, and observe heterogeneous impacts for low and high beta portfolios.

One of the observations over the cross-section of stocks is that the historical risk-return trade-off is flat or inverted: within the CAPM, we would expect that stocks with high systemic risk would outperform their low risk counterparts, but results have shown otherwise. It is an empirical fact that interest rates have been declining over the recent decades, and there is evidence that interest rate movements affect portfolio choice. The question then arises whether there are heterogeneous impacts to the interest rate for high and low beta portfolios, as the anomaly arises from the observation that low beta portfolios outperform their high beta counterparts. We want to find the origin of this so-called "anomaly", which we believe is linked to the behavior of portfolios to interest rates.

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There is some evidence that in the context of Sharpe's market model [SHA 64] the differing exposures to interest rate movements are not captured by systematic risk, but by an alpha effect that is heterogeneous over portfolios. We observe that low beta portfolios outperform high beta portfolios at times of low interest rates: we saw a steady decrease in interest rates over 1980–2010, which matches the period of low beta outperformance. However, a model that estimates the interest rate effect as a structural break would fail to take the one period nature of the CAPM into account, and the resulting effect on the *ex ante* expectations set by the model. This relates directly to the setting of the interest rate target by the Federal Open Market Committee (FOMC); movements in the target rate are gradual, almost constant in magnitude, and highly persistent.

Hence, we propose a method where we use sign changes in interest rates to capture the underlying macroeconomic policy implications for actual reactions of investors. The heterogeneous impact can be quantified through the effect on the intercept of the CAPM, indicating a violation of the CAPM assumptions and suggesting a change in behavior around a zero change. We validate the threshold with a grid search along the likelihood function of our data, and link the asymmetry in the portfolio returns to the persistence of interest rate sign changes.

At the source is the trade-off between being implicitly long or short bonds in times of interest rate changes, and the mismeasurement that occurs if we do not account for the term structure. This is at the heart of the argument in this chapter, which is that the type of interest rate used is dependent on the composition of investors in the market. Investors differ in their degree of risk aversion, and we argue that this is pronounced through either a spread between a borrowing and lending rate, or investing on different parts of the yield curve. The argument follows from the observation of inverted yield curves in times of recession, and suggests that the anomaly arises from exogenous macroeconomic influences.

There are two lines of argument as to why low and high beta portfolios react differently: first, the opportunity cost when the interest rate decreases makes safer investments more attractive, and second, the interest rate is a reflection of real economic conditions and economic health, which particularly impacts firms that have more gearing. We do not see a similar switch in high beta portfolios as of the heterogeneous gearing across firms in a high beta portfolio: firms that are riskier are generally more equity financed in absolute terms rather than leveraged on debt.

As firms with a lower market beta usually have a higher gearing ratio, we expect that increases in the interest rate affect their performance more than firms with a higher market beta; low beta portfolios will have a lower return when interest rates increase, but see a higher return when the interest rate is decreasing. Thus, interest rate changes affect low beta portfolios asymmetrically because of the underlying composition of debt.

We combine the literature on leverage constraints with macroeconomic factors and studies relating to the term structure of interest rates (see [EST 96] and [BAL 10]), where we distinguish portfolios as heterogeneous investors as in [BRE 93]. We argue that the term structure of interest rates and the impact of heterogeneous risk aversion across investors lead to a discrepancy between portfolio returns, and that the anomaly arises for a failure to account for this effect. The chapter focuses on two potential explanations of the low beta anomaly, namely interest rate sign changes and failure to account for the interest rate term structure.

13.1. Literature review

The anomaly has been recognized empirically in many applications (see, for instance, [BLA 72a, BLA 72b, FAM 92, HAU 75]). Baker and Wurgler [BAK 11] provide an extensive review in favor of the low beta anomaly. Also, see Ang *et al.* [ANG 06], who find that stocks with higher idiosyncratic risk earn lower returns in all cases considered.

The causes of the anomaly and how to quantify them are at the heart of the literature: for instance, approaches using mismeasurement and volatility premiums on high risk stocks [DIB 12, KLE 13], the impact of unobservables and leverage on the returns [FAM 96, COC 13, FRA 11], approaches using cumulative prospect theory from [KAH 92] to model lottery preferences and different preferences in the loss domain [COR 08, BAR 08, BHO 11, LEV 12, KUM 09] and manager behavior perspectives [CHE 97, SRI 98, ASN 12].

We focus on the literature relating to unobservables and underlying leverage, and combine it with macroeconomic factors and studies relating to the term structure of interest rates [EST 96, BAE 10], where we distinguish portfolios as heterogeneous investors as in [BRE 93]. We argue that the different portfolio return distributions for interest sign changes lead to a discrepancy between low and high beta portfolio returns.

Di Bartolomeo [DIB 12] and Klepfish [KLE 13] argue that high-frequency arithmetic rates of returns are mistakenly compared to the geometric rates of return over longer period, leading to a volatility premium. For instance, we can show that a discrete return adjusted for a volatility premium can be expanded as a Taylor series:

$$E_t(\frac{P(t+1)}{P(t)}-1)\text{-}E_t(\ln(P(t+1))-\ln(P(t))=\frac{1}{2}(\mu^2-\sigma^2)+o(\mu^3)$$

The symbol $o(\mu^3)$ means that the remainder is of order three in the instantaneous mean. We note too that under these assumptions, as long as the instantaneous mean is small, we require that μ be greater than σ in absolute value for arithmetic expected returns to be greater than geometric ones. Not accounting for this factor causes substantial differences between arithmetic and geometric returns, particularly in their average volatility. Hence, portfolios with a higher beta would underestimate the expected return if the volatility bias is not taken into account. Related is the work by Haugen and Wichem [HAU 74] who explore the impact of holding duration of risky versus riskless assets on their relative price volatility.

Mispricing can also occur through the effect of unobservable factors, as in the three factor model by Fama and French [FAM 96]. This model uses three stock specific factors that offer potentially orthogonal dimensions of risk and a return [SCH 11] premium for investors willing to take the risk with these factors [COC 13]. The factor premiums capture effects formerly incorporated in a CAPM intercept, which implies that the higher low risk return is not an anomaly but a mismeasurement of missing factors.

This is related to leverage constraints on portfolio choice. Frazzini and Pedersen's [FRA 11] explanation of this phenomenon follows from the preference of investors to carry more risk than the market can provide, but leverage is costly to obtain. In turn, these investors buy high beta stocks instead of leveraging, driving up the cost for high beta stocks relative to low beta counterparts. An extension using option theory is provided by Cowan and Wilderman [COW 11]. In the context of our simple model, explicitly levering low beta simply gives the high beta portfolio due to two fund money separation so we will not pursue this explanation.

The riskiness of leverage strategies is determined by the underlying risk-free rate: interest rates can affect the portfolios through the effect of maturity premia and the borrowing constraints of investors. The yield curve shows the range of interest rates across bonds of the same risk and liquidity but with differing maturities. It is argued in previous work by Estrella and Mishkin [EST 96] that the slope of the yield curve is a good predictor of recessions in the US as the sign gives an indication of whether the economy is slowing down or the money supply is tightening. In economic turmoil, it is possible that the yield inverts: as the long-term interest rate represents the risk-adjusted average of the expected future short-term interest rates and the long-term interest rates will fall, but by a smaller amount than the short-term interest rates. Others confirming this result are Adrian *et al.* [ADR 10], Bernanke and Blinder [BER 92], Bernard and Gerlach [BER 98] and Rudebusch and Wu [RUD 04], who find evidence in favor of the prediction power of the term structure.

Furthermore, the magnitude of changes in the target interest rate has been remarkably constant, regardless of the sign of the respective change (see, for instance, [COI 11, GOO 05, GUR 05]). Also, Coibion and Gorodnichenko show that there is substantial persistence in the target rate set by the FOMC, which implies that there are cumulative, non-independent expectations of interest rate changes. The leverage argument provides substantial insight as to how portfolio returns may differ with regard to their interest sensitivity, with more importance to the gearing on debt of the firms underlying the portfolio that causes the anomaly. As the gearing ratio is an indicator of the debt structure of a firm, there are heterogeneous responses to interest rate movements over high and low beta firms. We reconcile the above approaches to argue that failure to account for interest rate movements leads to substantial mispricing which causes the low beta anomaly.

13.2. The anomaly and interest rates

Let μ_i, μ_m be the expected arithmetic rates of return on asset *i* and the market *m*, respectively. Let β_i, r_f be the population beta of asset *i* with respect to the market *m* and the riskless rate of return, respectively. The CAPM states:

$$\mu_i - r_f = \beta_i(\mu_m - r_f) \quad [13.1]$$

We will look at this relationship to see how changing conditions influence the price of the asset. We can conceive of this as being the following things within the model framework: (1) multiple changes, (2) changes in the risk premium, (3) changes in expectations of future earnings and (4) changes in aggregate risk aversion.

Noting that at time t, $\mu_i = \frac{E_t(P_{i,t+1})}{P_{i,t}} - 1$, where $E_t(P_{i,t+1})$ is the expectation held at time *t* of the price of asset *i* at time *t*+1, an amount that would take into account expected capital gains and dividends:

$$P_{i,t} = \frac{E_t(P_{i,t+1})}{1+r_f+\beta_i\,(\mu_m - r_f)} \quad [13.2]$$

Suppose, we were to consider a change in the market expected rate of return and a simultaneous change in the riskless rate of return. We denote these changes by $d\mu_m$ and dr_f, respectively. Let the change in the price be dp_{it}. Thus:

$$dP_{it} = \frac{dP_{it}}{dr_f} dr_f + \frac{dP_{it}}{d\mu_m} d\mu_m$$

$$dP_{it} = \frac{-E_t(P_{i,t+1})}{(1+r_f+\beta_i(\mu_m - r_f))^2}\left(dr_f + \beta_i\left(d\mu_m - dr_f\right)\right) \qquad [13.3]$$

Since the terms to the left of the brackets are unambiguously negative, we can see that a total change in the risk premium $\left(d\mu_m - dr_f\right)$ that is positive, say 2% with an asset with a beta of 0.5 will decrease prices as long as the associated interest rate fall is less than 1%. There is a difference in the response across portfolio types: as high beta portfolios are linked to being short bonds while low beta ones are long bonds, the latter carry a different sensitivity to the interest rate. By going long on the riskless bond, low beta portfolios see an increase in their relative return in times of interest decreases, while high beta portfolios see a decrease under similar conditions.

Ross [ROS 71, ROS 76] developed a theory of asset pricing following the attack on the conclusions reached by the CAPM as equity returns are not normally distributed and the model is not empirically validated. Arbitrage pricing theory (APT) follows from the notion that, for any financial asset, there is no single systematic risk factor but rather a combination of many. One of the main implications of the APT is the principle of diversification, meaning that idiosyncratic risk is not present for well-diversified portfolios.

$$\mu_i - r_f = \beta_{i1}\left(\mu_{m1} - r_f\right) + \cdots + \beta_{ik}\left(\mu_{mk} - r_f\right)$$

Burmeister *et al.* [BUR 03] provide an overview of the methods in which risk factors can be included in the empirical justification of the APT. Again, empirical specifications of the APT are subject to the critique of Fama and French [FAM 96] as we can think of an infinite set of factors that might have an influence on the expected returns: hence, there is a need for a proper theoretical foundation of the factors. For instance, interest rate risk is identified as a strong potential risk factor. We write the CAPM as follows:

$$\mu_i = \beta_i \mu_m + (1 - \beta_i)\, r_f$$

Inspecting the CAPM above, it is clear that, if we are in equilibrium, a fall in the interest rate will lower the expected rate of return for a low beta asset and raise the expected rate of return for a high beta asset. A possible explanation of a failure of modeling this in the CAPM lies in the difficulties of using a one period model with a time series of data, and the failure to provide insights into disequilibria.

By decomposing the CAPM to incorporate the risk-free rate directly, we see that macroeconomic interest rate movements have a direct impact on the portfolio returns. Our contribution is empirical but has a theoretical basis: interest rate movements follow from the CAPM as a subcase of the APT and we estimate the potential difference in impacts for low and high beta portfolios. Following from the observation that the magnitude of interest rate changes is fairly constant, we argue that interest rate sensitivity is captured by the sign changes and cumulative persistence of the target rate.

A rise in the interest rate is equivalent to a fall in the price of "cash" and shorting such an asset will increase the value of the portfolio, the high beta stock. We argue that the cost of taking on gearing is related to interest rate movements: when the relative cost of borrowing increases, firms underlying a low beta portfolio which generally take on more debt are more affected than firms that are mostly equity financed: investment moves toward (away) high (low) beta portfolios, driving up (down) the price and return of these products.

13.3. Model specification

In keeping with an APT interpretation, we extend the traditional CAPM analysis by including a term that captures the relative leverage of portfolios to the risk-free rate. In order to test for heterogeneous impacts for high and low beta portfolios, we study two portfolios with differing beta exposures.

$$r_t = \alpha + \beta \; r_{mt} + V_t \qquad [13.4]$$

For a time series regression on a single portfolio, the ordinary least squares estimator (OLS) will be unbiased and efficient if the characteristics of our error term and estimator follow the Gauss–Markov assumptions. Under a correct CAPM specification, we should find that the intercept term α is insignificant in the specification. However, many attempts at CAPM modeling have concluded that this is not the case, particularly for low beta stocks. To capture why we would see a non-zero intercept, we estimate the CAPM again but model the changes in the interest rate directly as an extra factor:

$$r_t = \alpha + \beta \; r_{mt} + \gamma \Delta r_{ft} + V_t \qquad [13.5]$$

We expect that portfolios with different degrees of systematic risk are affected asymmetrically: low beta portfolios are expected to be negatively affected by the positive changes in the interest rate, while high beta returns are expected to increase.

Rather than modeling the magnitude of interest rate changes, we are more interested in the effect of interest sign changes on the portfolio intercept and market beta as the magnitudes of changes in the rate are constant over time. A structural break analysis at the point of major change in interest rate movements only gives us information on the effect on different samples rather than the actual change in expectations. We propose a threshold analysis where we estimate the CAPM based on the sign of the interest rate change around a reference point *c*:

$$i_t = \begin{cases} 1 \; if \; \Delta r_{ft} > c \\ 0 \; if \; \Delta r_{ft} \leq c \end{cases}$$

The reference point takes a natural value of zero when we are interested in the sign of interest rate changes. We estimate the threshold using a grid search upon the likelihood function with refined tolerances as a robustness check. We estimate the model with interaction terms with the market premium to test whether interest rate changes also affect systemic risk of a portfolio.

$$r_t = \alpha_1 + \beta_1 \; r_{mt} + \alpha_2 i_t + \beta_2 i_t * r_{mt} + V_t \qquad [13.6]$$

13.4. Empirical analysis and results

As the CAPM is a one period theory of portfolio choice of a representative agent, we need to be clear on which interest rate would correspond to the dominating factor. We estimate the model using the 10-year bond rate as well as a mixed equilibrium rate. We argue that there is no distinct difference between the monthly T-bill rate and the 10-year bond rate when it comes to their general movements over the time period, but in terms of changes and volatility there is a major difference. The short-term rate is much less volatile than the long-term rate, which can have substantial differences in a one period model such as the CAPM. Hence, even though interest rates in general may have been declining over the recent decades, what matters is the change over the time frequency which explains our preference for a sign change indicator rather than a structural break analysis.

We use long run industry level data to analyze beta effects. The source of the data is the monthly industry level Fama–French industry level returns from Kenneth French's Website. We use 43 industry groupings from 1953.01 to 2012.12 to calculate full sample betas. Some initial rolling calculations on the data found five industries that had betas less that 1 (defensive) and nine with betas greater than 1 (aggressive). The defensive industries are food products, tobacco, oil, utilities and telecoms. The aggressive industries are building materials, fun and entertainment, construction, steel, machinery, electrical equipment, chips, lab equipment and financials. Then, we build market capitalization-weighted portfolios of the high beta and low beta industries.

The rationale for this methodology could also be construed in Bayesian terms. We could argue that we have prior beliefs about the nature of certain sectors, for example, we think of utilities as defensive and computers as aggressive. The reason for taking this approach is that it avoids the high degrees of uncertainty in estimated beta. Our empirical approach simply supports what could be justified by prior beliefs. Summary statistics are available upon request, where the numbers reported show noticeable differences between arithmetic and geometric returns. We also report the medians and standard deviations of geometric returns. In all periods, and overall, the standard deviations of high-beta portfolios are higher than those of low-beta portfolios.

We estimated the CAPM by regressing portfolio excess returns on an intercept and market excess returns, and present our results in the first panel of Table 13.2. We would expect the intercept to be zero if the CAPM holds; interestingly, the low beta portfolio has a positive intercept, while the high beta portfolio does not. This demonstration shows the returns to low risk portfolios based on a CAPM theory of risk. Investing in low beta portfolios gives us an extra 3.68% per annum relative to what the CAPM suggests.

Panel 1	*Interest* ↑	*Interest* ↓	*HIB* ↑	*HIB* ↓	*LOB* ↑	*LOB* ↓
Mean	0.016	-0.016	0.568	0.803	0.003	1.354
Standard Dev	0.016	0.018	5.891	5.829	3.514	3.630
Skewness	2.504	-3.224	-0.592	-0.340	-0.533	0.021
Kurtosis	10.903	15.475	3.407	0.880	1.384	1.259

Panel 2	*Pre*	*Post*	*HIBPre*	*HIBPost*	*LOBPre*	*LOBPost*
Mean	6.088	6.232	0.666	0.708	0.547	0.825
Standard Dev	2.948	2.553	5.467	6.230	3.445	3.813
Skewness	1.146	0.581	-0.048	-0.747	0.150	-0.495
Kurtosis	0.935	-0.048	0.739	2.883	1.456	1.414

Table 13.1. *Moments of 10-year rate, HIB and LOB conditional on interest changes*

Panel 1	α	$t(\alpha)$	β	$t(\beta)$	γ	$t(\gamma)$	R^2	-
Equation (4)								
HIB	-0.007	-0.102	1.274	81.138	-	-	0.902	-
LOB	0.307	4.096	0.696	40.857	-	-	0.699	-
Equation (5)								
HIB	-0.011	-0.154	1.282	81.324	10.740	3.569	0.903	-
LOB	0.313	4.254	0.681	40.267	-17.585	-5.448	0.711	-
Panel 2	α	$t(\alpha)$	β	$t(\beta)$	δ	$t(\delta)$	R^2	–
Sample Break								
HIB	-0.061	-0.612	1.273	80.863	0.113	0.792	0.903	-
LOB	0.404	3.911	0.702	40.982	-0.201	-1.332	0.711	-
Panel 3	α_2	$t(\alpha_2)$	β	$t(\beta)$	$\alpha_1 - \alpha_2$	$t(\alpha_1 - \alpha_2)$	$\beta_1 - \beta_2$	$t(\beta_1 - \beta_2)$
Equation (6)								
HIB $\beta\alpha$	-0.270	-2.852	1.273	57.730	0.545	3.977	0.011	0.347
HIB α	-0.280	-2.928	1.278	82.087	0.551	4.053	-	-
Equation (6)								
LOB $\beta\alpha$	0.766	7.381	0.693	29.466	-0.923	-6.321	-0.010	-0.297
LOB α	0.770	7.496	0.688	41.424	-0.929	-6.402	-	-
Panel 4	α_1	$t(\alpha_1)$	α_2	$t(\alpha_2)$	β	$t(\beta)$	R^2	-
Double Alpha								
HIB	0.269	2.789	-0.282	-2.928	1.278	82.087	0.904	-
LOB	-0.159	-1.540	0.770	7.496	0.688	41.424	0.716	-
Panel 5	α_1	$t(\alpha_1)$	β	$t(\beta)$	$\alpha_1 - \alpha_2$	$t(\alpha_1 - \alpha_2)$	–	–
Estimated (6)								
HIB c1	-0.282	-2.928	1.278	82.087	0.551	4.053	-	-
HIB c2	-0.307	-3.033	1.279	82.001	0.549	4.018	-	-
HIB c3	-0.255	-2.907	1.280	82.267	0.624	4.486	-	-
Estimated (6)								
LOB c1	0.770	7.496	0.688	41.424	-0.929	-6.402	-	-
LOB c2	0.648	6.870	0.687	41.028	-0.860	-5.745	-	-
LOB c3	0.750	7.520	0.686	41.294	-0.947	-6.506	-	-

Table 13.2. *CAPM results per specification*[1]

By including the change in the interest rate (10-year bond rate) as in equation [13.5], we see an increase in the explanatory power for the low beta model and significance for both specifications. The estimates are of opposite signs, which

1 Numbers are estimates of coefficients of variables, including constant and market risk, and the 10-year rate, respectively. Values of the t-statistic above 1.645 indicate significance above the 5% level, while values above 2.326 indicate significance above the 1% level.

confirms our hypothesis that the low beta portfolio is negatively affected by positive changes in the risk-free rate, while the opposite holds for high beta portfolios. We expect alpha to be significantly different from zero and negative for a portfolio with low beta, and insignificant for high beta portfolios. Table 13.2 shows that alpha is a significant factor for low beta portfolios, albeit not negative. The negative sign is captured by the estimate on the changes in the interest rate.

In the second panel of Table 13.2, we allow for a structural break in 1983 when interest rates started to decrease. From the results, we see that there is no significant difference in the two samples[2], and no evidence of an increase in systematic risk for either portfolios in the different interest rate regimes. There is some significance on the 15% level for low beta portfolios, but this only results in a double alpha effect rather than a systematic change. Table 13.1. shows the moments for the interest sign changes (Panel 1) and interest magnitude changes (Panel 2); clearly, a structural break is unable to pick up the asymmetry in mean returns in the way a sign change does. The moments for interest rate changes are remarkably similar across positive and negative times for both the sign and magnitude specification. The abnormal return for low beta portfolios for negative interest changes is more visible with sign changes, where we see a positive return of 1.354 for negative changes and 0.003 for positive changes.

Next, we turn to the model specification in equation [13.6]. Using the indicator setup, we are able to pinpoint the impact of the sign of interest rate changes from a reference point, and absorb these changes in double alpha and/or double beta effects. It is clear from Table 13.1 that the distribution of positive and negative changes is quite different over the two sample periods: before 1983, there were significantly more positive interest rate changes than from 1984 onward, which can be explained by the rise of monetarism and the focus on inflation fighting by the chairman of the Federal Reserve under Reagan's administration. Furthermore, the number of "ups" and "downs" is remarkably similar in structure in that the proportion of decreases prior to 1983 is approximately the same as the proportions of increases post-1983. This also suggests that our data set represents a fairly complete epoch of history as the overall proportion of ups, taken over both periods, is very close to 50%. Together with the relatively constant magnitude of the changes in the target interest rate set by the FOMC, this provides evidence that the sign of the change is more important for the expected return.

Panel 3 of Table 13.2 presents our results for interest sign changes. We observe that the impact of the sign of interest rate changes is not captured by a two beta model for both low and high beta portfolios. Instead, the impact is captured by a two

2 Structural break within equation [13.5] around 1983–1. Estimates are the coefficients of the constant, market risk and changes in the 10-year rate, respectively.

alpha model for both portfolios[3]. Focusing on the alpha effect-only models, we see that alpha becomes a significant factor in the high beta portfolio if we include the sign of interest rate changes (see Panel 4). The estimates for beta in both portfolios hardly change when we include the sign changes, suggesting that systemic risk itself is not affected.

Continuing with our discussion of Panel 4, we demonstrate the impact on low beta portfolios as an example of the total effect on the intercept when including sign changes. We see that the intercept is positive (0.770) whenever we have a negative change in the interest rate as alpha is positive and the indicator takes value zero. This result is in line with the observation that returns of low beta portfolios are positively affected by negative changes in the interest rate. Whenever we see positive changes in the rate (and the indicator takes value unity), alpha for low beta portfolios is negative (–0.159) which confirms our hypothesis that low beta portfolios are asymmetrically affected.

The opposite mechanism holds for high beta portfolios: a decrease in the rate leads to a decrease in the return (–0.282) and an increase leads to a positive change (0.269). Again, this confirms our hypothesis that low beta portfolios outperform high beta counterparts in times of interest rate declines. We found some evidence for a lower alpha in the period leading up to 1983 for low beta than in the period after 1983 and thus gives support to the argument listing interest rates as a factor in low beta outperformance. Interest rates are a significant factor in low beta outperformance and extend the result to the one period CAPM model.

We checked our estimates of our preferred model, the double alpha specification from equation [13.6], for robustness by estimating the reference point using a refined grid search over the likelihood function to find the global minimum. We find the possible minimum and maximum value of the threshold and start by estimating the model for each step starting from the minimum, and computing the sum of squared residuals at each point. Then, we find the optimal threshold by minimizing the Residual Sum of Squares (RSS) function[4].

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3 Estimates of the double alpha model in equation [13.6], rewritten to reveal the underlying significance of the alpha parameters. We test whether alpha 1 and alpha 2 are statistically different, and we find that a Wald test rejects equality for both high (15.394) and low (39.967) beta portfolios.

4 The distribution of the estimates is non-standard and can be estimated using bootstrapping methods. We use three refinement scales (steps of 0.01, 0.001 and 0.0001 which we denote by c1, c2 and c3). We find that the points are not significantly different from zero for the largest refinement scales, but they are for the finest scale (minimum at 0.0034, 95% confidence interval of 0.0648, 0.1021). But, this distance is so close to zero that we do not change our results.

The results are presented in Panel 5. We see that there is not a significant difference from zero, and the estimates are robust to the refinement level. The results support our original model. We find that there is strong evidence that the alphas are for both portfolios, but with opposite signs depending on the interest rate changes. Figure 13.1 shows the behavior of the RSS function for the low beta portfolio, and shows a clear minimum at, or very near, the reference point. We obtain a similar result for the high beta portfolio.

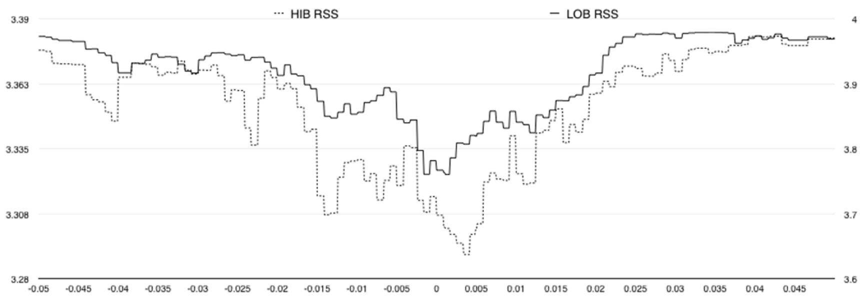

Figure 13.1. *Residual sum of squares (RSS) to the threshold level for the low-beta portfolio*

Hence, the preferred specification is equation [13.6] where we only allow for a double alpha effect. We see that the sign of the interest rate change is the most significant in distinguishing the effects for both portfolios: whenever we see an increase in the interest rate from the reference point, low beta portfolios will be negatively affected while the opposite holds for high beta portfolios. The strategy with low beta portfolios means being implicitly long on the riskless asset. Empirically, whenever we see a shift in the risk-free asset, low beta portfolios are more affected than high beta portfolios. The estimates show that there is a significant alpha impact in this specification for high beta portfolios, which can be explained by portfolio rebalancing after underlying interest rate movements.

As a robustness check, we estimate model [13.6] including positive changes in the interest rate (dyield+) and negative changes in the rate (dyield-). Positive changes are collected in dyield+ as their actual value, where negative or null values are set to zero (similarly for negative changes). A Wald test testing for the equivalence of the effects and bringing us back to [13.5] shows that the parsimonious model is equivalent, suggesting that there is no difference in upward or downward movements of interest rates for this particular model.

Time-varying estimates are computed over a rolling window without overlap (Figure 13.2). We see, unsurprisingly given the construction methodology, that the result for beta is stable and shows that low beta portfolios indeed see a lower systematic risk than high beta portfolios, except for very specific periods. We see that low betas spiked above high beta portfolios in 1994 during the bond price crash: after a long recession with falling inflation, the cycle turned aggressively in this year after economic recovery and a rise in the federal interest rate. The estimates for alpha are less consistent, but show a clear distinction between high beta and low beta portfolios and mean changes over specific periods. We see that the behavior of the low beta alpha mirrors that of high beta and observe similar implications for the interest indicator variable.

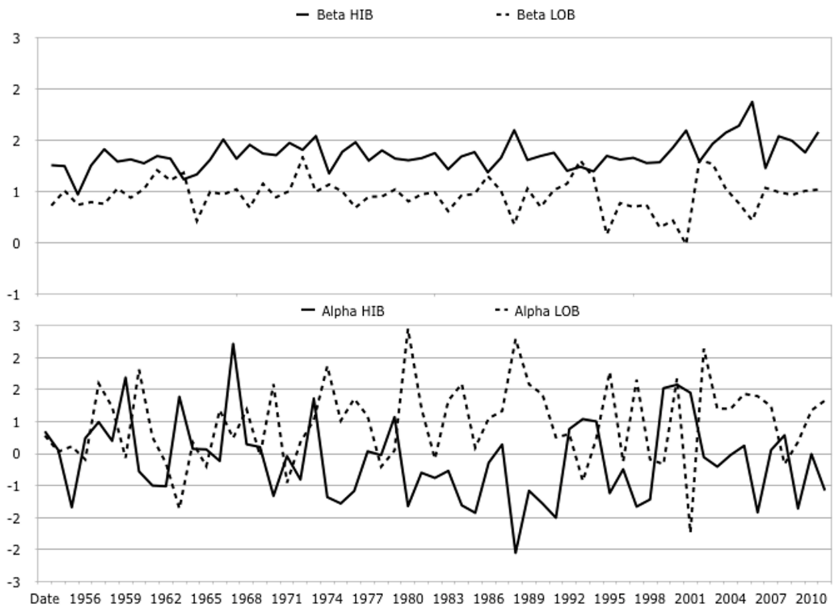

Figure 13.2. *Time-varying estimates of equation [13.6]*

13.5. The anomaly and interest maturity mismatch

One immediate difficulty with the CAPM is that, in its static form, it is not especially informative about what interest rate we should be using. The CAPM, as discussed above, is a one period theory and the interest rate used would correspond to the holding period of the representative agent. This chapter presents an explanation as to why we might find significant interest rate returns within a CAPM framework. The motivation comes from a traditional theory of interest rate demand often known as the preferred habitat hypothesis. In this model, investors have at their disposal a bond of a particular maturity, reflecting, perhaps the duration of their liabilities or other considerations. We will capture this by building a CAPM-type model which assumes that there are two agents, both of whom are mean variance optimizers, both confronted by the same set of risky assets, both believing in the same asset price distribution with identical means and variances but having as choice of riskless asset a short-rate bond in one instance and a long-rate bond in the other.

The optimization problem they face is:

$$U = \omega' E(r) - \frac{\lambda}{2}\omega' \Sigma\omega - \theta(\omega'\imath - 1)$$

where θ is the Lagrange multiplier and λ is the coefficient of absolute risk aversion, ω is a vector of portfolio weights chosen to maximize [13.1] and i is a vector of ones. The vector of expected rate of return of the risky assets is E(r) = μ, and the covariance matrix of returns is given by Σ. We note the following result. The optimal mean-variance weights in the presence of a budget constraint with known parameters are given by*:*

$$\omega = \frac{1}{\lambda}\Sigma^{-1}\mu - \frac{(\beta-\lambda)}{\lambda\gamma}\Sigma^{-1}i$$

We define $\alpha = \mu'\Sigma^{-1}\mu$, $\beta = \mu'\Sigma^{-1}i$, $\gamma = \imath'\Sigma^{-1}\imath$. The expected utility associated with this case is given by, substituting the first into the second equation and simplifying. The maximized value, V, is given by:

$$V = \frac{\alpha\gamma-(\beta-\lambda)^2}{2\lambda\gamma}$$

If we ignore the budget constraint in the optimization, then the optimal portfolio becomes $\omega = \frac{1}{\lambda}\Sigma^{-1}\mu$ and *E(r)*= $\frac{\alpha}{2\lambda}$. Formally, the optimal portfolios where i = 1,2 for short and long rates (r_{1f} and $r_{2f}$are the short and long rates, respectively) are given by:

$$w_i = \frac{1}{W_0^i}\lambda_i^{-1}\Sigma^{-1}\left(E(r) - r_{if}\right).$$

This is the same result as given above except that individuals differ in terms of initial wealth, absolute risk aversion and riskless rates of return. Defining societal wealth as W_{m0}:

$$W_{m0} = W_0^1 + W_0^2$$

Thus, societal investment in the different assets (i.e. the market portfolio) is equal to w, and where $\lambda = ((\lambda_1)^{-1} + (\lambda_2)^{-1})^{-1}$ is societal risk aversion.

We can distinguish two types of agents, with a different risk aversion λ_i. Both agents would choose the same market portfolio, but have a different slope of the riskless rate. The interest rate for investor (1) is lower than the rate for investor (2): in normal economic conditions, this would imply that investor (2) invests on the longer part of the yield curve. Therefore, we can see that it is far from trivial which interest rate we should be using when we depart from the assumption of homogeneous risk aversion.

Therefore, the optimal portfolio weights are:

$$w = \frac{1}{W_{m0}}\Sigma^{-1}E(r)\left(\left(\frac{1}{\lambda_1}+\frac{1}{\lambda_2}\right)\right)-\frac{1}{W_{m0}}\Sigma^{-1}(i)\left(\left(\frac{r_{1f}}{\lambda_1}+\frac{r_{2f}}{\lambda_2}\right)\right)$$

Now,

$$\text{Cov (r,w'r)=}\Sigma w = \frac{1}{W_{m0}}\left(E(r)\left(\left(\frac{1}{\lambda_1}+\frac{1}{\lambda_2}\right)\right)-(i)\left(\left(\frac{r_{1f}}{\lambda_1}+\frac{r_{2f}}{\lambda_2}\right)\right)\right) = aE(r)+bi$$

$$Var(w'r) = w'\Sigma w = \mathrm{a}\mu_m + b.$$

Therefore, $\beta = \frac{\mathrm{aE(r)+bi}}{\mathrm{a}\mu_m+b}$.

Thus, $\mathrm{aE(r)} + \mathrm{bi} = \beta(\mathrm{a}\mu_m + b)$.

Dividing both sides by a, we arrive at:

$$E(r) - \frac{\left(\frac{r_{1f}}{\lambda_1}+\frac{r_{2f}}{\lambda_2}\right)}{\left(\frac{1}{\lambda_1}+\frac{1}{\lambda_2}\right)}i = \beta\left(\mu_m - \frac{\left(\frac{r_{1f}}{\lambda_1}+\frac{r_{2f}}{\lambda_2}\right)}{\left(\frac{1}{\lambda_1}+\frac{1}{\lambda_2}\right)}\right)$$

This we call the heterogeneous interest rate CAPM. Defining the relative risk tolerance of the short-rate investors as δ_1 with the relative risk tolerance of long-rate investors being δ_2, it follows immediately that $\delta_1 + \delta_2$=1. The interest rate term in the heterogeneous interest rate CAPM now becomes $\delta_1 r_{1f}+\delta_2 r_{2f}$ and we can write our CAPM as:

$$\mathrm{E(r)} - (\delta_1 r_{1f}+\delta_2 r_{2f})i=\beta(\mu_m-(\delta_1 r_{1f}+\delta_2 r_{2f}))$$

The question that naturally arises is: would we expect the long-rate investors to be more risk averse than the short-rate investors? We would think this to be the case so that the short-rate investors would tend to dominate; that is $\delta_1 > 50\%$.

Suppose, we now run a conventional short-rate CAPM. We would assume the constraint:

$$E(r) - r_{1f}i = \beta(\mu_m - r_{1f})$$

instead of the true model [13.1]. This misspecification would lead to additional terms:

$$\text{E(r)-}\ r_{1f}i = \beta(\mu_m\text{-}r_{1f}) + \beta(r_{1f}\text{-}(\delta_1 r_{1f} + \delta_2 r_{2f}))\ \text{-}r_{1f}i + \text{i}(\delta_1 r_{1f} + \delta_2 r_{2f})$$

$$\text{E(r)-}\ r_{1f}i = \beta(\mu_m\text{-}r_{1f})\text{+}\beta((1\text{-}\delta_1\ )r_{1f}\text{+}\delta_2 r_{2f})) + \text{i}((\delta_1 - 1)r_{1f}\text{+}\delta_2 r_{2f})$$

$$\text{E(r)-}\ r_{1f}i = \beta(\mu_m\text{-}r_{1f})\text{+ }(\beta - i)(1\text{-}\delta_1\ )r_{1f}\text{+}(\beta + i)\delta_2 r_{2f}.$$

This equation gives us the misspecified CAPM and shows how interest rates can occur as a result of the misspecification. If δ_1 is near 1, and β is near i, we might expect the short rate to have a coefficient close to zero, while the long rate should be typically much larger.

Of course, reality is much more complex and the precise nature of the misspecification could involve almost any point in the term structure. It is worth noting that the equilibrium discussed above generalizes to K different rates where each one will be weighted by the relative risk tolerance of the investors who use the particular discount factor. Furthermore, these relatives' weights will a dd to 1.

13.6. Model specification

While the interest rate impact will be very difficult to estimate with any degree of conviction, we can consider two polar cases, the monthly T-bill rate and the 10-year bond rate. In a world of nominal prices, rather than real prices, these correspond to holding periods of 1 month, consistent with the rebalancing interval of institutional investors, and a holding period of 10 years which would correspond to medium-to-long-term investment. Incorrectly assuming one rate or the other to be correct throws up additional terms in the regression. In our analysis, we make use of the mixed equilibrium riskless rate, which is in line with the use of possible bond yield curve effects.

To further analyze the impact of specific investing horizons, we use a weighted average of the rates based on the ratio of a particular type of investor to the total investors. It is an established fact in the literature that large institutional investors and professionally managed funds trade on higher frequency (the short end of the yield curve) than smaller, independent investors [SHA 00, DIA 91, COH 75]).

$$\mu_t - \left(\delta_2 r_{1,ft} + (1-\delta_2) r_{2,ft}\right) \\ = \alpha + \beta\left(\mu_{mt} - \left(\delta_2 r_{1,ft} + (1-\delta_2) r_{2,ft}\right)\right) \\ + \gamma\left(\delta_2 r_{1,ft} + (1-\delta_2) r_{2,ft}\right) + u_t$$

Here, δ_2 represents the share of smaller investors who invest mostly on the long rate $r_{2,ft}$ as described earlier. We assume that 70% of the agents invest on the short rate, representing a large share of professional traders. We test whether there are changes to the market beta in this specification, as well as to the interest rate exposure γ.

Next, we go deeper into the potential misspecification. The case is as follows: if we estimate the traditional CAPM using the short rate, we create a misspecification that could lead to potential bias. We want to test whether this bias is indeed present, and whether wrongly assuming a short rate is a potential cause of the low beta anomaly. We can rewrite the misspecification as:

$$\text{E(r)-}\ r_{1f} i = \beta(\mu_m\text{-}r_{1f}) + \delta_2\left((\beta - i) r_{1f} + (\beta + i) r_{2f}\right)$$

$$\text{E(r)-}\ r_{1f} i = \beta\mu_m + (\delta_2 - 1)\beta r_{1f} - \delta_2 r_{1f} + \delta_2(\beta + 1) r_{2f}$$

Now, imagine the case for a low beta portfolio with β = 0.67 and a high beta portfolio with β = 1.25. The equations are formed as follows:

$$\text{E}(\text{r}_{\text{low}}) - r_{1f} i = 0.67\mu_m + (\delta_2 - 1)0.67 r_{1f} - \delta_2 r_{1f} + \delta_2(1.67) r_{2f}$$

$$\text{E}(\text{r}_{\text{high}}) - r_{1f} i = 1.25\mu_m + (\delta_2 - 1)1.25 r_{1f} - \delta_2 r_{1f} + \delta_2(2.25) r_{2f}$$

The effect on the returns is dependent on the composition of traders. If δ_2 is small, e.g. the composition of short rate, traders is high and the long rate effect is negligible. When δ_2 is large and the proportion of long rate investors is high, the long rate is significantly affecting the portfolio returns and creating the misspecification effect. Denoting $\theta_1 = (\delta_2 - 1)\beta - \delta_2$ and $\theta_2 = (\beta + 1)\delta_2$, we test whether θ_2 is insignificant to ensure no misspecification is present. We estimate the model as follows:

$$r_r - r_{1ft} = \alpha + \beta\mu_{mt} + \theta_1 r_{1ft} + \theta_2 r_{2ft} + v_t \qquad [13.7]$$

We expect that larger investors tend to be more sensitive to the short-term rate, while smaller investors are more affected by the long-term rate. The result for portfolios is dependent on the main investors in the market: given also that large investors are generally less risk averse, their presence in the market would lead to a higher demand for portfolios with a higher beta. In our specification, we assume that 70% of the market is dominated by large institutional investors.

13.7. Results

First, we estimate the model for the yield curve specification for both the low and high beta portfolios. After that, we turn to the estimates of the misspecification equation [13.7]. As a robustness check, we allow for multiple values for δ_2 to see whether the misspecification occurs for other investor proportions.

We find that using the 1 month T-bill rate gives insignificant results for the change in the interest rate, which is explained by the frequency of rebalancing of portfolios by institutional investors. In a correct specification of the CAPM, the market premium is the only risk factor. In Panel 1 of Table 13.3, we use the contemporaneous slope of the yield curve as our interest rate variable. In the first specification, we test for the change in the yield curve directly, and we show that there is no significant impact on either the low beta or high beta portfolios. Given that 70% of the investors are assumed to invest on the short rate, this result is not remarkable. When we include the sign change specification, we observe that the sign changes are not as significant any more for high beta portfolios, but still very significant for the low beta set. This confirms our hypothesis that portfolios with low beta are negatively affected by positive interest rate movements, but the impact does not reverse when interest rates decline as we saw with the long rate estimations in the previous section.

Panel 1	α	$t(\alpha)$	β	$t(\beta)$	γ	$t(\gamma)$	ϕ	$t(\phi)$
Yield Curve								
HIB	-0.110	-1.221	1.273	80.954	0.972	0.891	-	-
LOB	0.310	-4.092	0.701	40.840	-1.062	-0.900	-	-
HIB	-0.283	-1.252	1.272	81.231	-	-	0.243	1.732
LOB	0.481	-4.762	0.702	41.402	-	-	-0.381	-2.560
Panel 2	α	$t(\alpha)$	β	$t(\beta)$	θ_1	$t(\theta_1)$	θ_2	$t(\theta_2)$
Equation (7)								
HIB	-0.147	-2.096	1.281	80.540	10.580	3.467	-0.390	-0.353
LOB	0.467	6.210	0.682	39.965	-17.286	-5.280	0.142	0.119

Table 13.3. *CAPM results for different interest rate specifications*

Panel 2 of Table 13.3 presents results for the mixed interest rate return case. To remind the readers, the excess returns and market premium are based on the mixed interest rate as in equation [13.7], and we use the 1 month rate (coefficient θ_1) and the 10-year bond (θ_2). To explain the anomaly, we would require the long rate to have a significant impact, while the short rate should have a coefficient close to zero (particularly when the share of high-frequency investors is high and beta is near 1). We observe this is false and that the expected return of low beta portfolios is actually smaller when we include the long rate: for this period and market, the anomaly cannot be explained by the misspecification of interest rates at least under the assumption that we have made for investor relative shares.

It is possible that the share of short rate investors is not close to 70%, but actually higher or lower. As a robustness check, we repeat the analysis of the misspecification for different values of δ_2. We can rewrite θ_1 and θ_2 of Panel 2 of Table 13.3 into the direct "impacts" of the rates:

$$\frac{\theta_1+\delta_2}{\delta_2-1} = \gamma_1 \text{ and } \frac{\theta_2-\delta_2}{\delta_2} = \gamma_2$$

In our case, θ_2 is insignificant so we focus on the effect of θ_1 instead. In our specification of a share of 30% of investors on the long rate, we see that γ_1 is 27.16 for low beta and –17.06 for high beta. When we assume that the proportion of investors on the short rate is zero, we see a sign change in the interest rate effect: low beta portfolios are positively affected by the short rate (17.28), while high beta is negatively affected (–10.58). When we increase the share to 50%, low beta is negatively affected (–55.32) while high beta is positively affected (33.13). The misspecification is not present for any value of δ_2, as it is fully dependent on the significance of θ_2, the coefficient on the long rate. In our case, the long rate is not significant and therefore the effect diminishes.

In this section, we explored the alternative explanation that the low beta anomaly is caused by the composition of interest rate maturities. We find that there is not enough evidence of misspecification in the CAPM to suggest that the anomaly is caused by the proportion of investors on different parts of the yield curve, and that the yield curve specification actually removes the double alpha effect we observed in the previous section. Additionally, the outperformance of low beta is no longer observed in the sign change specification. However, agents only differ in the risk-free rate and still invest in the same market portfolio in the framework presented in this section. The next section provides an extension where we allow for different lending and borrowing portfolios, and where the agents are distributed as combinations of these two portfolios. The implication of this framework is that we do not have a single defined market, which might provide additional insights to the investor composition effect.

13.8. Concluding remarks

This chapter compares different specifications with macroeconomic factors by allowing for threshold CAPMs driven by interest rate movements. From the structural break results, we see that the differing exposures to interest rate movements are not captured by a heterogeneous beta model, but by a double alpha effect for low beta portfolios. However, this method fails to find the impact of actual interest rate changes on the slope and intercept of the two models when there are different changes in the same period.

In our proposed specification, using the sign of the interest rate change (validated by a reference point check using a grid search upon the likelihood function of our specification) rather than the actual change, we find that alpha is negative for low beta portfolios whenever the interest rate is rising and that it is positive whenever the rate is decreasing. In line with the previous results, we find significant evidence of outperformance of low beta portfolios based on interest rate movements and underperformance of high beta portfolios. There is no systematic effect of the interest rate on beta itself. This is evidence that the outperformance of low beta portfolios is not related to their systematic market risk but to interest rate factors that influence the intercept of the CAPM.

We show that the opaque nature of the definition of the riskless asset is a complicating factor. We find evidence that the slope of the yield curve has a significant and differentiating impact on low and high beta portfolios by using a simple general equilibrium model. We consider 1 month, 10-year rates and an equilibrium combination of the two based on an estimated relative share of investors. We might expect that the appropriate rate for the CAPM is the 1-month rate as this would reflect the rebalancing period of institutional investors. What we find empirically is that we see similar results for the slope of the yield curve and the long-term rate.

When we test a misspecified version of the CAPM based on a mismatch in maturity levels and investor preferences, we observe that the short-term interest rate does not have a significant impact on the excess returns of the portfolios, in line with theory. However, we expect the sign of the long-term rate to be positive in both cases. We find that the coefficient for the low beta portfolio is of the opposite sign, resulting in a rejection of the hypothesis that the anomaly arises from this particular form of mismeasurement. However, the analysis might differ if we include more securities of different maturities.

The main force behind the anomaly is likely to be attributed to exogenous macroeconomic factors influencing the risk-free rate. Monetary policy over the last

30 years has favored low beta strategies by increasing the price of bonds and it is fair to say that these macroeconomic factors shape our results, and are the main drivers behind off-equilibrium movements of returns. Hence, our model provides a link between macroeconomic (yield curve related) factors and the origin of the low beta anomaly. It seems that the underlying exposure to the risk-free asset has to be considered for a model consistent with the CAPM implications. To call out of equilibrium movements, an anomaly in the social sciences seems unwarranted.

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