

REPORT TO THE AER

RISK, ASSET PRICING MODELS AND WACC

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Executive summary

This report addresses issues related to understanding the risks in asset pricing models in the context of the cost of equity, the cost of debt and the WACC. It also considers the implementation of asset pricing models in practice. The report also considers two models that are not asset pricing models - the Fama French three factor model and the dividend growth model. These are not asset pricing models as they do not explicitly identify the risks that are priced, but they can be used to estimate the required return on equity.

The clear message from these models is that it is only the covariance between the firm's cash flows and the systematic risk factors faced by portfolio investors that determine required returns. Some risks change the expected values of the cash flows and this should be accounted for when estimating the expected cash flow of the business, not by adjusting the required return. Some risks change the variance of cash flows, but this should not affect the required return of portfolio investors if it can be diversified away. It is only the non-diversifiable (covariance) risk that is priced.

Unfortunately, the existing suite of asset pricing models do not provide a consensus on what the systematic risk factors are. Indeed, in some models the risk factors are not defined. Some models have only one risk factor, others have multiple risk factors. Factor loadings and, in multiple factor models, risk premiums can even be negative, which is counter intuitive as it means that some risk exposures can actually reduce required returns.

Of the models discussed, the capital asset pricing model (CAPM), the Fama and French three factor model and the dividend growth model (DGM), have been investigated or applied in regulatory settings. However, only the CAPM and DGM appear to have been used by regulators. These latter two are also the models that are most easily implemented. Except in the USA, the CAPM has tended to dominate the DGM in regulatory practice, with the DGM largely used as a cross check to aspects of the CAPM's application.

With respect to the WACC, the report makes the critical point that the objective is not to estimate the cost of the firm's finance. Rather the WACC is used as an instrument to measure the required return on the firm's investment, as it is the risk of the firm's investment that drives the WACC. Ideally, the covariance between the investment's cash flows and the systematic risk factors would be used to directly estimate the WACC. Unfortunately, this is rarely feasible and measuring the risks and returns of the securities that firms have issued is used instead.

1 Risks, Asset Pricing Models and WACC

This report addresses issues related to understanding the risks in asset pricing models in the context of the cost of equity, the cost of debt and the WACC. The scope of the analysis is limited to a selection of asset pricing models on the basis of advice provided by the AER. We do however add one model to the AER's list by considering a very general form of asset pricing model known as the Stochastic Discount Factor model. This report also comments on the practical implementation of the different asset pricing models.

1.1 Risk and the cost of capital

In common parlance, risk refers to the chance that dire things can happen. In finance, however, risk means that future outcomes are not known with certainty. Hence, the cash flows that actually occur in the future can differ from those that were expected. By definition, if cash flows differ from what was expected then so do rates of return.

The focus in finance is on the risk of variability in the magnitude of future cash flows, and, in some cases requires consideration of the possible states of the world when those cash flows are to be received. States of the world can matter in that a high cash payoff in a good state of the world is good, but a high cash payoff in a bad state of the world may be better. If so, it is not just the unknown magnitude of the cash flows that matters, but also the relationship between that risk and the various possible states of nature (booms and busts). These risks matters to investors because they create uncertainty about future wealth and so future consumption. Indeed, one way of classifying asset pricing models is whether their focus is on the risk to future wealth or the risk to future consumption.

Investors willingly accept risk in the expectation of achieving a higher return than would otherwise be the case. Yet, not all risk is rewarded. As the Nobel Prize winner in finance, Bill Sharpe says, "If there is a reward for risk, it has got to be special." (Burton, 1998, p.2). Financial economists focus on risks that are 'priced'. This refers to the risks that asset pricing models show to affect investors' required returns, or equivalently, their expected returns in equilibrium. We note that the 'in equilibrium' qualifier is important, but confusingly, it is often dropped and financial economists often just refer to expected returns.

Priced risks are those that validly enter the determination of the discount rate (required return) in a valuation or regulatory determination. Some risks are not compensated by extra returns because they can be diversified away and so are not priced. These diversifiable risks can affect value, but

do so by their effect on the expected cash flow rather than the discount rate. For example, there may be some risk of an idiosyncratic event, say an oil spill, which will create substantial costs that reduce the expected cash flows to a project. So the correct approach is to account for such events in the expected cash flow. Despite this, a common and bad practice is to add a ‘fudge factor’ to discount rates. This discount rate adjustment adds an extra risk premium to allow for negative events (‘risks’) that have not been accounted for when estimating the expected cash flow. In other words, the expected cash flow is not really the expected cash flow, as it is upwardly biased, and increasing the discount rate attempts to compensate for this. Adding fudge factors to discount rates is a bad practice as it drives a wedge between the theoretically correct discount rate and the discount rate actually used and also because it is likely to lead to error. A discount rate adjustment will be non-linear in its effect, as adjustments to the discount rate compound through time. Whereas, the correct cash flow adjustments may well be linear in nature and possibly declining, rather than increasing, through time.

We stated above that one distinguishing feature of asset pricing models is whether their focus is risk to future wealth or risk to future consumption. Other distinguishing features are whether the model considers investment as single period decision or a multi-period decision and the number of risk factors considered. In multifactor models, the premiums on risk factors can be negative. In asset pricing models, it is also possible for the covariance of returns with risk factors to be negative. As a consequence, some risk exposures may provide the counter intuitive result of actually reducing the required returns.

2 Risks to be compensated in the WACC

2.1 The required return on investments

The weighted average cost of capital (WACC) provides a particular way of estimating the required return on an investment by a company. The principles for the required rate of return are that: it should reflect the risks for which investors require compensation; it should be forward-looking since it is to apply to cash flows in the future; and most importantly, it should reflect the opportunity cost of the investment. That is, the investment should be expected to return as much as an equivalent-risk security traded in the capital market, otherwise the investor would be better off not investing, but rather putting their money into the capital market instead. If capital markets are reasonably efficient, competition should ensure an equilibrium in which securities of equivalent risk offer

the same expected return. Thus, in equilibrium, the expected return on a security is the return that investors require for that security's level of risk.

In principle then, what we first need to do is to measure the risk of the investment. We then discount the expected future cash flows from the investment at the *current* equilibrium expected return in the capital market for securities with the investment's level of risk. The word 'current' is important here. In any required return calculation, we should be using current values because if capital markets are efficient current values contain the best information available on future values. In particular historic values for the rate of return on equity, or interest rates, are not relevant *except* to the extent that they help us estimate the *current* rates. Since current interest rates are readily observable, historic interest rates typically have no place in determining the required rate of return. If the current interest rates differ from historic rates then there will have been windfall gains or losses that are already reflected in the current value of equity.

In selecting the relevant opportunity cost from the capital market it will be important to match both risk and maturity. It is evident that there is a term structure of interest rates.¹ Other things being equal, this will induce a term structure of discount rates. Consequently, it is not just risk, but also maturity that is likely to determine the opportunity cost of investment and hence the required return.

2.2 The use of security returns

In practice, we encounter a problem in applying the first in-principle step of determining the investment's risk. Determining the risk of the assets that constitute the investment is difficult. It is particularly difficult when the portfolio of assets that constitute the investment is not regularly traded, as is commonly the case. Since securities are regularly traded, the commonly adopted solution is to look to the cost and risk of the portfolio of securities that has been used to finance the portfolio of assets. It is, however, vitally important to remember that we are only using the securities as an instrument to measure the required return for the investment's level of risk. Thus, the costs of the portfolio of securities that do not relate to the risk of the investment, such as the transaction cost of making a security issue, do not belong in the discount rate. They are best accounted for as a reduction to the investment cash flow.

Unfortunately, the use of the securities as an instrument to *measure* the required return (cost of capital) has led to confusion. In particular, it can lead to the mistaken belief that it is the financing package that *determines*

¹ As noted later, it is much less clear whether there is a term structure of required returns to equity.

the required return. A moment's thought will reveal that this implies that the investment inherits the characteristics of the portfolio of securities issued to finance the investment. That is, the risk of the investment (assets) is determined by the risk of the securities. Clearly it is the other way around. The risk of the *portfolio* of securities (but not individual securities) and the risk of the portfolio of assets are the same. The portfolio of securities inherits the risk characteristics of the assets. Ultimately, all the cash flow that goes to service the securities has to be the cash flow that the assets generate. There is no cash flow from anywhere else - no assets means no cash flow. As such, the expected return and risk for the portfolio of issued securities has to match the expected cash flow and risk of the assets.²

In the section above, the importance of matching the required returns to the maturity of the investment was raised. Given the maturity of investment, the following question arises - what if the maturity of debt a firm issues differs from the maturity of the investment? In this case, if there is a term structure in returns, the required return on the firm's portfolio of issued securities will not match the required return given the risk of the investment and its maturity. For example, what if the firm uses shorter term debt that it plans to roll over until the maturity date of the investment? The consequences then depend upon the term structure of interest rates. If the term structure is upward sloping then the required return on the portfolio of issued securities will understate the required return on the investment and vice versa.

Assuming that the pure expectations hypothesis explains the term structure, for example, it is upward sloping only because future short-term rates are expected to be higher than current short-term rates. Then the expected to cost of debt from following a strategy of rolling over shorter term debt is the same as if the firm matched the maturity of its debt to the maturity of its investment, but there is extra cost from exposure to refinancing risk and more transactions costs. Alternatively if the upward sloping term structure is partly explained by a liquidity premium, then rolling over short-term debt is cheaper for the firm, but an offsetting cost is exposure to refinancing risk and more transactions costs.

Whatever, the financing choices of the firm, the key point of the present value principle is that it is the *capital market* discount rate for assets of the relevant risk and maturity that should be used in valuing investments. Even if the firm faced capital rationing and faced limited access to capital markets, as long as its investors have good access to capital markets, it is the capital market that determines the required return.

² In imperfect markets the financing package may influence the total cash flow, for example via tax shields, and hence have valuation effects. However, these are generally second order effects.

2.3 WACC measurement and risk

The WACC attempts to measure the required return on the company's portfolio of issued securities in order to infer the required return on the company's investment. It is a portfolio return formula, as is shown by equation (3) below.

The problem now boils down to correctly measuring the weights and the required returns. There are several different ways that the required returns can be measured, for example, with or without tax adjustments, and in nominal or real terms. Consequently there are several different ways to measure the WACC. The trick is to apply the consistency principle and match the WACC in the denominator to the definition of the cash flow in the numerator of the discounted cash flow calculation.³ Common practice is to use an after tax WACC that allows for interest tax shields via a reduction to the cost of debt and to apply this as a discount rate to cash flows calculated before allowance for the interest tax shield. However, a critical assumption of this approach is that the firm maintains its market value leverage ratio. In other words as the market value of the firm changes, the firm has to adjust the level of debt and equity to keep the leverage ratio constant. For a firm facing a corporate tax rate t , the after tax WACC ($r_{\text{after-tax}}$) is given by

$$r_{\text{after-tax}} = w_b \times r_b(1 - t) + w_e \times r_e \quad (1)$$

Where the weights for debt and equity are w_b and w_e respectively and sum to one, r_b is the cost of debt and r_e is the cost of equity, t is the corporate tax rate.

Allowing for differential taxes on dividends (including imputation) and capital gains, the Dempsey and Partington (2008) version of the after tax WACC is given by:

$$r_{\text{after-tax}} = w_b \times r_b(1 - t)q + w_e \times R_e \quad (2)$$

where q is the ratio of the market value of dividends to the face value of dividends and R_e is the cum-dividend return appropriate to discounting prices. When this approach is used, however, it is necessary to scale the cost of internally generated investment finance (retained earnings) by q when computing the net present value, in order to properly reflect the opportunity cost of internally generated funds. If q is equal to one, this

³ While the current brief is to discuss the WACC, we note that the adjusted present value method (APV) is a possible alternative. Since the AER currently uses the plain vanilla WACC, they are already a long way down the road to the APV. Alternatively, we could make all the risk adjustments to the cash flow. For example, we could use the certainty equivalent version of the CAPM, where we adjust the cash flow according to the covariance of the cash flow with the return on the market. Discounting in such models is only for time and so is done at the risk free rate.

approach reduces to the traditional approach. If q is not equal to one, which seems likely, the traditional approach is wrong.⁴

The practice of the AER has been to use a nominal plain vanilla WACC, which has no adjustments for the tax shield effects of debt financing. Tax effects of financing are accounted for in the cash flow.⁵ Assuming only two sources of finance, debt and equity, the plain vanilla WACC can be written as:

$$r_{\text{vanilla}} = w_b \times r_b + w_e \times r_e \quad (3)$$

This plain vanilla WACC has some attractive features, one of which is that the adjustments for tax shield benefits of debt financing do not appear in the WACC. Consequently, the WACC depends solely on asset risk and so we can write the WACC in terms of an asset pricing model. Thus if we believe the CAPM is the correct asset pricing model we can write the plain vanilla WACC as a function of the CAPM (see equation(8)).

This neatly answers the question, what risks should be compensated by the WACC. The answer is that it is contingent on which asset pricing model that you favour. If you use the CAPM, then the risk is the covariance (beta) risk of the returns on the portfolio of assets with the return on the market. Alternatively, you can take an entirely pragmatic approach and say that we do not know for sure what the risks of the investment are, but that their total required compensation is reflected in the plain vanilla WACC. However, this still leaves you with the problem of measuring the WACC without using an asset pricing model.

An attractive feature of the plain vanilla WACC is that we do not need to worry about constant capital structure weights.⁶ Indeed if we can agree on the *unlevered* equity beta, which is equal to the asset beta, we do not need to know either the capital structure weights, or the cost of debt. We can derive the required return on the investment by inserting the value for the asset beta directly into the CAPM equation.

In reality, in order to derive the unlevered equity beta we usually need to start from a known capital structure and known cost of debt and equity. In the case of the plain vanilla WACC, however, we can dispense with the

⁴ Fortunately the traditional approach involves two offsetting errors. For example if q is less than one the present value of the project is overstated by the traditional approach, but in computing the NPV this is offset because the cost of investment financed with profit retention is understated. Furthermore, internal finance is the major source of investment funds for established firms.

⁵ The implicit assumption in this approach is that the interest tax shield has the same risk as the firm, but there is no explicit allowance for the risk that the government might change the corporate tax rates or the rules on the tax deductibility of interest.

⁶ As mentioned above, the assumption of a constant capital structure weights and consequent rebalancing of the capital structure as the value of the firm changes is a critical feature of the commonly used after tax WACC.

usual formulas for unlevering the equity beta. The plain vanilla WACC is a direct estimate of the unlevered cost of equity and this simplifies things considerably. We just substitute that estimate of WACC, the risk free rate and the market risk premium into the CAPM equation and back solve for the unlevered equity beta.

The mechanics of computing the WACC described in equation (3), are comparatively simple. However, getting the required estimates of the current cost of equity, the current cost of debt and the current market value weights is difficult. The cost of equity cannot be directly observed and is particularly difficult to estimate. The cost of debt based on the promised return is comparatively easy to estimate and so it is widely used. The problem is that if there is significant default risk the promised cost of debt overstates the true expected return and so overstates the cost of debt. This is a problem that is typically ignored in the WACC calculation, where the promised return on debt (internal rate of return on the debt based on promised cash flows at the current price of the debt) is commonly used. For regulated businesses with low default risk, however, the overstatement is unlikely to be substantial. For example, Cooper and Davydenko (2007) provide estimates of an overstatement that is only of the order of 20 basis points for a typical firm with investment grade debt.

It is unambiguously clear from the discussion in Part 4 of this report that modern finance theory specifies that the risk to be compensated via the WACC is the non-diversifiable, or systematic, component of total risk (in simple terms, that risk which cannot be eliminated by holding stocks in a well diversified portfolio). This risk is measured as covariance, or equivalently beta, risk. Which covariance risks are admissible depends upon which asset pricing model is favoured. However, as a practical matter it is difficult to implement models other than the Sharpe-Lintner CAPM in determining required returns. In which case, risk is the covariance of returns on the asset with returns on the market for capital assets. Unfortunately, even measuring this covariance is fraught with problems. One of which is that it might be systematically time varying, That is, the covariance between stock returns and market returns may change systematically between booms and busts with stocks moving more closely with the market when it goes down.⁷

While it is clear from the asset pricing models that covariance risk is the key risk, it is instructive to consider some of the factors that drive the covariance risk. It is also instructive to consider the games that can be played in relation to risk and wealth transfers. The differing nature of debt and equity also requires some consideration. We address these issues below.

⁷ A similar issue may arise with other factor portfolios such as size and momentum.

2.4 The risks of equity

2.4.1 Business risk

The key risk of equity is the business risk, i.e. the risk inherent in the asset. According to the asset pricing models, this depends on the covariance of returns on the asset with returns on the risk factor portfolio(s), which is the market portfolio in the CAPM model.

In the case of the CAPM, the determinants of asset risk are understood to be the inherent volatility of the returns and how strongly they correlate with the business cycle. More volatility and higher correlation with the business cycle leads to a higher covariance risk for revenue and thence for the asset. This covariance risk can be levered up by substituting fixed costs (operating leverage) for variable costs. Thus, more volatile and more cyclical revenues and more fixed costs, tend to increase the asset beta.

In the case of the regulated businesses, revenue betas would be expected to be low. However, their extensive capital investment represents fixed costs that lever up the revenue beta. It is the low revenue betas that enable them to utilise substantial fixed costs without excessive risk.

2.4.2 Financial risk

The equity beta can be levered up to a level above the asset beta by the addition of further fixed costs in the form of payments to debt-holders. Since the shareholders are residual claimants to the cash flows more debt, and increasing prior claims of debt holders on the cash flow, gives rise to increased risk to the shareholders. This risk is known as financial risk. The extra leverage so created increases both the variance and covariance of shareholder cash flows and returns. The combination of business risk and financial risk is captured in the beta for levered equity.

Introducing leverage also introduces the possibility of financial distress (default), which reduces the expected cash flow to shareholders. To the extent that the debt holders face a positive probability of default, they are bearing some of the business risk and this is an increasing function of the probability of default. As part of the business risk is transferred from the shareholders to the debt holders, the rate of increase in the extra compensation that shareholders require for increases in leverage slackens. Financial risk keeps increasing with more debt and so does the default risk. Shareholders still require extra returns for the financial risk, but this is increasingly offset by the reduction in the business risk that they face due to risk transfer to debt holders. Indeed, if leverage and default risk become high enough, so much of the business risk gets transferred to the debt-holders that the required return on equity starts to come down.

As discussed in Section 3, while interest rate risk is a systematic risk factor, it is unclear how it covarys with financial risk. What we can say is that given the low default risk in regulated utilities, these financial risk effects are unlikely to be substantive in normal market conditions. We provide an extensive discussion of the relation between the cost of debt and the cost of equity in a prior report to the AER, Mckenzie and Partington (2011b).

We discuss the risk for shareholders of agency costs below but we note here that more debt reduces agency costs. This is because debt forces managers to distribute cash to lenders. This generates more discipline in the use of cash, thus mitigating the free cash flow problem that we discuss below.

2.4.3 The risk of agency costs for equity

Agency costs arise because the managers act as agents in managing the assets on behalf of the shareholders. The shareholders want the managers to act in the shareholders' interests and maximise value, but the managers will naturally pay some attention to their own interests. Since the managers control the assets they have the opportunity to enjoy the perquisites of stewardship. This may impose costs on the shareholders and reduce the value of their shares. For example, managers may engage in empire building investments that increase their salary, power and prestige, but that cost more than they are worth. Agency costs therefore reduce the expected cash flow. Because the magnitude of the agency costs is uncertain they also increase the cash flow's risk. As these risks can be diversified away however, they are not priced.

Agency costs are likely to be smaller in private firms because of less diffuse ownership and consequent closer monitoring of managers. While utilities are large firms, they are regulated firms and this distinction is important. One consequence of this regulation is that agency costs are likely to be lower for regulated entities. It is well accepted that monitoring of the firm, such as audits, is a way of reducing agency costs, although the cost of monitoring is considered to be part of the agency costs that shareholders have to bear. Regulated firms are subject to extra monitoring by the regulator and part of the cost is borne by the public purse, so the shareholders benefit from some free monitoring. The regulated return, the focus on efficient investment and the public and private scrutiny during the regulatory process are likely to reduce both the magnitude and risk of agency costs.

It is also the case that agency costs increase with information asymmetry between managers and outsiders. The regulatory process is likely to reduce this information asymmetry and if so this will also help reduce agency costs and risk.

There is another reason why agency costs are likely to be lower in regulated entities. It is well recognised that agency costs are likely to be greater where there is a strong free cash flow and large amounts of cash piling up in the firm. Mountains of cash are difficult to protect against misuse. In the case of utilities, cash and liquid asset balances tend to be small (smaller than in many other industries). Less surplus cash means smaller agency costs and risks.

Related to the concept of agency costs is the risk of tunnelling, which is where a dominant shareholder, or group of shareholders, expropriates wealth from the company at the cost of the other shareholders. Such risks are likely to be small in regulated businesses.

2.4.4 Liquidity risk for equity

Liquidity risk arises from a mismatch between the investor's need for cash and the maturity of the asset. Liquidating a position in the asset before maturity may involve costs in both finding a counterparty and negotiating a price at which the asset can be liquidated. The consequence is a reduction in the expected cash flow and hence a reduced value for the asset. Since the size and timing of the liquidity costs are uncertain, there is an increase in risk. In the case of listed equity for example part of the cost of trading is the bid ask spread and there may be market impact costs, where the act of trading moves prices. Such liquidity costs may be exacerbated at times when market liquidity is generally low.

The impact of liquidity risk depends on the investor's time horizon for trades. For short term traders, liquidity costs such as the bid-ask spread may be substantial. In contrast for investors with a long time horizon, costs like the bid-ask spreads are unlikely to be a substantial component of total returns.

Unfortunately, there is no agreed standard for the measurement of liquidity. For example, one suggested liquidity measure is bid-ask spread, another is market-depth, and another is serial correlation in returns. Trading volume and turnover have also been used and so on. Unsurprisingly, therefore, there is no real consensus on the pricing of liquidity risk. However, in a much cited paper, Pastor and Stambaugh (2003) suggest that aggregate liquidity is a systematic risk factor in equity returns and that stocks liquidity betas are a significant determinant of required returns.

Liquidity is obviously lower for equity in private companies than it is for equity in public listed companies. It has been suggested that the liquidity premium for private companies is of the order of two to three percent (see Franzoni, Nowak, and Phalippou, 2011 and Anson, 2010). The question for regulators is whether they should provide compensation for illiquidity that rises from the choice of organisational form, which

presumably is compensated for by other benefits of being a private company. As suggested earlier, as long as investors have access to the capital market the appropriate cost of capital rate is the rate in the capital market.

2.5 Risks of debt

In principle, the required return of debt could be determined by using one of the asset pricing models described in Section 4. In practice this is rarely done. The key risks for debt holders are systematic (beta) risk, credit risk and liquidity risk. Because debt returns are typically given as promised rates of return, the three risks are mixed together in the return and are often difficult to disentangle. Thus, the difference between the return on government bonds and corporate bonds, i.e. the credit spread, reflects incremental systematic risk, credit risk and incremental liquidity risk.

Systematic risks arise because the valuation of all debt is affected by changes in variables, such as the level of interest rates and changes in the rate of inflation. With respect to interest changes, both changes in the level of interest rates and the term structure are potential risks. Interest rate and inflation rate changes affect the value of both government debt and corporate debt and therefore we anticipate considerable commonality between the systematic risk of government and corporate debt. It is possible, however, that corporate debt has more systematic risk than government debt. In McKenzie and Partington (2011, p.6) we suggest this extra systematic risk is likely to be small:

In our opinion, the systematic component of the credit spread is likely to be small. There are two reasons for this. First, debt betas tend to be small. Second, to the extent that there is systematic risk in debt returns, there is likely to be some commonality in the systematic risk of government bonds and risky debt. Consequently, any systematic risk component in the credit spread is only the systematic component over and above the component that the risky bonds share with government bonds. We therefore conclude that the credit spread will have a substantial default risk component. In other words, the credit spread is called the credit spread for a reason, it reflects the risk that creditors will lose some or all of their money.

The components of credit risk are the risks of default and the risk of rating downgrades. In the case of downgrades, the debt is facing a growing risk of default and consequently a declining value. It is not clear how much of default risk is systematic and how much of the default risk can be diversified away. If we take the usual assumption under the CAPM that all default risk is diversifiable, then this leads to a decrease in the expected cash flows, but no change in required returns.

As explained earlier the cost of debt is usually estimated based on promised returns and so overstates the expected return by the expected losses due to default. However, the credit risk for regulated utilities is likely to be relatively small under normal market conditions as the default risk is small and the risk of credit migrations for utilities is lower than for most stocks, Kadam and Lenk (2008).

A more precise estimate of the component of return for credit risk might be obtained from the credit default swap market. Using data from the US credit default swap market Longstaff, Mithal and Neis (2005) find that the majority of the credit spread is due to default risk. Nonetheless, they also find that there is a substantial time varying liquidity component to the credit spread.

Liquidity risk is likely to be an issue for corporate bonds, because the corporate bond market is much thinner than the government bond market, particularly in the Australian market. In a recent working paper, Bianchi, Drew, Roca and Whittaker (2013) provide estimates for the default risk premium and liquidity risk premium for Australian corporate bonds that are of a similar magnitude. However, given the difficulty of disentangling the components of the credit spread, it is probably as accurate, and certainly simpler and more transparent, to use the promised return as the cost of debt for utilities.

2.5.1 The risk of agency costs for debt

The risk of agency costs for debt arises because the shareholders have the control rights to the assets and also control the firm's capital structure. This allows shareholders to engage in strategies that transfer wealth from the debt holders to the shareholders. Risk shifting is one example, the firm starts with a conservative capital structure and then sharply increases leverage, so the value of the pre-existing debt falls. A similar effect can be achieved by increasing the risk of the firm's assets. The reduction in the value of debt is a wealth gain to shareholders. In the case of traded debt, this gain can be immediately realised by buying back the debt in the market at a discounted price rather than ultimately repaying it at face value. The potential for this type of behaviour increases the risk of debt but also imposes costs on shareholders. The costs to shareholders come in the form of restrictive debt covenants, designed to prevent actions detrimental to the debt-holders' interests and in higher interest rates.

This risk to debt holders, and hence the cost to shareholders, is probably smaller in the case of regulated businesses. First, the debt is investment grade and therefore the risk of financial distress is low. It is well understood that the temptation to play games like risk shifting is low when financial distress risk is low. The problem increases with financial distress risk and rises sharply when the risk of financial distress is high.

Second the regulatory process and reduced information asymmetry are likely to reduce the prospects for such opportunistic behaviour.

3 Regulatory treatment of risk factors – rate of return or regulated cash flows?

In a related report (Frontier, 2013), there is discussion of the main potential risks to a regulatory network. These risks are reproduced in Table 1, below. Whatever the risks, the fundamental issue is what do they do to cash flow? If they affect the expected cash flow, then they should be accounted for in the expected cash flow. If they also affect the covariance of cash flow with systematic risk factors then they should be accounted for in the discount rate.

Table 1
Summary of potential risk factors for a regulated network

Business risks	Financial risks
Demand risk	Refinancing risk
Input price risk	Interest rate reset risk
Cost volume risk	Illiquidity risk
Supplier risk	Default risk
Inflation risk	Financial counterparty risk
Competition risk	
Stranding risk	
Political / regulatory risk	
Other business risks	

Source Frontier (2013)

On the basis of the asset pricing models considered in Section 4, if the risks only affect the variance of cash flows, but not covariance with systematic risk factors, they do not affect the required rate of return. The clear message of the discussion in Section 4 is that investors are only compensated for bearing risks that are priced. Recall that priced risks are those that validly enter the determination of the discount rate (required return) in a valuation model. Some risks are not compensated by extra returns because they can be diversified away. These diversifiable risks

may affect value, but if they do it is by their effect on the expected cash flow rather than the discount rate.

Clearly, some risks in Table 1 reflect market wide (systematic) factors, such as inflation risk, interest rate risk and liquidity risk, and so are likely to affect investors required returns. It is, however, important to understand that because a firm faces a risk related to a systematic factor, such as refinancing risk, this does not necessarily mean it cannot be diversified away. For example, making portfolio investments in both borrowers and lenders diversifies away the investor's exposure to the firm's refinancing risk. Some risks such as cost-volume risk and stranding risk have substantial firm or industry specific elements so that some, or all, of the risk could be readily diversified away by a portfolio investor.

We strike a fundamental difficulty at this point as there is no reliable way to determine the nature of the relationship between any risk factor in Table 1 and systematic risk factors (or more formally, how any of the risk factors in Table 1 covary with systematic risk factors). As we explain in Section 2, it is so difficult to directly measure the risk of investments, that we measure the risks and returns of securities instead. All of the risks in Table 1 get bundled up into the business cash flow through effects on the mean, variance and covariance of the cash flows. The covariance then determines the required return on the securities. In general, provided the expected returns on securities are correctly measured, any risks from Table 1 relevant to the discount rate are captured in the total required return in the capital market for the investment's risk class.

The problem then becomes measuring the required return on securities correctly. This is difficult because exactly what constitutes systematic risk factors is far from clear. The basic CAPM and the Black CAPM model, both clearly argue that market risk is priced, due to its link to wealth. The consumption CAPM (C-CAPM) has a similar interpretation, except it is the covariance with aggregate consumption that is important. The Merton Intertemporal CAPM (I-CAPM) tells us that a range of unspecified *state variables* are priced, but is unclear as to exactly what these variables are.

The Fama and French model is more precise inasmuch as it tells us that size and book to market are important and these factors have been economically interpreted as proxies for illiquidity and the risk of financial distress. It is important to note, however, that it would be wrong to automatically assume that these factors are priced. To understand why, Smith and Walsh (2013, p. 75) recount the parable of Ferson et al (1999) in which:

an empirical anomaly, based on the position in the alphabet of the names of companies, is used to create a factor that is used in asset pricing. The use of anomalies such as this gives a workable method of coming up with ex post efficient portfolios. However, this says

nothing about asset pricing as there are an infinite number of ex post efficient portfolios ... this method of constructing ex post efficient portfolios is in effect picking the low lying fruit. Armed with a vector of ex post average returns and a historical variance-covariance matrix, any competent analyst could derive the entire range of ex post efficient portfolios...

The point is that just because a factor may be used to identify an *ex post* efficient portfolio, does not mean that the factor is priced (and this includes other factors such as momentum).

To our minds, this discussion gets to the heart of the source of confusion in terms of understanding what risks drive expected returns. Most of the asset models surveyed in this paper theoretically allow for multiple risk factors to be priced. Even the Sharpe-Lintner CAPM, which in its most popular form has only one risk factor, does allow for the possibility of multiple factors. Sharpe himself makes this point in Burton (1998), stating that,

... you didn't have to assume only one factor. Th(e) basic result comes through in a much more general setting. There could be five factors, or 20 factors, or as many factors as there are securities.

Sharpe goes on to say

I'd be the last to argue that only one factor drives market correlation. There are not as many factors as some people think, but there's certainly more than one.

The problem lies in identifying which of these risk factors are priced *ex ante*. Subrahmanyam (2010) documents at least fifty variables that have been used to predict the cross-section of stock returns. He notes that these variables are generally motivated by either Wall Street folk lore, behavioural biases, market frictions, or theory. Yet, it is unclear exactly how many of these factors are priced. As Subrahmanyam (2010) says:

It is important to understand which of these effects are robust and which do not survive changes to sample and/or method. Of those that do survive then, it is important to understand the correlation structure between the variables.

Certainly the arguments as summarised in Smith and Walsh (2013) would suggest that market risk is the only factor as:

(t)he market portfolio is important and special because it is the only portfolio which we can specify ex ante to be an efficient portfolio.

On the other hand, an argument can be made for an interest factor on the basis that interest rates have been used as a state variable in the I-CAPM model. Further, the Fama and French type proxies for liquidity and default risk, may also be relevant. However, if we assume that they are priced risk factors, then we open up a veritable Pandora's Box in terms of

the estimation difficulties we now face. Taking the Fama and French factors as an example, a cursory examination of the of studies in this area (discussed in Section 4), reveals that the significance of size and book to market variables exhibits substantial variation both within and across studies and, in some cases, has been associated with negative risk premiums. Thus, it is entirely unclear exactly what the reward is for exposure to these risk factors. Further, an entirely different set of issues arise in the discussion of how to quantify the firm's exposure to these risks, which are analogous to the well documented problems that exist in estimating the firms exposure to market risk. Thus, including additional risk factors, even if it could be theoretically justified, does not necessarily mean that a superior estimate of the cost of capital will result. It is entirely possible that the estimation errors associated with these additional factors could mean that a more complex model produces a less reliable result.

Note that we do acknowledge the possibility that periods during which these risk factors change, may be associated with similar changes in the cost of equity. For example, McKenzie and Partington (2013, p.9) note that while default risk does not enter into the computation of CAPM required returns, it does not rule out periods of increasing default risk being associated with an increase in risk aversion in the equity market, or an increase in the market price of risk, and either could lead to an increase in the market risk premium.

In our opinion, ad-hoc adjustments to the WACC (like Ofwat's allowance of a small firm premium for water companies) are problematic. It is not clear whether such adjustments take you closer too, or further away from the true required return, particularly as the most recent Australian evidence would suggest a negative adjustment (i.e. a reduction in required returns for small utilities) which is counter intuitive. However, it is entirely reasonable to account for increased transactions costs for smaller scale security issues as a cash flow cost to the business.

4 Asset pricing models

This section of the report introduces and discusses a number of asset pricing models as specified by the AER (the Appendix presents a discussion of the Stochastic Discount Factor (SDF) model, which is a very general model that encompasses many of the asset pricing models discussed in this section). While these models are most commonly applied in the context of pricing equity, it should be emphasised that they are general asset pricing models. As such, they apply to both equity and debt. In fact, they apply to any asset.

4.1 The Consumption CAPM (C-CAPM)

Since its development in the late 1970's and early 1980's, the C-CAPM has drawn high praise from financial academics. For example, Lettau and Ludvigson (2001, p.1241) argue that “(a)s a measure of systematic risk, an asset’s covariance with the marginal utility of consumption has a degree of theoretical purity that is unmatched by other asset pricing models”. Campbell and Cochrane (2000, p.2864) rank the C-CAPM as a major advance in finance theory and note that “all current asset pricing models are derived as specializations of the consumption-based model”. Thus, just as with the SDF model, the C-CAPM can be shown to capture the implications of complex dynamic inter-temporal multifactor asset pricing models.

In essence, the C-CAPM assumes that investors seek to minimise the variance in their consumption stream and so, effectively maximise their lifetime utility of consumption function (which is assumed to increase at a marginally decreasing rate with higher levels of real consumption). The relevant risk parameter in the C-CAPM is beta, which measures the contribution of an asset to the variance in aggregate consumption. In that sense, the C-CAPM incorporates not only the risk of wealth volatility (as captured by the CAPM beta), but also the risk of changes in reinvestment opportunities over time.

While the C-CAPM is theoretically appealing, its poor empirical performance has meant that it has remained largely on the fringes of asset pricing theory (and in this sense it is somewhat similar to the SDF model). In fact, Campbell and Cochrane (2000, p.2864), go so far as to argue that the consumption based model has failed “the test of time” and CAPM, and its multifactor extensions, perform better.

The main problem in operationalising the C-CAPM lay in the choice of a suitable proxy for consumption. The standard measure of consumption for US based studies is personal expenditure on nondurable goods and services from the National Income and Product Accounts. Alternative expenditure-based proxies have also been used (see Parker and Julliard, 2005, and Jagannathan and Wang, 2007). Irrespective as to the choice of measure, the explanatory power of the C-CAPM model is quite low and it typically compares poorly to the more traditional wealth based models in explaining returns. Campbell and Cochrane (2000) use simulation analysis to explore why this might be the case and finds that, while in a perfect world both do well, in an imperfect world the market return is a superior indicator of the state of the discount factor. We note that the recent publication by Savov (2011) has had greater success in explaining the equity risk premium, the risk free rate and the cross section of expected returns. Savov (2011) finds a strong positive relationship between municipal solid waste growth (garbage is used as a proxy for

consumption) and the market return. However, a cost of capital based on garbage statistics seems an unlikely basis for determining the regulatory return.

We are unaware of any instances in which the C-CAPM has been used in the determination of the cost of capital for utilities in regulatory proceedings. However, we note that a recent paper by Ahern, Hanley and Michelfelder (2011) argues in its favour as a way of moving beyond the CAPM and discounted cash flow models in this context. They present empirical estimates of the cost of capital based on the C-CAPM for public utility stock and bond indices in the US and find they are reasonably stable and comparable to the estimates derived using the more traditional methods. However, the empirical implementation involves recasting the C-CAPM as a volatility model. We are aware of no Australian studies that attempt to apply the consumption CAPM.

4.2 The Sharpe-Lintner CAPM

A conventional textbook representation of the CAPM takes the form:

$$E[r_i] = r^f + (E[r^m] - r^f)\beta_i \quad (4)$$

where m is the market portfolio with expected return $E[r^m]$, the risk-free return is r^f and β_i is a measure of non-diversifiable risk for stock i . The most common interpretation of beta is that it measures the sensitivity of an asset's return to variations in the market return. More formally, beta in the CAPM is the covariance risk of an asset in the market portfolio measured relative to the variance of the market return (which is itself the average covariance risk of assets), i.e.

$$\beta_i = \frac{\text{Cov}(r_i, r^m)}{\text{Var}(r^m)} \quad (5)$$

In the words of Sharpe, “(t)he key insight of the Capital Asset Pricing Model is that higher expected returns go with the greater risk of doing badly in bad times. Beta is a measure of that.” (Burton, 1998)

Thus, it is covariance risk that matters in the CAPM (the denominator in the measure of beta) and Brown and Walter (2013) provide the intuition as to why the focus is solely on covariance:

The total risk of a portfolio comprising N individual investments contains N^2 covariance terms, of which N are usually referred to as variances. As N becomes large, the number of covariance terms ($N^2 - N$) dominates the variance terms. For example, the variance of a portfolio comprising 50 stocks has 2,450 covariance terms (or 98% of all terms that collectively determine the risk of the portfolio) and only 50 individual stock variances.

The market portfolio is the focus of the CAPM and it is sufficiently large such that the variance of any individual asset makes an insignificant

contribution to the portfolio variance. Thus, variance risk is essentially eliminated in a well-diversified portfolio and all that is left is covariance risk.

Rearranging (4) it follows that,

$$E[r_i] = (1 - \beta_i)r^f + \beta_i E[r^m] \quad (6)$$

Thus, a two-fund separation equilibrium prevails, in which investors will hold only two funds, the risk-free fund paying r^f and the market portfolio M paying $E[r^m]$.

In the case of a levered firm, the CAPM predicts that the cost of debt, r_b is given by

$$E[r_b] = r^f + (E[r^m] - r^f)\beta_b \quad (7)$$

Similarly, the cost of equity r_e is given by

$$E[r_e] = r^f + (E[r^m] - r^f)\beta_e \quad (7A)$$

A portfolio consisting of both a debt security and an equity security, with weights for debt and equity of w_b and w_e respectively (that sum to one), will thus have a rate of return

$$r_p = w_b \times r^f + (E[r^m] - r^f)\beta_b + w_e \times r^f + (E[r^m] - r^f)\beta_e \quad (8)$$

This can be written equivalently as

$$r_p = w_b \times r_b + w_e \times r_e$$

which is the plain vanilla version of the WACC.

Without doubt, the CAPM is the most widely used model for estimating the cost of equity in regulated companies. Support for the CAPM can be found in Price Waterhouse Coopers (2009, p.2) who state that it is “the most appropriate framework for calculating the cost of equity”. The Water Services Regulation Authority (2010, p.N4) argue that “although the CAPM has its limitations, it is the most robust way for a regulator to measure the returns required by shareholders”. Further, the Civil Aviation Authority (2001, p.4) argue that the CAPM, “is an industry standard specifically in the context of estimating appropriate return benchmarks for regulated industries.”

A comprehensive survey on the use of CAPM is provided by Sudarsanam, Kaltenbronn and Park (2011) who survey the regulatory practices of the USA, Canada, Germany, Australia, New Zealand and the UK. They find that only Australia and Germany have aspects of the cost of equity calculation prescribed in statute. For five of the six countries surveyed the CAPM plays some part in the determination of the cost of equity. Table 2 reproduces the summary table in Sudarsanam, Kaltenbronn and Park (2011) and highlights the importance of the CAPM either as the primary model or the secondary model used to determine the cost of equity. We

note that Wright, Mason and Miles (2005, p.76) argue that “there is at present no one clear successor to the CAPM for practical cost of capital estimation”, while acknowledging that other approaches may provide a useful cross check. Further, Green, Lopez and Wang (2003) report that the CAPM is used (in conjunction with the comparable accounting earnings method and the discounted cash flow model) to determine the cost of equity capital for the US Federal Reserve Banks.

Table 2
Primary and secondary models used by regulators to estimate the cost of equity

	Australia	Germany	New Zealand	USA	Canada	UK
Regulator	AER	FNA	CC	NYSPUC	OEB	Ofgem
Primary model	CAPM	CAPM/RPM	CAPM	DDM	RPM	CAPM
Secondary model	CAPM					
Other use of DDM	Cross-check on MRP		Cross-check on MRP		Cross-check on MRP	Cross-check*

Source: Sudarsanam, Kaltenbronn and Park (2011)

Notes: * - on the overall cost of equity but not for individual firms, RPM = Risk Premium Model, DDM = Dividend Discount Model.

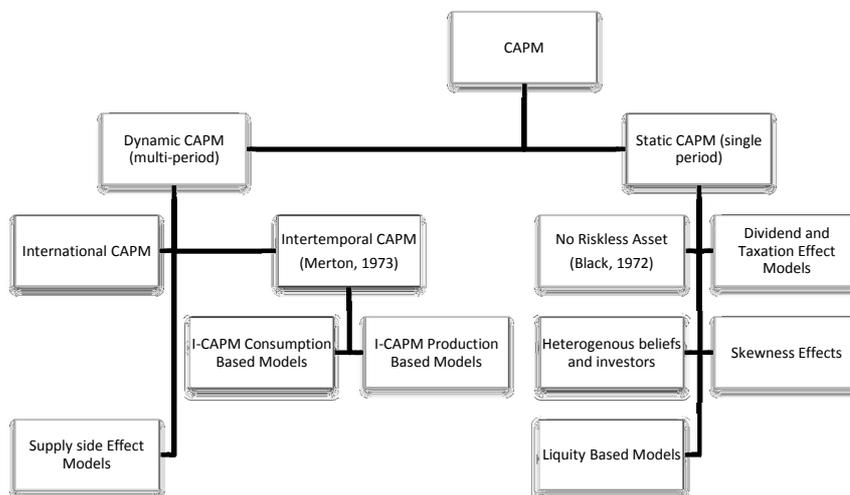
Despite its extensive use, the CAPM is not without theoretical and empirical difficulties. There are numerous, well documented difficulties associated with implementing the CAPM (for a detailed discussion of these issues in the regulatory context see McKenzie and Partington, 2011, 2012a). The main difficulty arises from the fact that the CAPM is a forward looking model - the implication is that we need information about *expected* returns, and *expected* variances and *expected* covariances of all possible assets in the market portfolio are required to estimate each firm’s *future* beta. Of course, applications of the CAPM typically use actual data and assume the future reflects the past. While attempts have been made to use expectations data (see Brav, Lehavy and Michaely, 2005), this raises a whole host of ancillary issues related to what these expectations are really capturing (see So, 2013).

A further difficulty is that the market portfolio is only vaguely defined. For example, Roll (1977, p.137) takes the market portfolio to be defined as a value-weighted combination of all assets. Fama and French (2004) note that under this definition, the market portfolio can in principle include not just traded financial assets, but also unlisted equities, debt, real estate, natural resources, art, precious metals, and so on.

Data availability places an obvious limit on the range of assets that are included in the market portfolio and the choice has typically been to focus on some form of equity index. Efforts have been made to broaden the set of assets included in the market portfolio (most notably Stambaugh, 1982, and more recently Doeswijk, Lam and Swinkels, 2012), however, drawing on a point made in Rolls (1977) critique, Diacogiannis and Feldman (2013, p.28) argue that “it is meaningless to use inefficient benchmarks to implement regressions of CAPM, which is designed to use efficient benchmarks”. As noted by Brown and Walter (2013), this means that attempts to refine empirical tests of the CAPM are ‘fundamentally flawed’ since the exact composition of the true market portfolio is unknown.

The empirical shortcomings of the CAPM have led researchers to consider alternative specifications in the search for a better asset pricing model. Shih, Chen, Lee and Chen (2013) survey this literature, distinguishing between developments that extend the standard single period CAPM and those that extend the CAPM to a multiple investment period framework. The literature surveyed in this paper is summarised in Figure 1 and reveals that a wide variety of extensions to the CAPM framework have been proposed. Of these, we have been asked to consider the Black CAPM and the Merton I-CAPM. Figure 1 is useful as it allows us to place these models in the context of the overall literature.

Figure 1
Summary of CAPM literature



Source: modified from Figure 1 in Shih, Chen, Lee and Chen (2013)

4.3 The Black CAPM

The standard CAPM model shows that the market portfolio is the only portfolio of risky assets known to be efficient *ex ante*. This result is derived under a number of assumptions, the most critical of which is that investors can borrow unlimited amounts at the risk free rate. In the event that this assumption does not hold, then the market portfolio may not be the only mean-variance efficient portfolio of risky assets and different portfolios may be suitable for different investors depending on their particular level of risk aversion. Black (1972) derived a modified version of the CAPM, in which the assumption of unlimited borrowing was relaxed. In its place, it was assumed that investors could engage in short selling and all proceeds of the stocks sold would be available for investing. Under the Black CAPM, the market portfolio is efficient once more, but it is not unique. Black's version of the CAPM can be summarised as:

$$E[r_i] = E[r_z] + (E[r^m] - E[r_z])\beta_i \quad (9)$$

where r_z ($r^f < r_z < r^m$) represents the return on a portfolio that has zero covariance with the return on the market portfolio, i.e. $\beta_z = 0$. Thus, the key distinction between the CAPM and the Black CAPM is the replacement of the risk free rate with the return to a zero beta portfolio.

In equilibrium, investors will again hold only two funds, the zero- β fund paying $E[r_z]$ and the market portfolio M , paying $E[r^m]$. The measure of non-diversifiable risk is given by,

$$\beta_i = \frac{\text{Cov}(r_i, r^m)}{\text{Var}(r^m)} \quad (10)$$

Clearly (5) and (10) are identical so the implication is that (7), (7A) and (8) would obtain with $E[r_z]$ in place of r^f . Therefore, in equilibrium, investors will only be compensated for bearing non-diversifiable risk.

A problem with the Black CAPM is that the assumption made about the proceeds of short selling does not accord with how the stock lending markets work.⁸ In the real world, short sellers are required to post collateral when lending stock in the form of cash and/or equity. As this key assumption to the Black model does not hold, the efficiency of the market portfolio is again lost. As noted by Markowitz (2005, p.19), these departures of efficiency can be considerable and the market portfolio can have almost maximum variance among portfolios with the same expected value. In this case, there is no representative investor and expected returns are not linear functions of risk.

⁸ See Faulkner, M. (2004) "An Introduction to Securities Lending" available at www.bankofengland.co.uk/markets/Documents/gilts/securitieslending.pdf

McKenzie and Partington (2012b, pp.7-8) provide an in depth discussion of the use of the Black CAPM in the context of its application to regulated utilities and argue:

At issue here is the benchmark for measuring risk premiums/excess returns and the method of estimation for that benchmark. Should the benchmark be the risk free rate, for which the usual proxy is the return on a government security? Alternatively, should the benchmark be the rate of return on the zero beta portfolio in the Black CAPM, for which the proxy is an estimate of the rate of return on a zero beta portfolio?

The near universal practice in measuring the risk premium/excess returns is to benchmark using the risk free rate as proxied by the yield on a government security. The widespread nature of this approach suggests that there are good reasons to prefer the risk free rate as the benchmark. As we subsequently demonstrate there are indeed good reasons to prefer the risk free rate.

Using the yield on a government security as a proxy for the risk free rate is generally accepted. The measurement of the yield is relatively simple and transparent. The input variables can be readily observed and error in the measurement of the resulting yield is little or nothing.

In contrast, there is no generally accepted empirical measurement of the zero beta return in the Black CAPM. This is because the empirical measurement of the zero beta return is neither simple, nor transparent. There are many possible zero beta portfolios that might be used and the return on these portfolios is not directly observed, but has to be estimated. In the estimation process for the zero beta return, there are also inputs that cannot be observed and they too have to be estimated. The resulting estimate of the zero beta return is sensitive to the choices made in regard to the input variables and methods of estimation. As a result the measurement error can be large and the result ambiguous.

We refer interested readers to the original document for further discussion and elaboration on these points.

In terms of the regulatory use of the Black CAPM, to the best of our knowledge, there has not been a regulatory body that has relied on the Black CAPM to estimate the cost of equity. We note that the NERA (2012) report arguing for the use of the Black CAPM in regulation does not cite any use of the Black CAPM in practice, instead arguing for the possibility that it may be implicitly used.

4.4 The Merton Intertemporal CAPM (I-CAPM)

In both the CAPM and the Black variant, investors care only about the wealth their portfolio produces at the end of the investment period. In the I-CAPM, however, investors are also concerned with the

opportunities they will have to invest (or consume) the payoff. These opportunities vary with future state variables, which capture expectations about income, consumption and investment opportunities. Equilibrium in this model suggests that investors expected returns will reflect not only market risk, but also compensation for bearing the risk of unfavourable shifts in the investment opportunity set.

Thus, the I-CAPM model potentially has a number of betas, which at the limit is theoretically equal to one plus the number of state variables needed to describe the relevant characteristics of the investment opportunity set. While investors in this model still prefer high expected return and low return variance, they are also concerned with the covariances of portfolio returns with the state variables.

The main problem with operationalising the I-CAPM is that it is not easy to identify the state variables that affect expected returns. We do know that at least one element of the investment opportunity set is directly observable - the interest rate – and it varies stochastically over time.

The simplest form of the I-CAPM model assumes that the distribution of returns is lognormal and the stochastic behaviour of r^f gives rise to a three-fund separation, with investors holding the risk-free fund paying r^f , the market portfolio M paying $E[r^m]$, and a third fund N , that covaries negatively with r^f . The required return on the i^{th} asset in this situation is

$$E[r_i] = r^f + \delta_1(E[r^m] - r^f) + \delta_2(E[r^n] - r^f) \quad (11)$$

where r^n is the instantaneous return on an asset displaying perfect negative correlation to r^f , δ_1 and δ_2 are weights given by

$$\delta_1 = \frac{\beta_{im} - \beta_{in}\beta_{nm}}{1 - \rho_{nm}^2} \quad \text{and} \quad \delta_2 = \frac{\beta_{in} - \beta_{im}\beta_{nm}}{1 - \rho_{nm}^2}$$

and

$$\beta_{ik} = \frac{\text{Cov}(r_i, r^k)}{\text{Var}(r^k)}. \quad (12)$$

where K indexes the factors. It is clear from this equation that the form of β has not changed, with the exception of the movement from rates of return over discrete intervals of time to instantaneous rates of return (the I-CAPM is a continuous time model). This implies that, in equilibrium, investors will only be compensated for bearing non-diversifiable risk and so equations (7), (7A) and (8) would apply with appropriate adjustment of the asset pricing model.

In terms of the regulatory use of the I-CAPM, to the best of our knowledge, there has not been a regulatory body that has relied on this version of the CAPM to estimate the cost of capital.

4.5 The Fama and French Three Factor model

In the remaining part of Section 4, we discuss the Fama and French three factor model and the dividend growth model. Strictly speaking, these are not asset pricing models as they contain no theory about equilibrium expected returns. These models have nonetheless been used to provide estimates of the cost of equity.

It should be said at the outset that the origins of the Fama and French (1993) three factor model are empirical and were derived based on the analysis of US stock data. The model builds on Fama and French's (1992) search for variables that would statistically explain subsequent returns. Fama and French's (1992) search was not just data-mining across a mass of variables. The relatively small set of variables, selected for inclusion in the search, were ones that had been identified in prior empirical work as having explanatory power with respect to returns. In their 1992 paper, Fama and French found that both firm size and a firm's book to market ratio helped explain subsequent returns. It was this discovery that led them to include a size factor and a book to market factor in their 1993 factor model of returns. They showed that his model did a good job empirically in explaining returns on portfolios. The Fama and French (1993) model is therefore solidly rooted in data analysis and, as a consequence, there is no clear theoretical foundation to identify the risk factors, if any, that the model captures. There have, however, been attempts at ex-post rationalisation of what the risk factors might be.

The Fama and French model can be written as:

$$E[r_i] = r^f + \beta_{im}(r^m - r^f) + \beta_{is}(r^s - r^b) + \beta_{ib}(r^h - r^l)$$

where, r_i is the rate of return on security i , r^f is the risk free rate of return, $r^m - r^f$ is the market risk premium, $r^s - r^b$ is the size premium often written as SMB (small minus big), $r^h - r^l$ is the value premium often written as HML (high minus low), β_{im} is the beta factor (factor loading) for security i on the market factor and similarly for the size and value factors. We note that the definition of β_i in this model is of the same form as (5) and so result analogous to equations (7), (7A) and (8) are again obtained and the same conclusion holds, i.e. only non-diversifiable risk is compensated in equilibrium. However, the question still remains, what risks do the factors in the model capture?

4.5.1 What risk do the factors capture?

The three factors in the model are the market factor, the size factor and the book to market factor. The market factor was well known from the CAPM. The size effect was also a well known CAPM anomaly and referred to the fact that, relative to their beta, small firms have historically offered higher returns than big firms. As such, a natural measurement of

the size factor was to take the difference in returns between a portfolio of small firms and a portfolio of large firms.⁹ Such spread portfolios are equivalent to a long position in securities that have a high exposure to the “risk” factor and a short position in securities that have little or no exposure. The main contribution of the Fama and French model was to popularise the book to market or value factor as an explanatory variable in asset pricing. Firms with a high book to market ratio, known as value stocks, offered a higher return than the firms with a low book to market ratio, known as growth stocks. So, a natural choice was to measure the value factor as the difference in return between a portfolio of high book to market ratio stocks and a portfolio of low book to market ratio stocks.

A risk explanation for the size factor sometimes offered is that it reflects a liquidity premium. Small stocks are known to be less liquid than large stocks and are therefore more difficult and expensive to trade. Small stocks also have higher bid-ask spreads than large stocks and the market impact of trades is higher. Consequently, it is argued that investors demand a higher return to compensate themselves for the cost of liquidity and the risk that they will need to liquidate their investment at a time when liquidity costs are high.

A risk explanation for the book to market factor sometimes offered is that it is a financial distress factor that is being priced. A high book to market ratio is suggestive of a company where the market value of the assets have fallen sharply and the book values have not caught up. Such companies are at higher risk of financial distress and perhaps this is what is being priced.

It may be that the size and book to market are risk factors which are missing from the CAPM, but it is not clear why. Another possibility is that they are not risk factors themselves, but are proxies for missing risk factors. It might also be the case that the factors proxy for changing state variables, which reflect the dynamic variation of investment opportunities in the spirit of Merton’s inter-temporal CAPM. For example, Fama (1996) notes that, while the size and book to market factors are not state variables, they may reflect unidentified state variables that produce non-diversifiable risks in returns that are not captured by the market return.¹⁰

Another interesting possibility is that the Fama and French factors reflect higher moments of the return distribution, such as skewness and kurtosis

⁹ More recent history, in the USA is that the size effect is getting smaller and may even have disappeared. In Australia some studies suggest that it is negative.

¹⁰ We note that a body of research exists that is aimed at replacing the size and book to market variables with economic variables that relate more readily to investors’ concerns (see *inter alia* Petkova, 2006, Brennan et al, 2004).

that are ignored in the mean variance theory that underlies the CAPM.¹¹ Chung and Schill (2006) show that when the higher order co-moments of the return distribution are included in the asset pricing model then the Fama and French factors become statistically insignificant.

4.5.2 Do the factors mean anything?

It is possible that the Fama and French factors do not mean anything at all, but are merely the result of capitalising on chance. If the data are tortured by statistical analysis for long enough it will give up a story, but the significance tests will have no substance. However, the Fama and French model has been found to work in markets other than the much analysed US market. We discuss the results of studies in the Australian market below. The applicability of the model in new datasets suggests that the Fama and French results are not just chance. However, because there is no theory, it can also be suggested that the model's success may not be due to the explanation of equilibrium returns, but rather that the model explains pricing errors arising from investors' behavioural biases.

An important note of caution about the interpretation of models like the Fama and French model was sounded by Ferson, Sarkissian and Simin (1999). They take the case of a return anomaly, based on a firm attribute, such as firm size, where the anomaly has nothing to do with risk, but is a consequence of either data mining or behavioural bias. They then show that spread portfolios, long on one attribute say small stocks and short the other attribute say large stocks, can appear to be a priced covariance risk in a factor model, even though the attribute has nothing to do with risk. Smith and Walsh (2013) note that the existence of such *ex-post* factors such as size and book to market, neither supports nor contradicts the CAPM. They make the point that just because you can create *ex-post* efficient portfolios does not mean that the factors you use are priced *ex-ante*. Another note of caution was sounded by Black (1993) who argued that such studies are about explaining variance rather than expected returns.

¹¹ While beyond the scope of this report, we note that models have been developed that specify a representative investor whose preferences extends beyond the mean and variance of returns (see Zhang, 2012, and references therein). A parallel literature also exists which examines the empirical performance of asset pricing models that include the third (see Harvey and Siddique, 2000) and fourth (see Fang and Lai, 1997, and Dittmar, 2002) moments of the return distribution. The general form of such as model is:

$$E[r_i] - r_f = \alpha_1 \text{Cov}(r_m, r_i) - \alpha_2 \text{Cov}(r_m^2, r_i) + \alpha_3 \text{Cov}(r_m^3, r_i)$$

where the α 's are the market prices of systematic variance, systematic skewness and systematic kurtosis respectively. If the asset has more or less skewness, or kurtosis, than the market, this will be priced into expected returns.

4.5.3 Are other factors priced?

The search for priced factors continues. One well recognised return anomaly is the momentum effect. This refers to a short run phenomenon whereby there is a continuation of returns, in that winners tend to continue to win and losers tend to continue to lose. This motivated Carhart (1997) to add a momentum factor to the Fama and French three factor model, with a winner minus loser (UMD) factor. However, the question of whether this is a risk factor is left open, as Carhart (1997, p.61) observes, “*I employ the model to "explain" returns, and leave risk interpretations to the reader.*” Fama and French (2004) argue that the short lived nature of the momentum effect means that it is irrelevant when estimating the cost of capital.

The term spread and the credit spread were added to the Fama and French model by Green, Lopez and Wang (2003). This was motivated by their application, which was to estimate the cost of capital for banks. However, they concluded that the augmented Fama and French model was not a substantive improvement on the CAPM.

4.5.4 Australian evidence

Haliwell, Heaney and Sawaki (1999) found some support for the Fama and French model using Australian data, but the evidence for a size factor was more compelling than the evidence for a book to market factor. A subsequent study by Faff (2004) provided qualified support for the Fama and French model, but there was a negative risk premium for the size factor. These results are typical of the early Australian studies, inasmuch as there is consistent evidence for a size effect, albeit with varying sign, but the evidence for the book to market effect is less convincing. The early studies, however, were hampered by relatively small firm sample sizes, covering relatively short time periods. This was due to difficulties in obtaining data, particularly the book values required to calculate the book to market ratio. A recent study, Brailsford, Guant and O’Brien (2012), uses a much more comprehensive data set involving a substantial amount of hand collected data. This paper provides stronger support for the three factor model than the previous research, particularly with respect to the book to market factor. However, the size factor again had a negative risk premium, although statistically speaking it was not significantly different from zero.

4.5.5 Applications of the Fama and French model

The Fama and French model (often in conjunction with a momentum factor), has become an almost standard control in academic studies that attempt to measure abnormal returns in event studies and to evaluate trading strategies using US data. A key reason for this popularity is that

Ken French makes much of the required data, particularly the factor premiums, freely available on his website. In other countries, the use of the Fama and French model is less prevalent because of the costs of data acquisition and computation. Note that the use of the Fama and French model as an experimental control in academic studies of returns does not necessarily require a belief that the factors control for risks. In the words of Carhart (1997, p.61):

Alternately, it may be interpreted as a performance attribution model, where the coefficients and premia on the factor-mimicking portfolios indicate the proportion of mean return attributable to four elementary strategies...

In terms of practice, there is little evidence of use of the Fama and French model by companies to estimate their cost of capital. In the case of Australia, the survey evidence of Truong, Partington and Peat (2008) suggests that the CAPM dominates and that the Fama and French model is not used. In their (1997) paper, industry costs of equity were estimated by Fama and French using both their three factor model and the CAPM. They tended to favour the results of their three factor model, but there was not a lot in it. Their main concern was the woefully imprecise estimates provide by both models.

The Fama and French model has been used in attempts to estimate the cost of capital for regulated utilities in the USA (see Schink and Bower, 1994, and Chetrien and Coggins, 2008), and in the UK (Europe Economics, 2007 and 2009). The general regulatory preference, however, has clearly been for the use of the CAPM. This is not surprising when we consider evidence such as that of Europe Economics (2007), who analysed the factor premiums over time and reported that they change sign and that they are often not significantly different from zero. Indeed the return on the book to market factor was never significantly different from zero. Furthermore, in estimating the factor loadings for a regulated entity (Heathrow and Gatwick airports), the only significant factor loading was on the market factor. Similar results were obtained in a study of regulated water companies by Europe Economics (2009). Europe Economics (2007, p.47) conclude:

The results of the investigation are not encouraging for the use of the Fama and French model in regulatory price review setting.

4.5.6 Conclusion on the Fama and French model

In summary, the Fama and French three factor model provides no clear guidance on exactly what are the risk factors that are priced. There are also some somewhat arbitrary choices that must be made in measuring the

factor risk premiums as the return to the spread portfolios.¹² Furthermore the empirical evidence suggests ambiguity about the magnitude of the premiums and even their sign. Despite these issues, the Fama and French three factor model has been used as a method to estimate the cost of equity. However, to do so requires significant effort in estimating factor risk premiums and factor loadings with no clear evidence that an improved estimate of the cost of capital results relative to the simpler CAPM.

Green, Lopez and Wang (2003) established the method that the US Federal Reserve used to estimate the cost of equity for US banks and we leave the last word on multifactor models, such as the Fama and French model, to them:

Multibeta models could be employed to calculate the equity cost of capital used in the PSAF. However, because there is no consensus on the factors, adoption of any particular model would be subject to criticism. Because the academic literature shows that multibeta models do not substantially improve the estimates, the gain in accuracy would likely be too small to justify the burden of defending a deviation from the CAPM. We therefore do not recommend using multibeta models to calculate the cost of equity capital in the PSAF. Nevertheless we present some numerical results based on the Fama and French (1993) model. These results indicate that any additional accuracy provided by multibeta models is clearly outweighed by the difficulties in specifying and estimating them. (p.73)

4.6 The Dividend Growth Model (DGM)

4.6.1 Implied cost of capital models

At the outset of any discussion on the use of DGM, it is important to recognise that there are several dividend growth models, each of which varies depending on the assumptions made about the growth path for dividends and the terminal value used in the model. The second point to be made is that in estimating the cost of equity, dividend growth models form a sub-class of implied cost of capital models. As the name suggests, these models make no explicit statement about what determines the cost of capital and, in particular, they are silent on the risks that are to be compensated through the cost of capital.

¹² One procedure is to take the top 30 percent of the firms and bottom 30 percent of firms. For example, the difference in returns to a portfolio of the bottom 30 percent of firms by book to market and the top 30 percent by book to market gives the return to the book to market factor, but other choices would be just as valid.

These implied cost of capital models are based on discounted cash flow valuation models, although it is possible to recast such models in terms of accounting data. The implied cost of capital of a firm is the internal rate of return that equates its current stock price to the present value of dividends, future cash flows, residual incomes, or abnormal earnings, depending on which valuation models are used. Given the valuation model and all inputs except the cost of capital, it is possible to solve the model and obtain the value of the unknown variable, which is the implied cost of capital.

In the implied cost of capital approach, the cost of capital is exogenous to the model. Some would argue that this is a strength of these models. They do not rely on a theoretical asset pricing model, but rather extract a forward looking cost of equity capital direct from the data. However, the validity of the cost of capital estimates depends upon choosing the right valuation model and using the right inputs.

4.6.2 Inputs for the DGM

In the case of the dividend growth model, the usual input data is price, expected dividends, dividend growth rates and their pattern through time. When applying this model, it is net dividends that are required, which allow for share repurchases and contributions of capital. This latter factor is particularly neglected and, given that many companies now have dividend reinvestment plans, it is likely that there will be ongoing capital contributions by owners to be accounted for, as well as subscriptions of extra capital via rights issues and placements.

In the Australian context, it may also be necessary to consider the value distributed to investors via the imputation tax credits that accompany dividends. Whether this is required depends on the application of the consistency principle, as we discuss in a later section of our report. However, once we admit that tax considerations may matter, the composition of returns becomes important to the dividend growth model. This is because dividends and capital gains are often taxed at different rates. Thus, it is not just the magnitude of the cash flow that may matter to investors, but the form it comes in, i.e. dividends or capital gains. This not only has implications for the appropriate dividend growth model, it also has implications for measuring the net dividend, since share repurchases are considered a return of equity capital. Similarly, whether shareholders participate in a Dividend Reinvestment Plan or a share bonus plan have different tax consequences. Whether these are first order or second order effects is an open question and may vary across firms.

There is also a neglected value factor in dividend growth models, which is the value of options, particularly growth options. Recall that the valuation in a dividend growth model is based entirely on the cash flow distributed as dividends. Therefore, it neglects any value that arises from options to

change the way the business operates as circumstances unfold. For example, there might be value in the option for regulated entities to choose to gold-plate the network, or to adopt cheaper technology, as the opportunity arises. The value of such options is not captured in dividend growth models and, to the extent that such options are significant contributors to firm value, the implied cost of equity will be understated by the dividend growth model.

4.6.3 The Gordon growth model

The dividend growth model commonly considered in the regulatory context is the Gordon growth model which may be written as:

$$P_0 = \frac{E[D_1]}{r - g}$$

where P_0 is the estimate of the equity value, $E[D_1]$ is the expected dividends next period, r is the required return on equity, g is the expected growth rate.¹³ This equation can be rearranged to give:

$$r = \frac{E(D_1)}{P_0} + g \quad (13)$$

The Gordon growth model can be derived assuming that future dividends are known with certainty. If dividends are assumed to be uncertain, equation (13) is only correct under certain assumptions about the nature of the uncertainty (and continuing to assume a constant discount rate). Armitage (2005, Chapter 12) outlines two possibilities as follows:

Suppose

$$Div_t = Div_1[1 + E_0(g)]^{t-1} + e_t$$

for some date t periods ahead, where $E_0(e_t) = 0$ and $E_0(e_t, e_{t-1}) = 0$. Div_1 is certain and the expected value of g is constant. Then

$$\begin{aligned} E_0(Div_t) &= E_0[Div_1(1 + g)^{t-1}] + E_0(e_t) \\ &= Div_1(1 + g)^{t-1} \end{aligned}$$

in which case equation (13) can be used; the expected dividend can be treated as a certain dividend. A second possibility is if

$$Div_t = Div_{t-1}(1 + g + e_t)$$

where $E_{t-1}(e_t) = 0$ and $E_{t-1}(e_t, e_{t-1}) = 0$. In this case dividend realisations form a random walk with upward drift. Taking expectations,

$$E_{t-1}(Div_t) = E_{t-1}[Div_{t-1}(1 + g + e_t)]$$

¹³ In the Gordon model, everything (prices, dividends and earnings) grow at the same rate.

$$= Div_{t-1}(1 + g)$$

in which case

$$E_0(Div_t) = Div_1(1 + g)^{t-1}$$

assuming Div_1 is certain. Again, equation (13) can be used. But the equation will not be correct if there is serial correlation across future dividends, nor if Div_1 is uncertain and correlated with $1 + g$.

Equation (13) provides a simple, rough and ready model that may be used to estimate the cost of equity capital for regulated businesses. In this form, however, it may be too rough even to act as a reasonableness check. As is evident from the model, the long run growth rate (which is unobservable and is usually based on some assumption such as matching the expected growth rate in the economy) is critical to the estimated magnitude of the cost of capital. Price volatility can also have a substantial impact, as sharp changes in the current price can have a dramatic effect on the dividend yield.

More refined models may be used such as equation (14), where dividends are forecast out to some time horizon and then the Gordon growth model is used to estimate the horizon value of the share. Other multi-horizon versions of (14) are possible allowing for more complex patterns in growth. Whatever the model, the same conclusion applies in that the results are usually sensitive to assumptions about growth rates.

$$P_o = \sum_{t=1}^n \frac{E(D_t)}{(1+r)^t} + \frac{\frac{E(D_n)}{r-g}}{(1+r)^n} \quad (14)$$

Despite the problems of the DGM, it is relatively easy to form cost of capital estimates using this method and it has had considerable use by regulators in the USA (see Sudarsanam, Kaltenbronn and Park, 2011). Indeed, in time gone by, before the advent of the CAPM, the DGM was commonly advocated for estimating the cost of capital. However, its primacy in this role was ended by the ascendancy of the CAPM.

4.6.4 Risk in the DGM

The DGM provides no explicit guidance on risk, but inspection of the model (such as that presented in equation (14)) reveals that the only stochastic variable is the expected dividend. While the dividend growth rate is usually written as a fixed parameter in the model, the reality is that the growth rate is uncertain and this translates into ongoing uncertainty of the magnitude of the dividend through time. Thus, the risk that is recognised in the DGM, and therefore presumably driving the required return, is uncertainty over future cash flows in the form of dividends.

This is consistent with asset pricing models where uncertainty over future cash flows is the key risk.

Since Miller and Modigliani (1961) there has been an ongoing debate about whether dividend policy can affect value. While it is now accepted that this is possible, it is important to note that this is not because dividend policy affects risk. Rather taxes and transactions costs can potentially drive a wedge between the market and face value of dividends and if they do this is likely to affect the value of the firm.

4.6.5 The effect of taxes

Accounting for the effect of differential taxes on dividends and capital gains is tricky but Dempsey and Partington (2008) provide a simple method to deal with this. The method relies on knowing the ratio of the market value of dividends to the face value of dividends q . The value of the firm's equity is then given by:

$$P_0^{ex} = \sum_{t=1}^T \frac{E[D_t]q}{(1+R)^t} + \frac{E[P_T^{ex}]}{(1+R)^T} \quad (15)$$

where the superscript 'ex' indicates an ex-dividend price. Multiplying the dividend by q converts the dividend to a market value. Consequently the discount rate R must be a discount rate appropriate to discounting market values (prices) and is defined as the cum-dividend return:

$$R \equiv \frac{E[P_{t+1}^{cum}] - P_t^{ex}}{P_t^{ex}} \quad (16)$$

Where the superscript 'cum' indicates a cum dividend price. This discount rate can be obtained from a particularly simple version of the *after tax* CAPM:

$$R = r^f q^b + \beta(r^m - r^f q^b) \quad (17)$$

Where the returns represent cum-dividend returns and q^b is the ratio of the market to face value of interest.

4.6.6 Conclusion on the DGM

While the DGM provides no explicit guidance on the risks that investors should be compensated for, implicitly, they are compensated for uncertainty about future cash flows in the form of dividends, which in turn is driven by uncertainty about the firm's cash flow. The DGM approach gives rise to models that are readily implemented, however, the resulting estimate of the cost of equity will be sensitive to the choice of model and to assumptions about the growth rate in dividends.

Appendix – The Stochastic Discount Factor model

The SDF model states that the price of an asset is equal to the expected discounted value of the asset's payoff. This seems straightforward enough. However, as the name of the model implies the discount rate is stochastic. The basic intuition of the model is that both cash flows and discount rates may vary according to future states of the world.

Since both the cash flow and the discount rate are contingent on the possible future states of nature, both are uncertain at the current date. Discount rates will depend on the rate at which investors are prepared to trade current for future consumption in a given state of the world. Thus, for example, when the future state is recession consumption may be valued more highly at that time and, if so, the discount rate for that state is likely to be higher.

Risk in this model depends on the covariance of cash flows with the discount factor. Payoffs that have negative covariance with the discount factor (low payoffs when the discount factor is high) are less highly valued than payoffs that are certain. In other words, assets which have negative covariance with the SDF must offer a positive risk premium in order to induce investors to hold the asset.

More formally and following Smith and Wickens (2002), the price of an asset in period t is the expected discounted value of the asset's payoff in period $t + s$ based on information available in period t (for convenience and without loss of generality, assume $s = 1$ hereafter), i.e.

$$P_t = E_t[Y_{t+1}X_{t+1}] \quad (A1)$$

where P is the price of the asset in period t , $E[\]$ is the expectations operator and X is the payoff to the asset, which is unknown at t and is assumed to be a random variable. Y is the discount factor, which is $0 \leq Y \leq 1$. Both the future cash flow and the future discount factor are contained within the brackets of the expectation operator so the future values of both are uncertain.

Defining the assets gross return as $R_{t+1} = \frac{X_{t+1}}{P_t}$, equation (A1) can be rewritten as:

$$1 = E_t[Y_{t+1}R_{t+1}] \quad (A2)$$

Where the asset is risk free, R_{t+1} will be equal to the risk free rate of return, i.e. $1 + r^f$. As such, equation (A2) may be rearranged as

$$E_t[Y_{t+1}] = \frac{1}{1 + r^f}$$

and $E_t[Y_{t+1}] = \frac{1}{1+r^f} + \varepsilon_{t+1}$, where the error term is a random variable with a zero conditional mean. Thus, by definition, the discount factor, Y_t , is a stochastic variable.

For a risky asset, the excess return may be defined as

$$E_t[r_{t+1} - r_t^f] = -(1 + r_t^f)Cov(Y_{t+1}, r_{t+1} - r_t^f)$$

where the term on the right hand side is the risk premium, which must be non-negative and so, by implication, the covariance between the discount factor and the excess return is non-positive, i.e.

$$Cov(Y_{t+1}, r_{t+1} - r_t^f) \leq 0$$

Thus, in the SDF model risk arises from the negative covariance between the discount factor and the excess of returns over the risk free rate. When the stochastic discount factor is in a high (low) state, the present values of future cash flows, and so returns, are lower (higher).

Note also that under the no arbitrage assumption, it can be shown that $Y_{t,t+2} = Y_{t,t+1}Y_{t+1,t+2}$, i.e. a predictable terms structure of discount rates exists. One possible interpretation of this result is that a conditional equity market risk premium is called for. This issue has been addressed in a previous report by McKenzie and Partington (2011, p. 17), who note that:

... there are some compelling reasons to avoid the use of conditional equity market risk premium estimates. Firstly, Hathaway (2005) argues that there is no obvious term structure for equity returns, in the same way that bond yields have a term structure. In fact, he argues that it is 'safe' to say that the mean return per period is the same for short term investments as it is for long term investments (an argument therefore in favour of an unconditional approach).

We note that while theoretical models exist to support, upward sloping, flat, and downward sloping term structures for expected equity returns, recent empirical evidence supports a downward sloping term structure (see Binsbergen, Brandt and Koijen, 2012). However, this result is challenged by Boguth, Carlson, Fisher and Simutin (2012) who show that allowing for almost negligible pricing frictions (measurement errors) restores a flat term structure.

To operationalise the SDF model, a choice must be made about how to model the discount factor, Y_r . The model can either be implicit or explicit and can use observable or latent factors.

For example, the CAPM may be interpreted as an example of an implicit and observable SDF model. To see this, assume time separability¹⁴ and a discount factor $Y_{t+1} = \gamma_t(1 + r_{t+1}^m)$, where γ_t is the coefficient of relative risk aversion which is assumed to be $\gamma_t > 0$. In this case, where the market return is where r^m , the risk premium is:

$$E_t[r_{t+1} - r_t^f] = \gamma_t \text{Cov}_t(r_{t+1}^m, r_{t+1}) = \gamma_t \text{Cov}_t\left(\frac{\Delta W_{t+1}}{W_t}, r_{t+1}\right) \quad (\text{A3})$$

i.e. the risk premium is a function of the conditional covariance of the assets return with the market return (i.e. the standard CAPM), or equivalently, the rate of growth of wealth, i.e. $Y_{t+1} = \gamma_t \frac{\Delta W_{t+1}}{W_t}$. A more familiar representation can be derived if we define $\beta_t = \frac{\text{Cov}_t(r_{t+1}^m - r_{t+1}^f)}{\text{Var}_t(r_{t+1}^m)}$ (where Var_t is a conditional variance term) and $E_t(r_{t+1}^m - r_{t+1}^f) = \gamma_t \text{Var}_t(r_{t+1}^m)$, in which case we can re-express the SDF CAPM in equation (A3) in the standard form for the CAPM risk premium:

$$E_t[r_{t+1} - r_t^f] = \beta_t E_t[r_{t+1}^m - r_t^f]$$

that is to say, the expected excess return on a stock is beta (the quantity of risk) times the expected excess return on the market (the price of market level risk).

In the case of the consumption CAPM, the SDF (Y_t) depends on covariance between the payoff to the asset and consumption growth. More complex choices exist for the SDF, including having more than one risk factor. For example, a multi-factor version of CAPM and the C-CAPM may be expressed as

$$E_t[r_{t+1} - r_t^f] = \sum_i \beta_i f_{it}$$

where f_{it} are conditional covariances (often referred to as common factors), which in the cases above are related to r_{t+1}^m and $\Delta \ln C_{t+1}$ respectively. Thus, in the SDF model, assets are priced assuming the factors are linear functions of the conditional covariances between i factors and the excess return on the risky asset. The SDF model is extremely flexible and can be used for the analysis of both linear and non-linear asset pricing models (see Cochrane, 2001).

To estimate a particular asset pricing model, such as the CAPM, using its SDF representation the GMM (General Method of Moments) is typically used. The SDF, however, has not proven popular for empirical work.

¹⁴ Note that time separability of utility means that past work and consumption do not influence current and future tastes. This assumption may be violated, i.e. time non-separability, because of either habit persistence or durability (i.e. yesterday's consumption increases the agent's current utility).

Jagannathan and Wang (2002, p.2338) observe that the reluctance to use SDF models may be due to the perception that

...the generality of the SDF framework comes at the costs of estimation efficiency for risk premiums and testing power for model specification.

The previous literature referenced in Jagannathan and Wang (2002) certainly supports this notion. However, Jagannathan and Wang (2002) introduce a revised framework for comparing the SDF to the more traditional methods and find that the SDF method is as efficient at estimating risk premiums as the more traditional approach.

As noted by Smith and Wickens (2002), where the SDF model has proven useful is in the pricing of bonds and modelling of the term structure. The most common models employed in this context have been the explicit but unobservable factor models introduced by Vasicek (1977) and Cox, Ingersoll and Ross (1985). Both of their models assume that the stochastic discount factor can be expressed as a linear function of one or more random variables. In both of these models, the shape of the yield curve is a constant and curve shifts through changes in the short term rate. The Vasicek (1977) model, however, assumes the risk premium depends on the time to maturity, while the Cox, Ingersoll and Ross (1985) model allows for a conditional risk premium.

Thus, the generality of the SDF framework serves to highlight the point that a wide range of asset pricing models can be reduced down to various assumptions about how the SDF evolves.

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