Estimating β: An update

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1. Introduction and scope of the project

The focus of this report is the calculation of empirical estimates of the equity β for the regulated gas and/or electricity transmission/distribution networks. The consultant has also been asked to provide advice on the econometric aspects of the calculation of equity β. This report builds upon the methods for estimating equity β presented in two previous reports for the Australian Energy Regulator (AER), in 2008[1] and 2009[2].

The approach undertaken in this report is to estimate the equity β for the comparator set and then to delever/relever this estimate to produce an estimate of the equity β for the benchmark firm (at 60 per cent gearing).

1.1. Theoretical Background

The Capital Asset Pricing Model, (hereafter CAPM) predicts that the expected return to the i^{th} asset, $E(r_i)$, is given by

$$E(r_i) = r_f + \beta_i\left[E(r_m - r_f)\right], \quad (1)$$

Where r_f is the rate of return to the riskless security and $\beta_i = \frac{Cov[r_i, r_m]}{Var[r_m]}$. An investment in a security with β=1 can be thought of as being as risky as the decision to hold the market portfolio, while a security with β<1 is typically referred to as a defensive' investment. Finally, investors who decide to hold a security (or portfolio of securities) with β>1 will be compensated for their exposure to higher risk with increased expected returns.

Essentially the CAPM describes the excess expected return to the i^{th} asset, $E(r_i) - r_f$ as a risk premium. This risk premium may be written as a fixed price per unit of risk, $\lambda_i = \left[E(r_m - r_f)\right]/Var[r_m]$, multiplied by a quantity of risk, $Cov[r_i, r_m]$.

$$E(r_i) - r_f = \lambda_i Cov[r_i, r_m], \quad (2)$$

1 Henry, O.T., *Econometric advice and beta estimation*, 28 November 2008, available at: http://www.aer.gov.au/sites/default/files/Attachment%20C%20-%20Henry%20-%20Econometic%20advice%20and%20beta%20estimation.pdf .

2 Henry, O.T., *Estimating β*. 23 April 2009, available at http://www.aer.gov.au/sites/default/files/Attachment%20C%20-%20Henry%20-%20Estimating%20beta.pdf

1.2. Scope of the project

Table 1 summarises the scope of the project as defined by the AER. The table shows the various permutations that are to be undertaken in the empirical estimation of equity β. The table also includes areas where the consultant was asked to provide advice to the AER on conceptual (econometric) issues related to the generation of empirical estimates.

Table 1: Scope of the project

Issues	**Approach 1**	**Approach 2**	**Approach 3**	**Notes**
Regression details				
Regression equation	Either raw returns or excess returns (but not both)			Provide advice on the use of raw returns and excess returns.
Regression calculation	Ordinary Least Squares (OLS)	Least Absolute Deviation (LAD)		
Dataset details				
Estimation period	Longest available	Post tech boom, but excluding GFC	Last five years	Provide advice on the strengths and weaknesses of each period. Consultant determines the specific GFC dates
Return interval	Weekly			Disclose exact interval construction. Monthly intervals as a robustness check.
Data from Country	Australian			
Companies	Nine specified Australian gas/electricity firms			
Market index	ASX 300 Accum			
Risk free rate proxy	Short term Australian Government debt			Consultant determines the specific proxy

Return measurement	Continuous			
Leverage details				
Leverage benchmark	60 per cent			
Leverage calculation	None	Brealey/Myers (β_d=0)		Provide advice on leverage treatment
Reported figures	Equity β (relevered where appropriate)			Disclose gearing figures in an appendix
Analysis details				
Unit of analysis	Individual firms	Portfolio (constant weights)	Portfolio (time varying weights)	Provide advice on the interpretation on these results.
Portfolio construction (constant weight)	Value weighted	Equal weighted		
Blume adjustment	No			
Vasicek adjustment	No			
Confidence intervals	Report at 95%			
Standard errors	Reported			
R-squared	Reported			
Thin trading adjustment	Dimson			Provide advice on the interpretation of these results
Stability and robustness tests	Consultant's choice			Recursive Least Squares and Hansen suggested

1.3. Methodology

The first issue that the consultant was asked to determine was the choice of raw or excess returns. The continuously compounded raw return to asset *i* is defined as

$$r_{i,t} = \ln\left(P_{it} / P_{it-1}\right) \tag{3}$$

Here P_{it}, the price of asset *i*, has been adjusted for the payment of dividends so $r_{i,t}$ represents a measure of total return to the investor. Furthermore, let $r_{Mt} = \ln\left(A_{it} / A_{it-1}\right)$ represent the continuously compounded total return to the market portfolio, where A_{it} is the ASX300 accumulation index as specified by the AER. A conceptually valid estimate for the measure of undiversifiable risk can be obtained from the regression

$$r_{it} = \alpha + \beta r_{Mt} + \varepsilon_{it} \tag{4}$$

Where, the residual is $\varepsilon_{i,t} = r_{i,t} - \alpha_i + \beta_i r_{m,t}$. As the definition of $\beta_i = \dfrac{Cov\left[r_i, r_m\right]}{Var\left[r_m\right]}$, which coincides with the definition of the OLS estimator of β in (4), the use of raw returns is perfectly valid and widespread in the empirical literature.

Assuming that the risk free rate does not vary substantially with time, the data may be transformed to excess returns $R_{i,t} = r_{i,t} - r_{f,t}$; $R_{m,t} = r_{m,t} - r_{f,t}$ and estimates of β_i may be obtained from the regression

$$R_{i,t} = \beta_i R_{m,t} + \varepsilon_{i,t} \tag{4a}$$

Note that there is no need to include an intercept term in (4a). Note that (4a) may be rewritten as

$$r_{it} - r_{ft} = \beta_i\left(r_{m,t} - r_{f,t}\right) + \varepsilon_{i,t} \tag{4b}$$

Subtracting $r_{f,t}$ from both sides of (4b) and rearranging yields (4c):

$$r_{it} = \left(1 - \beta_i\right) r_{ft} + \beta_i\left(r_{m,t}\right) + \varepsilon_{i,t} \tag{4c}$$

Note that 4(c), or equivalently (4a) should yield estimates of β_i that are consistent with those obtained from (4) when the variance of the risk-free rate is low as the intercept term $(1-\beta_i)E(r_{ft}) = \mu$. For this reason, this report follows Henry (2008) and Henry (2009) in employing raw rather than excess returns.

At a practical level this approach overcomes the need to choose a suitable proxy for the risk free rate. Furthermore, the use of raw returns requires no assumptions about potential temporal variations in any chosen risk-free proxy. Of course, choosing a proxy for the risk-free rate can yield variations in the point estimates of the intercept $\hat{\mu}_i$ and slope $\hat{\beta}_i$ of the Security Market Line. For instance, figure 1 considers two outcomes, one using the approach (4) above such that $(1-\beta_i)E(r_{ft}) = \alpha$ and an alternative using a proxy such that $E(r_f^*) < E(r_{ft})$. In figure 1, the point estimate of the intercept calculated using r_f^*, $\hat{\alpha}_i^* < \hat{\alpha}_i$ and so the associated slope $\hat{\beta}_i^* > \hat{\beta}_i$ as both regression lines must pass through the sample means $\overline{R}_i, \overline{R}_M$. The distance $\hat{\beta}_i^*$ - $\hat{\beta}_i$ is only of interest if it is statistically significantly different from zero and the chosen proxy for the risk-free rate is credible. Where $\hat{\beta}_i^*$ lies within the 95% confidence interval for $\hat{\beta}_i$ then there is no evidence against the Hypothesis H_0: $\hat{\beta}_i^* = \hat{\beta}_i$ at the 5% level of confidence.

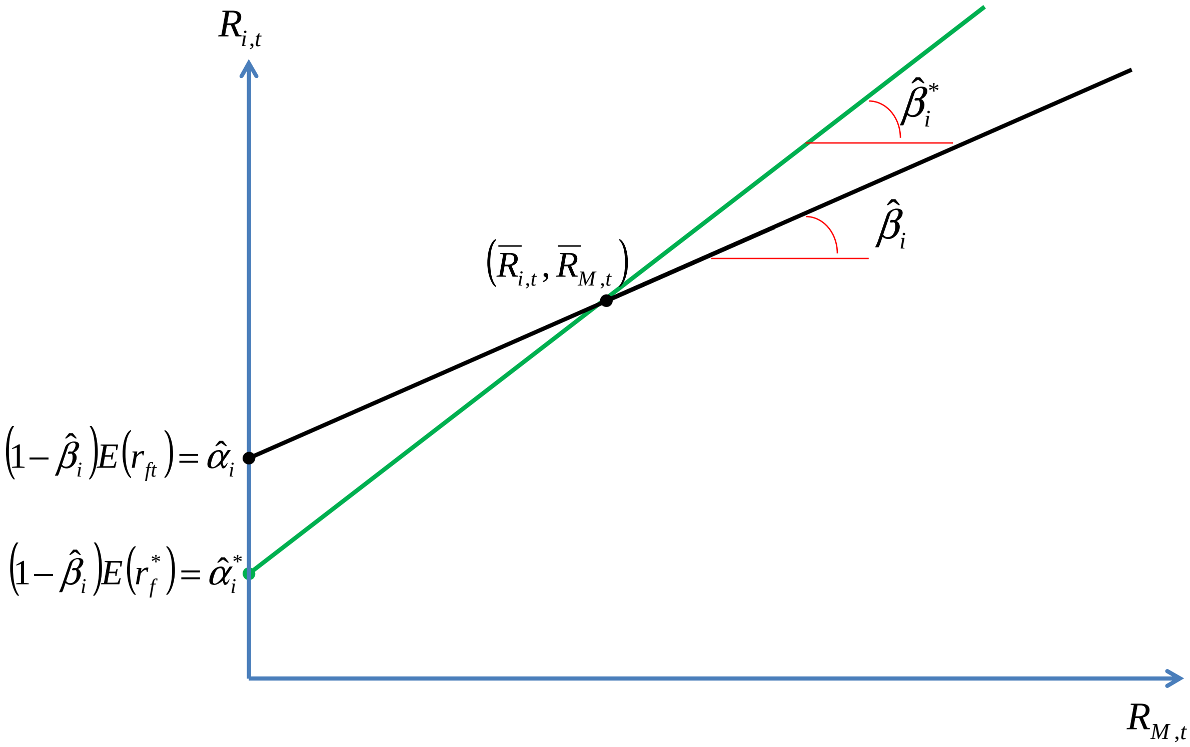

Figure 1: Alternative estimates of β_i.

At a practical level, the approach in this report and Henry (2008, 2009) avoids the need to choose a suitable, credible proxy for the risk free rate. Furthermore, the use of raw returns requires no assumptions about potential temporal variations in any chosen risk-free proxy.

The AER requires construction of ordinary least squares (OLS) estimates of β. This approach obtains estimates of the parameters of interest α_i and β_i, denoted as $\hat{\alpha}_i$ and $\hat{\beta}_i$, by minimising the sum of the squared residuals:

$$\sum_{t=1}^{T} \varepsilon_{i,t}^2 = \sum_{t=1}^{T} \left(r_{i,t} - \hat{r}_{i,t} \right)^2 = \sum_{t=1}^{T} \left(r_{i,t} - \hat{\alpha}_i - \hat{\beta}_i r_{m,t} \right)^2 \tag{5}$$

The AER also requires the construction of estimates of β using the Least Absolute Deviations (LAD) approach. There are some concerns about the validity of the OLS estimator of α_i and β_i in the presence of outliers. In such circumstances the estimates of α_i and β_i may vary with time. It is also possible that estimates of σ_i^2, the variance of the residual, $\varepsilon_{i,t}$, may be affected by the presence of outliers. The LAD estimator operates by minimising the sum of the absolute residuals:

$$\sum_{t=1}^{T}\left|\varepsilon_{i,t}\right| = \sum_{t=1}^{T}\left|r_{i,t} - \tilde{r}_{i,t}\right| = \sum_{t=1}^{T}\left|r_{i,t} - \tilde{\alpha}_i - \tilde{\beta}_i r_{m,t}\right| \tag{6}$$

By minimising the sum of the absolute values of the residuals rather than the sum of the squared residuals, the effect of the LAD estimator is to reduce the influence of outlying observations.

1.4. The Sample

Weekly data was collected from Datastream over the period 29 May 1992 to 28 June 2013, yielding a maximum possible 1102 price observations which can be used to construct a maximum of 1101 continuously compounded weekly returns. Not all of the returns series will have 1101 observations. Datastream provides these weekly price observations using the close on the last trading day within each week, defining the end of the week as Friday. Hence, the normal data is the Friday close price, but if Friday was a public holiday the Thursday close will be used.[3] Hence, in normal circumstances, the weekly return represents the change from Friday of one week to Friday of the next week. This implies that the most recent weekly observation in the data set is therefore for the week ending on Friday 28 June 2013

The sample periods and observation counts for the individual stocks are listed below. Again, the reader should note that the dates are to be read as the week ending on the stated Friday:

1. Alinta (AAN), available from 20/10/2000 to 17/08/2007: 357 observations
2. AGL Energy Limited (AGL), available from 29/05/1992 to 06/10/2006: 751 observations
3. APA Group (APA), available from 16/06/2000 to 28/06/2013: 681 observations
4. DUET Group (DUE), available from 13/08/2004 to 28/06/2013: 464 observations
5. Envestra Limited (ENV), available from 29/08/1997 to 28/06/2013: 828 observations

[3] Similarly, in the event that both Thursday and Friday were public holidays, the Wednesday close would be used.

6. GasNet (GAS), available from 21/12/2001 to 10/11/2006: 256 observations

7. Hastings Diversified Utilities Fund (HDF), available 17/12/2004 to 23/11/2012: 415 observations

8. Spark Infrastructure (SKI), available from 02/03/2007 to 28/06/2013[4]: 331 observations

9. SP Ausnet (SPN), available from 16/12/2005 to 28/06/2013: 394 observations

In addition to the analysis using a weekly sampling frequency, some regressions used a monthly sampling frequency as a robustness check. These monthly returns are calculated each month using the last closing price of the month.

The consultant was instructed by the AER to undertake the core set of regressions using two permutations of the regression calculation:

1. Ordinary Least Squares (OLS)

2. Least Absolute Deviation (LAD, sometimes referred to as Least Absolute Variation)

The use of LAD in addition to the (standard) OLS was intended to provide a robustness check on the underlying data with regard to data outliers. The consultant was not requested to provide expert advice or analysis on this design decision. The consultant also carried out recursive estimation of β using fixed and expanding windows. The results are presented graphically in a series of Appendices.

4 Note that the date range for SKI listed here excludes the period prior to 2/3/2007 where these stapled securities traded as instalment receipts (prior to the payment of the final instalment). The approach followed here is to deal with estimation of β for equities so the period prior to 2/3/2007 is excluded for consistency.

1.5. The Sample Period

The consultant was requested to undertake the core set of regressions using three permutations of the estimation period:

1. The longest period available

2. The period after the tech boom (also called the dot-com bubble) and before the GFC, then the period after the GFC

3. The last five years of available data

In so doing, the consultant was requested to provide advice on which of these estimation periods is preferable, including whether market conditions across each period provide a reasonable basis for generating an equity β estimate that is relevant to the AER's return on capital framework.

The AER considers that there is an established consensus on the start and end dates for the tech boom that affected Australian share prices (and therefore equity β estimation) from 1 July 1998 to 31 December 2001. Note that for weeks defined by their end Friday, the first week in the tech boom is the week ending on Friday 3 July 1998, and the last week is that ending on Friday 28 December 2001.

However, the AER acknowledges that there is no established consensus on the start and end dates for the GFC. The consultant was instructed to adopt the GFC start and end dates they consider most appropriate, and provide brief reasoning to support their position.

Consistent with the work presented to the AER in Henry (2008) and Henry (2009), the starting period of the GFC will be 1 September 2008. The consultant recognises that this choice is arbitrary, but does facilitate comparison between the results presented in Henry (2008) and Henry (2009) and the current report. This choice of date also coincides with an Organisation for Economic Cooperation and Development forecast on 2 September 2008 that the UK would enter recession, and the European Central Bank announcement of a cut in its growth forecast on 3 September 2008. On 7 September 2008 the US government announced the rescue of Fannie Mae and Freddie Mac. Finally, the collapse of Lehmann Brothers, at the time the largest bankruptcy in US history, occurred on 15 September 2008. This suggests that 1 September 2008 is not an unreasonable date to choose for the onset of the GFC. Therefore, we take the last week before the GFC as the week ending on Friday 29 August 2008, and the first week during the GFC is the week ending on Friday 5 September 2008.

Choosing an end date for the GFC is extremely difficult. This difficulty is exacerbated by the onset of the Euro Area crisis in 2009. Nevertheless, it is clear that the impact of the GFC on Australia was weakening by October 2009. On 3 June 2009 an unexpected announcement of a quarterly growth rate of GDP of 0.4% was taken to signal the fact that Australia had avoided recession. The Australian unemployment rate hit a six year peak at 5.8% in July 2009. By September 2009, the Australian dollar was at a 12-month high. On 3 October 2009, the Reserve Bank of Australia raised interest rates from 3% to 3.25%. Finally, the announcement on 29 October 2009 that between July and September 2009 the US economy had achieved an annual rate of growth of 3.5% suggests that the worst of the GFC had passed. Consequently, we will adopt the convention that the end date of the GFC, for the purposes of this report alone, was the end of October 2009. For the data sampled at a weekly frequency, the end of the GFC will be taken to be the week ending on Friday 30 October 2009. Consequently, the first week of the post-GFC sample is taken to be the week ending on Friday 6 November 2009.

The final date for the inclusion of recent data has been selected as the end June 2013 (as a recent date that is also the end of the financial year). The last weekly observation is for the week ending on Friday 28 June 2013.

1.6. Delevered/Relevered β

Let β_A and β_E represent the true asset and equity β, respectively. Assuming a debt β of zero, the delevering/relevering equation is

$$\beta_A = \beta_E \frac{E}{V} \tag{7}$$

Here E/V is the proportion of equity in the firm's capital structure. The average gearing level is calculated for the sample period used obtain estimates of the firm or portfolio β using data obtained from Bloomberg and provided to the consultant by the AER. The level of gearing is usually defined as the book value of debt divided by the value of the firm as represented by the sum of the market value of equity and the book value of debt. Define the average level of gearing as $\bar{G}$, then

$$\bar{G} = \frac{\bar{D}}{\bar{D} + \bar{E}} \tag{8}$$

Where D is the book value of net debt and E is the market value of equity. It is possible to show that the appropriate relevering factor that should be applied to the raw β estimates is:

$$\omega = \frac{1 - \bar{G}}{1 - 0.60} \tag{9}$$

If it is assumed that ω is constant and that the $\bar{G}$ is independent of $\hat{\beta}$ then, the relevered β, $\hat{\beta}_r$ has a mean of $\omega\hat{\beta}$ and a variance of $\omega^2\sigma^2_{\hat{\beta}}$. The results of the delevering/relevering process for individual stocks and portfolios are reported in the tables that follow, which also identify the $\bar{G}$ and ω used in each case.

1.7. Thin trading

Thin trading can create issues with the magnitude of the estimate of β. In effect, if the stock does not trade regularly, the OLS estimate of β tends to be biased towards zero. In the literature, there are two popular approaches to adjusting for thin trading. The Scholes-Williams[5] approach and the Dimson[6] approach. The consultant was instructed to calculate the Dimson adjustment by the AER, which involves estimation of the regression

$$r_{i,t} = \alpha_i + \beta_{i-1} r_{m,t-1} + \beta_i r_{m,t} + \beta_{i+1} r_{m,t+1} + \varepsilon_{i,t} , \tag{9}$$

Henry (2009) discusses the relative merits of the Scholes-Williams approach, which is based on the estimation of misspecified auxiliary regressions. The Dimson estimate of β, β_i^D is obtained from sum of the coefficients of the independent variables in equation (9).

This report also presents t-statistics used to test the hypothesis $H_0 : \beta_i^{OLS} = \beta_i^D$. These statistics are constructed as

5 Scholes, M. and J Williams (1977) "Estimating betas from nonsynchronous data" *Journal of Financial Economics*, 5, 309-327

6 Dimson, E. and P. Marsh (1983) "The stability of UK risk measures and the problem in thin trading", *Journal of Finance*, 38 (3) 753-784

$$t = \frac{\hat{\beta}_i - \beta_i^D}{s.e.\left(\hat{\beta}_i\right)} \qquad (10)$$

The statistics are constructed in this fashion to allow the use of the smaller OLS standard errors in the construction of the t-statistic. Given the absence of evidence of thin trading, the Dimson estimator is inefficient relative to OLS and so $s.e.\left(\hat{\beta}_i\right) < s.e.\left(\beta_i^D\right)$. This approach gives the greatest chance of rejecting $H_0 : \hat{\beta}_i = \beta_i^D$. An alternative approach would be to calculate the t-statistic as:

$$t = \frac{\beta_i^D - \hat{\beta}_i}{s.e.\left(\beta_i^D\right)}$$

In this situation, any t-statistic constructed will be opposite in sign and smaller in magnitude if $s.e.\left(\hat{\beta}_i\right) < s.e.\left(\beta_i^D\right)$. The approach followed maximises the chance of finding evidence against $H_0 : \hat{\beta}_i = \beta_i^D$.

1.8. Stability and sensitivity analysis

The consultant was asked to provide advice on the appropriate statistical tests (or other forms of analysis) to ascertain the stability and sensitivity of the empirical estimates presented in the report.

In Henry (2008) and Henry (2009), two approaches were implemented that specifically assess the structural stability of the regressions: recursive least square estimates and Hansen's test for parameter stability.

Recursive estimates of the parameters of interest may be obtained by allowing the sample to vary in a controlled fashion. There are two main approaches to recursive least squares. The first approach employs an expanding window of observations, while the second employs a fixed window that is rolled across the sample.

In the case of an expanding window, the first τ observations are used to form the initial estimate of α_i and β_i. An additional observation is then added to the estimation window and the resulting τ+1 observations are used to compute the second estimate of the coefficient vector. This process is repeated until all the observations in the sample have been employed yielding T-τ+1 estimates of α_i and β_i. These estimates and their associated standard errors may be plotted to detect evidence of time variation in the coefficient vector. Since the sample size is increasing from τ to T, the standard error bands will generally tighten as the sample size increases.

The moving window estimator employs τ observations from the sample of T observations. The initial estimates of α_i and β_i are obtained using the observations 1,2,3,… τ. Subsequent estimates are obtained using observations 2,3,… $\tau+1$ etcetera up until the final estimates obtained from observations T-τ to T. Again, these estimates and their associated standard errors may be plotted to detect evidence of time variation in the coefficient vector. Since the standard errors are calculated using τ observations the resulting standard error bands will generally be wider than those based on the full set of T observations.

Since the recursive estimates are only visual guides to the stability of the estimates, we also report Hansen's (1992) test for parameter stability.[7] This test examines the regression model (4) for evidence of instability in the residual variance, σ_i^2, the intercept, α_i, the slope coefficient β_i, and then a joint test for instability in all three measures. The null hypothesis of the Hansen (1992) test is that there is no instability in the parameter of interest, while the alternative is that there is instability in the parameter of interest. A joint test of the null hypothesis of no instability in α_i, β_i and σ_i^2 can be interpreted as a test for parameter stability in the model (3). Rejection of the joint null hypothesis indicates that the model suffers from parameter instability.

$$
\begin{aligned}
&H_0 : \text{The parameter (model) of interest is stable} \\
&H_1 : \text{The parameter (model) of interest is not stable}
\end{aligned}
$$

The test has a nonstandard asymptotic distribution which depends upon the number of coefficients being tested for stability. The decision rule is straightforward; in the absence of a significant test statistic, the investigator may be reasonably confident that either the model has not displayed parameter instability over the sample or that the data is not sufficiently informative to reject this hypothesis. In the presence of a significant test statistic, the

[7] Hansen, B.E. (1992) "Parameter Instability in Linear Models", Journal of Policy Modeling, 14 (4), 1992, pp. 517-533.

investigator may confidently conclude that the model is misspecified and prone to parameter instability.

One of the sample periods included in the core set of regression permutations is non-contiguous, encompassing data before and after the GFC. That is, the data vector omits observations from 1 September 2008 to 30 October 2009. Given the non-contiguous sample, recursive estimation and the Hansen tests are unlikely to be useful. For instance, a rejection of the null that the parameter or model of interest is stable may occur because the model is unstable, or because of the imposed break in the data from 1 September 2008 to 30 October 2009, or both.

2. Individual firm analysis

2.1. Estimation results: Individual firms

Tables 2–7 present estimates of equity β for individual firms. The primary analysis is presented in Tables 2–4 using a weekly sampling frequency. These three tables present different sample periods (the longest sample available for each firm; the period after the tech boom but excluding the GFC; and the most recent sample where data older than five years is excluded). This analysis uses both OLS and LAD regression calculations.

As a robustness check on the use of a weekly sampling frequency, Tables 5–7 use a monthly sampling frequency instead. The same three sample periods are presented (longest available, post tech boom but excluding GFC, last five years). This analysis is restricted to the OLS regression calculation.

Table 8 is a summary table of the individual firm analysis presented in Tables 2–7.

Table 2 presents results for the longest available sample period for each stock. The analysis ends on 28 June 2013, when the data was downloaded for analysis. These regressions use a weekly sampling period, both OLS and LAD regression calculations, delever/relever each firm to the benchmark gearing (60 per cent), and include 95% confidence intervals around the estimates.

Table 2: Delevered/relevered estimates of β
Longest sample available – Weekly frequency

	AAN	AGL	APA	DUE	ENV	GAS	HDF	SKI	SPN
Start	20/10/00	29/05/92	16/06/00	13/08/04	29/08/97	21/12/01	17/12/04	02/03/07	16/12/05
End	17/08/07	06/10/06	28/06/13	28/06/13	28/06/13	10/11/06	23/11/12	28/06/13	28/06/13
$\bar{G}$	0.3954	0.2967	0.5618	0.7651	0.7193	0.6662	0.4834	0.6604	0.6054
ω	1.5116	1.7582	1.0955	0.5872	0.7017	0.8345	1.2915	0.8491	0.9864
$\hat{\beta}$	0.8795	0.6812	0.5936	0.2826	0.3039	0.3139	1.0305	0.3285	0.2874
s.e	0.1597	0.1105	0.0561	0.0410	0.0363	0.0846	0.1553	0.0617	0.0623
$\hat{\beta}_u$	1.1925	0.8978	0.7036	0.3630	0.3750	0.4797	1.3349	0.4494	0.4095
$\hat{\beta}_l$	0.5664	0.4646	0.4835	0.2022	0.2328	0.1480	0.7260	0.2076	0.1653
$\tilde{\beta}$	0.5999	0.7193	0.5902	0.2138	0.2846	0.2252	0.7066	0.3195	0.2846
s.e	0.1608	0.1106	0.0561	0.0412	0.0363	0.0852	0.1561	0.0617	0.0623
$\tilde{\beta}_u$	0.9150	0.9361	0.7003	0.2946	0.3557	0.3922	1.0126	0.4404	0.4067
$\tilde{\beta}_l$	0.2848	0.5024	0.4802	0.1331	0.2134	0.0583	0.4006	0.1986	0.1625
N	356	749	680	463	826	255	414	330	393
R^2	0.0789	0.0484	0.1416	0.0933	0.0785	0.0516	0.0965	0.0796	0.0516

The average of the delevered/relevered OLS estimates, E($\hat{\beta}$), was 0.5223 with a maximum estimate of 1.0305 for HDF, and a minimum of 0.2826 for DUE. The median OLS estimate, M($\hat{\beta}$), was 0.3285. The evidence from Table 2 suggests that the majority of the OLS point estimates lie in the range 0.2826 to 1.0305.

The mean LAD estimate, E($\tilde{\beta}$) was 0.4382, with a maximum estimate of 0.7193 for AGL and a minimum of 0.2138 for DUE. The median LAD estimate, M($\tilde{\beta}$), was 0.3195. The evidence from Table 2 suggests that the majority of the LAD point estimates lie in the range 0.2138 to 0.7193.

With the exception of HDF, all the LAD estimates lie within the 95% OLS confidence interval for the relevant firm. This suggests that, in the main, the information provided by the LAD estimates is consistent with the evidence provided by the OLS estimates.

Taken together, the evidence from Table 2 suggests that the point estimates of equity beta lie in the range 0.21 to 1.04. The evidence in Table 2 is, broadly speaking, consistent with the outcomes reported in Tables 4.2 and 4.4 of Henry (2009).

Table 3 presents results for the period after the technology boom (also called the dot com bubble) but excluding the GFC. More specifically, the sample period starts using the week ending 4 January 2002 (following the instructions of the AER that the tech boom period is 3 July 1998 to 28 December 2001). The sample excludes the period 5 September 2008 to 30 October 2009 (both defined as weeks ending on each date), which in the consultant's view broadly coincides with the GFC. The sample period ends on 28 June 2013, in keeping with the earlier analysis. All available data for each stock is included within these overall period boundaries. These regressions use a weekly sampling period, both OLS and LAD regression calculations, delever/relever each firm to the benchmark gearing (60 per cent), and include 95% confidence intervals around the estimates.

	AAN	AGL	APA	DUE	ENV	GAS	HDF	SKI	SPN
Start	04/01/02	04/01/02	04/01/02	13/08/04	04/01/02	04/01/02	17/12/04	02/03/07	16/12/05
End	17/08/07	06/10/06	29/08/08	29/08/08	29/08/08	10/11/06	29/08/08	29/08/08	29/08/08
Start			06/11/09	06/11/09	06/11/09		06/11/09	06/11/09	06/11/09
End			28/06/13	28/06/13	28/06/13		23/11/12	28/06/13	28/06/13
$\bar{G}$	0.3954	0.2959	0.5513	0.7597	0.6947	0.6662	0.4751	0.6489	0.5920
ω	1.5116	1.7603	1.1218	0.6007	0.7632	0.8345	1.3123	0.8777	1.0200
$\hat{\beta}$	0.9956	0.7494	0.6354	0.2988	0.3656	0.3169	0.9046	0.3399	0.4684
s.e	0.1844	0.1686	0.0705	0.0422	0.0443	0.0850	0.1105	0.0772	0.0750
$\hat{\beta}_u$	1.3570	1.0799	0.7735	0.3815	0.4525	0.4836	1.1212	0.4912	0.6154
$\hat{\beta}_l$	0.6343	0.4189	0.4972	0.2161	0.2788	0.1502	0.6880	0.1886	0.3214
$\tilde{\beta}$	0.6136	0.5836	0.5340	0.2571	0.3354	0.2252	0.7236	0.4094	0.4664
s.e	0.1860	0.1692	0.0707	0.0423	0.0443	0.0856	0.1110	0.0773	0.0750
$\tilde{\beta}_u$	0.9782	0.9153	0.6725	0.3400	0.4224	0.3931	0.9412	0.5610	0.6134
$\tilde{\beta}_l$	0.2490	0.2520	0.3955	0.1743	0.2485	0.0574	0.5059	0.2578	0.3194
N	293	248	537	401	537	253	352	268	331
R^2	0.0911	0.0743	0.1319	0.1117	0.1128	0.0524	0.1607	0.0679	0.1060

Table 3: Delevered/relevered estimates of β

Sample from 2002 to present, excluding GFC – Weekly frequency

The average of the OLS estimates, E($\hat{\beta}$), was 0.5639 with a maximum estimate of 0.9956 for AAN, and a minimum of 0.2988 for DUE. The median OLS estimate, M($\hat{\beta}$), was 0.4684. The evidence from Table 3 suggests that the majority of the OLS point estimates lie in the range 0.2988 to 0.9956.

The mean LAD estimate, E($\tilde{\beta}$) was 0.4609, with a maximum estimate of 0.7236 for HDF and a minimum of 0.2252 for GAS. The median LAD estimate, M($\tilde{\beta}$), was 0.4664. The evidence from Table 3 suggests that the majority of the LAD point estimates lie in the range 0.2252 to 0.7236.

With the exception of AAN, all the LAD estimates lie within the 95% OLS confidence interval for the relevant firm. This suggests that the information provided by the LAD estimates is broadly consistent with the evidence provided by the OLS estimates.

Taken together, the evidence from Table 3 suggests that the point estimates of the equity β lie in the range 0.22 to 1.0. The evidence from Table 3 is, broadly speaking, consistent with the evidence presented in Table 2, both in terms of the average OLS estimate of the equity β and the range of estimates. The evidence in Table 3 is also, broadly speaking, consistent with the outcomes reported in Tables 4.2 and 4.4 of Henry (2009).

Table 4 presents results using only data from the last five years, from the week ending 4 July 2008 to the week ending 28 June 2013, for those firms which traded in this period. These regressions use a weekly sampling period, both OLS and LAD regression calculations, delever/relever each firm to the benchmark gearing (60 per cent), and include 95% confidence intervals around the estimates.

Table 4: Delevered/relevered estimates of β

Last five years sample – Weekly frequency

	AAN	AGL	APA	DUE	ENV	GAS	HDF	SKI	SPN
Start	*NA*	*NA*	04/07/08	04/07/08	04/07/08	*NA*	04/07/08	04/07/08	04/07/08
End			28/06/13	28/06/13	28/06/13		23/11/12	28/06/13	28/06/13
$\bar{G}$			0.6019	0.7725	0.7109		0.5057	0.6724	0.6289
ω			0.9952	0.5688	0.7229		1.2358	0.8190	0.9279
$\hat{\beta}$			0.5401	0.2443	0.3772		1.0192	0.2987	0.2727
s.e			0.0667	0.0519	0.0649		0.2184	0.0620	0.0698
$\hat{\beta}_u$			0.6708	0.3460	0.5044		1.4472	0.4202	0.4096
$\hat{\beta}_l$			0.4094	0.1425	0.2500		0.5912	0.1771	0.1359
$\tilde{\beta}$			0.5389	0.2169	0.3942		0.7771	0.2152	0.4056
s.e			0.0667	0.0520	0.0651		0.2190	0.0623	0.0705
$\tilde{\beta}_u$			0.6696	0.3189	0.5217		1.2062	0.3373	0.5437
$\tilde{\beta}_l$			0.4082	0.1149	0.2667		0.3479	0.0930	0.2675
N			260	260	260		229	260	260
R^2			0.2028	0.0791	0.1158		0.0876	0.0825	0.0559

In Table 4, the average of the reported OLS estimates, E($\hat{\beta}$), was 0.4587, with a maximum estimate of 1.0192 for HDF, and a minimum of 0.2443 for DUE. The median OLS estimate, M($\hat{\beta}$), was 0.3379. The evidence from Table 4 suggests that the majority of the OLS point estimates lie in the range 0.2433 to 1.0192.

The mean LAD estimate, E($\tilde{\beta}$) was 0.4246, with a maximum estimate of 0.7771 for HDF and a minimum of 0.2152 for SKI. The median LAD estimate, M($\tilde{\beta}$), was 0.3999. The evidence from Table 4 suggests that the majority of the LAD point estimates lie in the range 0.2152 to 0.7771.

All the LAD estimates reported in Table 4 lie within the 95% OLS confidence interval for the relevant firm. This suggests that the information provided by the LAD estimates is consistent with the evidence provided by the OLS estimates.

Taken together, the evidence from Table 4 suggests that the point estimates of the equity β lie in the range 0.21 to 1.02. The evidence from Table 4 is, broadly speaking, consistent with the evidence presented in Table 2 and Table 3 above. Furthermore, the evidence in Table 4 is, broadly speaking, consistent with the outcomes reported in Table 4.4 of Henry (2009).

Tables 5–7 present results using data sampled at the monthly frequency, rather than the weekly sampling frequency employed in Tables 2–4. Since these regressions function as a robustness check for the primary analysis (which used data sampled at the weekly frequency), only OLS estimates are reported. There was no systematic evidence of a statistically significant difference between the OLS and LAD estimates reported in Tables 2 to 4.

Table 5 presents results using the longest available sample period for each stock. It differs from Table 2 in that the results were generated using data sampled at a monthly sampling frequency and does not present LAD regression calculations in the interests of brevity.

Table 5: Delevered/relevered estimates of β									
Longest available sample – Monthly frequency									
	AAN	AGL	APA	DUE	ENV	GAS	HDF	SKI	SPN
Start	20/10/00	29/05/92	16/06/00	13/08/04	29/08/97	21/12/01	17/12/04	02/03/07	16/12/05
End	17/08/07	06/10/06	28/06/13	28/06/13	28/06/13	10/11/06	23/11/12	28/06/13	28/06/13
$\bar{G}$	0.3954	0.2967	0.5618	0.7651	0.7193	0.6662	0.4834	0.6604	0.6054
ω	1.5116	1.7582	1.0955	0.5872	0.7017	0.8345	1.2915	0.8491	0.9864
$\hat{\beta}$	0.7575	0.8356	0.6806	0.3334	0.3016	0.1684	0.2940	0.2508	0.3270
s.e	0.3243	0.1953	0.1136	0.0781	0.0834	0.1706	0.3677	0.1327	0.1160
$\hat{\beta}_u$	1.3930	1.2184	0.9033	0.4864	0.4651	0.5028	1.0146	0.5108	0.5543
$\hat{\beta}_l$	0.1219	0.4528	0.4580	0.1804	0.1382	-0.1659	-0.4266	-0.0093	0.0997
N	81	172	156	106	190	58	94	75	90
R^2	0.0646	0.0972	0.1890	0.1492	0.0651	0.0171	0.0069	0.0466	0.0829

The average of the OLS estimates, E($\hat{\beta}$), was 0.4388 with a maximum estimate of 0.8356 for AGL, and a minimum of 0.1684 for GAS. The median OLS estimate, M($\hat{\beta}$), was 0.3270. The evidence from Table 5 suggests that the majority of the OLS point estimates lie in the range 0.16 to 0.84. The average of the estimates of equity β reported in Table 2 was 0.5223, which exceeds the average estimate of 0.4388 reported for Table 5.

In summary, the evidence from Table 5 suggests that the point estimates of equity lie in the range 0.16 to 0.84. In that the individual estimates and their averages are greater than zero and less than unity, the evidence from Table 5 is, broadly speaking, consistent with the evidence presented in Table 2 (which presents the same analysis but at a weekly frequency). Furthermore, the evidence in Table 5 is, broadly speaking, consistent with the outcomes reported in Tables 4.1 and 4.3 of Henry (2009).

Table 6 presents results for the period after the technology boom but excluding the GFC. It differs from Table 3 in that it uses monthly sampling and does not present LAD regression calculations.

Table 6: Delevered/relevered estimates of β

Sample from 2002 to present, but excludes GFC – Monthly frequency

	AAN	AGL	APA	DUE	ENV	GAS	HDF	SKI	SPN
Start	04/01/02	04/01/02	04/01/02	13/08/04	04/01/02	04/01/02	17/12/04	02/03/07	16/12/05
End	17/08/07	06/10/06	29/08/08	29/08/08	29/08/08	10/11/06	29/08/08	29/08/08	29/08/08
Start			06/11/09	06/11/09	06/11/09		06/11/09	06/11/09	06/11/09
End			28/06/13	28/06/13	28/06/13		23/11/12	28/06/13	28/06/13
$\bar{G}$	0.3954	0.2959	0.5513	0.7597	0.6947	0.6662	0.4751	0.6489	0.5920
ω	1.5116	1.7603	1.1218	0.6007	0.7632	0.8345	1.3123	0.8777	1.0200
$\hat{\beta}$	0.7707	0.8573	0.7012	0.2944	0.2621	0.1668	0.5734	0.1251	0.5273
s.e	0.3963	0.3004	0.1551	0.0779	0.0871	0.1714	0.2350	0.1688	0.1341
$\hat{\beta}_u$	1.5475	1.4461	1.0052	0.4472	0.4327	0.5026	1.0341	0.4559	0.7901
$\hat{\beta}_l$	-0.0061	0.2685	0.3973	0.1416	0.0914	-0.1691	0.1127	-0.2057	0.2645
N	66	56	122	91	122	57	79	60	75
R^2	0.0558	0.1310	0.1456	0.1381	0.0702	0.0169	0.0717	0.0094	0.1748

The average of the OLS estimates, E($\hat{\beta}$), was 0.4754 with a maximum estimate of 0.8573 for AGL, and a statistically insignificant minimum estimate of 0.1251 for SKI. The median OLS estimate, M($\hat{\beta}$), was 0.5273. The evidence from Table 6 suggests that, ignoring insignificant estimates, the majority of the OLS point estimates lie in the range 0.26 to 0.86.

The evidence from Table 6 is, broadly speaking, consistent with the evidence presented in Table 5 and also Table 3 (which presents a similar analysis based on data sampled at the weekly frequency). Furthermore, the evidence in Table 6 is, broadly speaking, consistent with the outcomes reported in Tables 4.1 and 4.3 of Henry (2009). The relatively small sample size, and/or the non-contiguous nature of the sample underlying the results in Table 6 may explain the relatively poor explanatory power of the regressions and also lead to point estimates that are smaller in magnitude than those reported in Table 3.

The non-contiguous nature of the sample, coupled with the imprecision of the estimates reflected by relatively wide confidence intervals, some of which contain zero, suggests that the estimates presented in Table 6 are treated with a degree of caution.

Table 7 presents results using only data from the last five years, from the week ending 4 July 2008 to the week ending 28 June 2013, for those firms which traded in this period. It differs from Table 4 in that it uses monthly sampling and does not present LAD regression calculations.

Table 7: Delevered/relevered estimates of β

Last five years sample (July 2008 to June 2013) – Monthly frequency

	AAN	AGL	APA	DUE	ENV	GAS	HDF	SKI	SPN
Start	*NA*	*NA*	04/07/08	04/07/08	04/07/08	*NA*	04/07/08	04/07/08	04/07/08
End			28/06/13	28/06/13	28/06/13		23/11/12	28/06/13	28/06/13
$\bar{G}$			0.6019	0.7725	0.7109		0.5057	0.6724	0.6289
ω			0.9952	0.5688	0.7229		1.2358	0.8190	0.9279
$\hat{\beta}$			0.7715	0.3180	0.4282		0.1091	0.2072	0.3606
s.e			0.1254	0.1022	0.1850		0.5508	0.1394	0.1341
$\hat{\beta}_u$			1.0173	0.5184	0.7908		1.1887	0.4803	0.6234
$\hat{\beta}_l$			0.5257	0.1177	0.0656		-0.9706	-0.0660	0.0978
N			59	59	59		51	59	59
R^2			0.3990	0.1451	0.0859		0.0008	0.0373	0.1126

The average of the OLS estimates reported in Table 7, E($\hat{\beta}$), was 0.3658, with a maximum estimate of 0.7715 for APA, and a minimum of 0.1091 for HDF, an estimate that is statistically insignificantly different to zero. The median OLS estimate, M($\hat{\beta}$), was 0.3393. The evidence from Table 7 suggests that the majority of the significant OLS point estimates lie in the range 0.3180 to 0.7715.

The evidence from Table 7 is, broadly speaking, consistent with the evidence presented in Table 5 and Table 7. That is, the estimates are consistent with the view that the magnitude of the equity β was greater than zero and less than unity, although the confidence interval for APA includes unity. It is very difficult to determine a narrower plausible range of values given the imprecision of the individual estimates in the tables. For example, the 95% confidence interval reported for HDF and Ski contain zero. Similarly, the evidence from Table 7 is, broadly speaking, consistent with the evidence presented in Table 4 (which presents the same analysis but at a weekly frequency). Moreover, the evidence in Table 7 is, broadly speaking, consistent with the outcomes reported in Table 4.3 of Henry (2009). The limited sample size of a maximum of 60 monthly observations, coupled with the imprecision of the estimates reflected by relatively wide confidence intervals, some of which contain zero, suggest that the estimates presented in Table 7 are treated very cautiously.

The upper panel of Table 8 presents the average and median equity β estimates from Tables 2–7, while the lower panel reports the maximum and minimum point estimates. This includes analysis across three different sample periods, and three different permutations of the sample frequency and calculation types.

Table 8: Delevered/relevered estimates of β

Summary of individual firm equity β point estimates from tables 2–7

Regression details		Longest possible		Post tech, ex GFC		Last five years	
		Ave	Med	Ave	Med	Ave	Med
Weekly	OLS	0.5223	0.3285	0.5639	0.4684	0.4587	0.3379
Weekly	LAD	0.4382	0.3195	0.4609	0.4664	0.4246	0.3999
Monthly	OLS	0.4388	0.3270	0.4754	0.5273	0.3658	0.3393
Regression Details		Longest Possible		Post tech, ex GFC		Last five years	
		Min	Max	Min	Max	Min	Max
Weekly	OLS	0.2826	1.0305	0.2988	0.9956	0.2443	1.0192
Weekly	LAD	0.2138	0.7193	0.2252	0.7236	0.2152	0.7771
Monthly	OLS	0.1684	0.8356	0.1251	0.8573	0.1091	0.7715

Some caution should be exercised in interpreting these maximum, minimum, average and median values as in some cases the values include implausible point estimates which are statistically insignificantly different to zero.

Taken together, the evidence presented in Table 8 (which summarises the evidence from Tables 2–7) suggests that the point estimates of equity β lie, broadly speaking, in the range 0.2 to 0.8. Within this range, the average point estimates tend to cluster about 0.5, while the median point estimates are clustered about 0.4. .

2.2. Thin Trading analysis: Individual firms

Table 9 presents Dimson estimates of β for the longest available sample using a weekly sampling frequency. The standard OLS estimates underlying this analysis were presented in Table 2.

Table 9 Dimson's β – Firms

Longest available sample – Weekly frequency

	AAN	AGL	APA	DUE	ENV	GAS	HDF	SKI	SPN
β_{i-1}	-0.105	-0.040	-0.004	0.137	0.058	-0.102	0.176	0.081	0.067
s.e	0.108	0.063	0.051	0.069	0.052	0.102	0.121	0.073	0.063
β_i	0.591	0.387	0.539	0.489	0.434	0.390	0.806	0.389	0.286
s.e	0.108	0.063	0.051	0.069	0.052	0.102	0.121	0.073	0.063
β_{i+1}	0.107	0.006	-0.079	-0.067	0.032	-0.064	-0.038	0.012	-0.099
s.e	0.106	0.063	0.051	0.069	0.052	0.102	0.121	0.073	0.063
β_i^D	0.592	0.353	0.455	0.559	0.525	0.224	0.944	0.482	0.253
s.e	0.181	0.110	0.091	0.123	0.091	0.168	0.216	0.130	0.113
β_i^{OLS}	0.582	0.387	0.542	0.481	0.433	0.376	0.798	0.387	0.291
s.e	0.106	0.063	0.051	0.070	0.052	0.101	0.120	0.073	0.063
$\beta_i^{OLS} = \beta_i^D$	-0.097	0.554	1.694	-1.108	-1.769	1.498	-1.212	-1.308	0.609

Of the β_{i-1} and β_{i+1} coefficients in Table 9, only β_{i-1} for DUE is marginally significant. The results in Table 9 are consistent with the view that there is a paucity of evidence of thin trading in the sample of firms considered in this report. Moreover, none of the t-statistics used to test the hypothesis $H_0 : \hat{\beta}_i = \beta_i^D$ reported in Table 9 are statistically significant. This indicates that there is an absence of evidence of thin trading in the first sample period.

Table 10 presents Dimson estimates of β for the sample commencing after the tech bubble and excluding the GFC. The standard OLS estimates underlying this analysis were presented in Table 3.

Table 10 Dimson's β – Firms

Sample from 2002 to present, but excludes GFC – Weekly frequency

	AAN	AGL	APA	DUE	ENV	GAS	HDF	SKI	SPN
β_{i-1}	-0.173	-0.030	-0.079	0.002	0.005	-0.100	-0.044	-0.072	-0.021
s.e	0.126	0.096	0.063	0.070	0.058	0.103	0.086	0.089	0.074
β_i	0.682	0.431	0.568	0.507	0.475	0.390	0.688	0.385	0.447
s.e	0.125	0.096	0.063	0.070	0.058	0.103	0.085	0.088	0.074
β_{i+1}	-0.005	-0.058	0.005	0.055	-0.007	-0.065	-0.013	0.101	-0.106
s.e	0.123	0.096	0.063	0.069	0.058	0.103	0.084	0.087	0.073
β_i^D	0.504	0.343	0.494	0.564	0.474	0.225	0.631	0.414	0.321
s.e	0.204	0.159	0.109	0.121	0.100	0.168	0.150	0.155	0.129
β_i^{OLS}	0.659	0.426	0.566	0.498	0.479	0.380	0.689	0.387	0.459
s.e	0.122	0.096	0.063	0.070	0.058	0.102	0.084	0.088	0.074
$\beta_i^{OLS} = \beta_i^D$	1.269	0.866	1.147	-0.939	0.091	1.519	0.691	-0.303	1.886

None of the β_{i-1} and β_{i+1} coefficients in Table 10 are significant which is consistent with the view that there is a paucity of evidence of thin trading in the sample of firms considered in this report. Moreover, none of the t-statistics used to test the hypothesis $H_0 : \hat{\beta}_i = \beta_i^D$ reported in Table 10 are statistically significant. This indicates that there is an absence of evidence of thin trading in the second sample period.

Table 11 presents Dimson estimates of β for the sample using the last five years of available data. The delivered/relevered OLS estimates underlying this analysis were presented in Table 4.

Table 11 Dimson's β – Firms

Last five years sample (July 2008 to June 2013) – Weekly frequency

	AAN	AGL	APA	DUE	ENV	GAS	HDF	SKI	SPN
β_{i-1}	*NA*	*NA*	0.017	0.182	0.086	*NA*	0.201	0.111	0.093
s.e			0.067	0.090	0.090		0.178	0.076	0.075
β_i			0.538	0.441	0.535		0.834	0.370	0.294
s.e			0.068	0.091	0.091		0.179	0.076	0.076
β_{i+1}			-0.089	-0.123	0.094		-0.101	-0.043	-0.095
s.e			0.067	0.090	0.090		0.178	0.076	0.076
β_i^D			0.467	0.500	0.715		0.934	0.439	0.292
s.e			0.123	0.165	0.165		0.329	0.139	0.138
β_i^{OLS}			0.543	0.429	0.522		0.825	0.365	0.294
s.e			0.067	0.091	0.090		0.177	0.076	0.075
$\beta_i^{OLS} = \beta_i^D$			1.135	-0.769	-2.153		-0.621	-0.977	0.026

The results presented in Table 11 indicate that there is an absence of evidence of thin trading in the third sample period. Of the β_{i-1} and β_{i+1} coefficients in Table 11, only β_{i-1} for DUE is marginally significant. The results in Table 11 are consistent with the view that there is a paucity of evidence of thin trading in the sample of firms considered in this report. Moreover, while the t-statistic used to test the hypothesis $H_0 : \hat{\beta}_i = \beta_i^D$ for ENV is statistically significant, the estimates of β_{i-1} and β_{i+1} associated with this outcome are not significant. It is difficult to ascribe this outcome to thin trading given the insignificance of the relevant β_{i-1} and β_{i+1} estimates.

In summary, on the basis of the evidence in Tables 9-11, there is an absence of evidence for thin trading in the weekly data on the individual firms. This suggests that the uncorrected OLS estimate is the appropriate approach to measure β in Tables 2-4.

There was no evidence of thin trading in the monthly data. This is unsurprising given that there is no evidence of thin trading in the weekly samples.

2.3. Stability and sensitivity analysis: Individual firms

The appendix presents recursive estimates of β_i for each of the individual firms using the longest available sample period in graphical form. The estimates are produced using either a moving window with a fixed width of 1 year or an expanding window with initial width of 1 year.

Assessment of these graphs indicates that, irrespective of the construction of the recursion, the evidence for each firm is consistent. There is only very weak visual evidence of time variation in the estimates of β_i across the plots in the appendix. That is, there are relatively few occasions when the recursive estimates display sudden substantial jumps across all the cases considered. Moreover, there is no systematic evidence of regression to unity in the estimates of β. For example, the majority of figures 1.1 - 1.18 in the appendix suggest that the equity β lies somewhere between 0.2 and 0.8. This is consistent with the range of OLS point estimates reported in Table 2 of 0.2826 to 1.0305. In short, the recursive estimation provides no systematic evidence of parameter instability in the OLS estimates of β for individual firms.

Table 12 presents the marginal significance levels for the Hansen test calculated for the longest available sample.

Table 12 Hansen (1992) Structural Stability Tests

Individual Firms – Longest available sample – Weekly frequency

	AAN	AGL	APA	DUE	ENV	GAS	HDF	SKI	SPN
Joint	0.18	0.01	0.07	0.31	0.00	0.06	0.15	0.05	0.04
σ^2	0.91	0.00	0.01	0.05	0.00	0.01	0.21	0.00	0.03
α	0.84	0.74	0.68	0.88	0.44	0.87	0.35	0.33	0.49
β	0.03	0.08	0.37	0.84	0.07	0.92	0.74	0.93	0.05

Three out of nine joint tests reject the null of parametric stability. In the case of AGL (marginal significance level =0.01) and ENV (marginal significance level =0.00), this rejection is as a result of instability of the residual variance, σ_i^2. Instability in the estimate of σ_i^2 is a reasonably common event with asset returns, reflecting changes in the estimated level of asset-specific risk. In the case of SPN, the rejection of the null hypothesis of the Hansen joint test is very marginal given that the p-value is 0.04. Again it appears that this rejection of the joint null is down to instability in the residual variance, σ_i^2 (marginal significance level 0.03). However, in the case of SPN, the failure to reject the null of parametric stability in β is marginal as the p-value of this test is 0.05. A fourth joint test, for SKI, only marginally avoids the finding of a rejection of the joint null hypothesis (rounded result of 0.05). SKI is similar to AGL and ENV in that the individual parameter analysis finds instability in the residual variance (σ_i^2, marginal significance = 0.00) but not in the β (marginal significance = 0.93).

However, in 27 tests of individual parameters, on only three occasions is there evidence against the hypothesis that the parameter of interest is stable. In each of these three rejections, the evidence is consistent with the view that there has been instability in the estimated level of asset-specific risk.

The evidence in Table 12 is consistent with the view that there is no strong evidence of parameter instability in the estimated values for α and β presented in Table 2. This calls into question the necessity or even validity of omitting data due to concerns about structural instability arising from the tech boom and GFC.

Using the Hansen test, or any other approach, to test for instability in the estimated model with the non-contiguous post-Dotcom sample is tenuous. This sample period starts using the week ending 4 January 2002 (following the instructions of the AER that the tech boom period is 3 July 1998 to 28 December 2001). The sample excludes the period 5 September 2008 to 30 October 2009 (both defined as weeks ending on each date), which in the consultant's view broadly coincides with the GFC. The sample period ends on 28 June 2013, in keeping with the earlier analysis. There is more than a year of data omitted in each case in the interior of the sample. That is, the observation coinciding with the last week of August 2008 is immediately followed by the observation represent the first week of November 2009. A rejection of the null hypothesis of model/parameter stability may be due to the omission of more than a year of data from the interior of the sample space, or because the model is unstable, or both. For this reason, neither recursive regression results, nor stability tests were performed. Given the lack of evidence of instability in the estimated values for α and β presented in Table 2, there appears to be little necessity to partition the sample to allow for the tech boom and GFC.

Table 13 presents the results of the Hansen test for the last five years sample.

Table 13 Hansen (1992) Structural Stability Tests
Individual Firms – Last five years – Weekly frequency

	AAN	AGL	APA	DUE	ENV	GAS	HDF	SKI	SPN
Joint	*NA*	*NA*	0.00	0.02	0.00	*NA*	0.01	0.00	0.01
σ^2	*NA*	*NA*	0.00	0.00	0.00	*NA*	0.03	0.00	0.00
α	*NA*	*NA*	0.98	0.57	0.33	*NA*	0.57	0.38	0.58
β	*NA*	*NA*	0.58	0.64	0.73	*NA*	0.61	0.87	0.01

All of the joint tests for model stability reject the null of parametric stability. With the exception of SPN, each rejection is as a result of instability of the estimated residual variance, σ_i^2. Instability in the estimate of σ_i^2 is a reasonably common event with asset returns, reflecting changes in the estimated level of asset-specific risk. In the case of SPN, there is evidence of instability in the estimates of both σ_i^2 and β.

However, in eighteen tests of individual parameters, on only seven occasions is there evidence against the hypothesis that the parameter of interest is stable. In five of these seven rejections, the evidence is consistent with the view that there has been a change in the estimated level of asset-specific risk. For SPN, there is evidence of instability in the estimates of both σ_i^2 and β.

It is worth noting that the last 5 year sample spans the period of the GFC. There is no evidence of widespread instability in the estimate of β across the six firms considered in Table 13. This may simply reflect the lack of power associated with the Hansen test in small samples. However, there is no evidence of widespread parameter instability in the estimates of α and β reported in Table 12 for the full sample.

In short, on the basis of model instability, there does not seem to be convincing grounds to either omit data from the tech boom and GFC periods. Furthermore, there is no evidence to suggest that the last 5 year sample is superior to the full sample. The choice of a 5 year sample window is entirely arbitrary, particularly when more data is available. If the aim of the exercise is to provide credible estimates of the level of undiversifiable risk for the equity of interest, then data should only be excluded where necessary. The exclusion of data from the estimation process underlying Tables 3 and 4 appears unwarranted in the view of the consultant.

3. Portfolio Analysis

The AER instructed the consultant to construct estimates of β for two types of portfolios.

1. Portfolios with fixed weights. Empirically estimate the equity β for a portfolio comprising a number of individual firms, weighting each firm in a constant manner across the entire duration of the portfolio. There were two methods to determine the fixed weights:

    a. Equal weighting for each firm in the portfolio

    b. Value weighting each firm in the portfolio by its market capitalisation

2. Portfolios with time varying weights. Empirically estimate the equity β for a portfolio comprising a number of individual firms, changing the weighting on each firm across the duration of the portfolio.

3.1. Portfolio construction: Fixed weight portfolios

Consider a portfolio, *P*, containing two assets, *X* and *Y*, paying returns R_x and R_y, respectively. This portfolio has a constant proportion, *a*, of wealth invested in asset *X* and the remaining 1-a of wealth invested in *Y*. The expected return to *P* is given by

$$E(R_p) = aE(R_x) + (1-a)E(R_y) \tag{11}$$

It is useful to note that this point that (11) depends entirely on the fact that for a constant a and variable X $E(aX) = aE(X)$.

It is straightforward to show that the variance of return to *P* is given by

$$Var(R_p) = a^2 Var(R_x) + (1-a)^2 Var(R_y) + 2a(1-a)Cov(R_x R_y) \tag{12}$$

The fixed weight portfolios apply a constant weight to each firm that is a member of the portfolio across the duration of the portfolio. The consultant was instructed to use two different methods to determine these fixed weights:

1. Equal weighted portfolios. The weighting on each firm will be 1/*n*, where *n* is the number of firms in the portfolio.

2. Value weighted portfolios. The weighting on each firm will be proportional to the market capitalisation of the firm relative to the market capitalisation of the entire portfolio. In all cases, market capitalisation will be measured as the average across the portfolio duration. Hence, the weight on each firm will be $\bar{E}_i/\bar{E}_P$ where $\bar{E}_i$ is the average market capitalisation of the firm (across the relevant period) and $\bar{E}_P$ is the average market capitalisation of the entire portfolio (again, across the relevant period).

The consultant was instructed by the AER to examine the following portfolios (for both equal weighted and value weighted construction):

1. P1: APA, ENV from
   a. The longest available time period (16/6/2000 to 28/06/2013)
   b. As in (a) but excluding the tech boom and GFC period
2. P2: AAN, AGL, APA, ENV, GAS from
   a. The longest available time period (21/12/2001 to 06/10/2006)
   b. As in (a) but excluding the tech boom
3. P3: APA, DUE, ENV, HDF,SPN from
   a. The longest available time period (16/12/2005 to 23/11/2012)
   b. As in (a) but excluding the GFC period
4. P4: APA, DUE, ENV, HDF, SKI, SPN from
   a. The longest available time period (02/03/2007 to 23/11/2012)
   b. As in (a) but excluding the GFC period
5. P5: APA, DUE, ENV, SKI, SPN from
   a. The longest available time period (02/03/2007 to 28/06/2013)

b. As in (a) but excluding the GFC period

The consultant was not asked to provide expert advice on the rationale for preparing the portfolios.

3.2. Estimation results: Fixed weight portfolios

Tables 14–15 present estimates of equity β for fixed weight portfolios that give equal weighting to each firm in the portfolio. These tables present two different sample periods (the longest sample available for each firm; and the same longest sample available but excluding the tech boom and the GFC). This analysis uses both OLS and LAD regression calculations and a weekly sampling frequency.

Tables 16–17 present estimates of equity β for fixed weight portfolios that give value weighting (instead of equal weighting) to each firm in the portfolio. As with the equal weighted portfolios, two different sample periods are presented. The weights used in the construction of each portfolio, as calculated by the AER are presented in Annex A to this report.

Table 18 is a summary table of the fixed weight portfolio analysis presented in Tables 14–17.

Table 14 presents results for the longest available sample period for each fixed portfolio, equal weighting each constituent stock. The regressions use a weekly sampling period, both OLS and LAD regression calculations, delever/relever each portfolio to the benchmark gearing (60 per cent), and include 95% confidence intervals around the estimates.

	P1	P2	P3	P4	P5
Table 14: Delevered/relevered estimates of β *Fixed portfolio construction – Equal weighting* *Longest sample available – Weekly frequency*					
Firms	APA ENV	AAN AGL APA ENV GAS	APA DUE ENV HDF SPN	APA DUE ENV HDF SKI SPN	APA DUE ENV SKI SPN
Start	16/06/2000	21/12/2001	16/12/2005	02/03/2007	02/03/2007
End	28/06/2013	06/10/2006	23/11/2012	23/11/2012	28/06/2013
$\bar{G}$	0.6382	0.5101	0.6286	0.6381	0.6658
ω	0.9044	1.2246	0.9286	0.9047	0.8355
$\hat{\beta}$	0.4575	0.5198	0.5037	0.4759	0.3865
s.e	0.0355	0.0705	0.0468	0.0489	0.0381
$\hat{\beta}_u$	0.5271	0.6581	0.5954	0.5717	0.4612
$\hat{\beta}_l$	0.3880	0.3816	0.4120	0.3801	0.3119
$\tilde{\beta}$	0.4584	0.4097	0.5121	0.4989	0.4349
s.e	0.0355	0.0711	0.0469	0.0490	0.0383
$\tilde{\beta}_u$	0.5280	0.5491	0.6041	0.5949	0.5099
$\tilde{\beta}_l$	0.3887	0.2703	0.4202	0.4028	0.3599
N	680	250	362	299	330
R^2	0.1968	0.1796	0.2436	0.2419	0.2389

The average of the OLS estimates, E($\hat{\beta}$), was 0.4687 with a maximum estimate of 0.5198 for P2, and a minimum of 0.3865 for P5. The median OLS estimate, M($\hat{\beta}$), was 0.4759. The evidence from Table 14 suggests that the majority of the OLS point estimates lie in the range 0.38 to 0.52.

The mean LAD estimate, E($\tilde{\beta}$) was 0.4628, with a maximum estimate of 0.5121 for P3 and a minimum of 0.4097 for P2. The median LAD estimate, M($\tilde{\beta}$), was 0.4584. The evidence from Table 14 suggests that the majority of the LAD point estimates lie in the range 0.40 to 0.52.

All the LAD estimates lie within the 95% OLS confidence interval for the relevant portfolio. This suggests that the information provided by the LAD estimates is largely consistent with the evidence provided by the OLS estimates.

Taken together, the evidence from Table 14 suggests that the point estimates of equity lie in the range 0.38 to 0.52. The evidence from Table 14 is, broadly speaking, consistent with the evidence presented in Table 2 above. Furthermore, the evidence in Table 14 is, broadly speaking, also consistent with the outcomes reported in Table 5.3 of Henry (2009).

Table 15 presents results for the longest available sample period – but excluding the tech boom and the GFC – for each fixed portfolio, equal weighting each constituent stock. Note that there is one portfolio (P1) which encompasses both the tech boom (1 July 1998 to 31 December 2001) and the GFC (1 September 2008 to 31 October 2009). The other four portfolios are affected by only one of the exclusion periods – either the tech boom (P2, though only minimally) or the GFC (P3, P4 and P5). The regressions use a weekly sampling period, both OLS and LAD regression calculations, delever/relever each portfolio to the benchmark gearing (60 per cent), and included 95% confidence intervals around the estimates.

	Table 15: Delevered/relevered estimates of β				
	Fixed portfolio– Equal weighting				
	Longest sample available but excluding tech boom and GFC – Weekly frequency				
	P1	P2	P3	P4	P5
Firms	APA ENV	AAN AGL APA ENV GAS	APA DUE ENV HDF SPN	APA DUE ENV HDF SKI SPN	APA DUE ENV SKI SPN
Start	04/01/2002	04/01/2002	16/12/2005	02/03/2007	02/03/2007
End	28/06/2013	06/10/2006	23/11/2012	23/11/2012	28/06/2013
$\bar{G}$	0.6230	0.5101	0.6174	0.6250	0.6534
ω	0.9425	1.2246	0.9565	0.9374	0.8666
$\hat{\beta}$	0.4927	0.5209	0.5501	0.5317	0.4536
s.e	0.0420	0.0710	0.0453	0.0495	0.0427
$\hat{\beta}_u$	0.5749	0.6600	0.6390	0.6287	0.5374
$\hat{\beta}_l$	0.4104	0.3818	0.4613	0.4347	0.3699
$\tilde{\beta}$	0.4487	0.3819	0.5536	0.5604	0.5054
s.e	0.0420	0.0716	0.0454	0.0496	0.0429
$\tilde{\beta}_u$	0.5311	0.5223	0.6426	0.6576	0.5895
$\tilde{\beta}_l$	0.3664	0.2416	0.4645	0.4631	0.4213
N	537	248	300	237	268
R^2	0.2049	0.1797	0.3307	0.3292	0.2976

The average of the OLS estimates, E($\hat{\beta}$), was 0.5098 with a maximum estimate of 0.5501 for P3, and a minimum of 0.4536 for P5. The median OLS estimate, M($\hat{\beta}$), was 0.5209. The evidence from Table 15 suggests that the majority of the OLS point estimates lie in the range 0.45 to 0.56.

The average LAD estimate, E($\tilde{\beta}$) was 0.4900, with a maximum estimate of 0.5604 for P4 and a minimum of 0.3819 for P2. The median LAD estimate, M($\tilde{\beta}$), was 0.5054. Consequently, the evidence from Table 15 suggests that the majority of the LAD point estimates lie in the range 0.38 to 0.57.

All the LAD estimates lie within the 95% OLS confidence interval for the relevant portfolio, albeit that this is very marginal for P2. This suggests that the information provided by the LAD estimates is consistent with the evidence provided by the OLS estimates.

Taken together, the evidence from Table 15 suggests that the point estimates of portfolio equity β lie in the range 0.38 to 0.57. The evidence from Table 15 is, broadly speaking, consistent with the evidence presented in Table 2 above. While the estimates are significantly different from zero, the magnitudes of the estimates are much smaller than in Table 2 above or Table 14 of Henry (2009). Consequently the mean and median estimate and the associated range of point estimates are also smaller than their counterparts in Table 2 above or Table 14 of Henry (2009).

Table 16 presents results for the longest available sample period for each fixed portfolio, value weighting each constituent stock by its market capitalisation. The regressions use a weekly sampling period, both OLS and LAD regression calculations, delever/relever each portfolio to the benchmark gearing (60 per cent), and included 95% confidence intervals around the estimates.

Table 16: Delevered/relevered estimates of β

Fixed portfolio– Value weighting

Longest sample available – Weekly frequency

	P1	P2	P3	P4	P5
Firms	APA ENV	AAN AGL APA ENV GAS	APA DUE ENV HDF SPN	APA DUE ENV HDF SKI SPN	APA DUE ENV SKI SPN
Start	16/06/2000	21/12/2001	16/12/2005	02/03/2007	02/03/2007
End	28/06/2013	06/10/2006	23/11/2012	23/11/2012	28/06/2013
$\bar{G}$	0.6145	0.3756	0.6351	0.6450	0.6535
ω	0.9637	1.5611	0.9121	0.8875	0.8662
$\hat{\beta}$	0.4983	0.7032	0.4358	0.4198	0.3895
s.e	0.0384	0.1096	0.0426	0.0449	0.0405
$\hat{\beta}_u$	0.5736	0.9180	0.5193	0.5078	0.4688
$\hat{\beta}_l$	0.4230	0.4884	0.3523	0.3318	0.3102
$\tilde{\beta}$	0.5482	0.5823	0.4580	0.4566	0.4410
s.e	0.0385	0.1099	0.0428	0.0450	0.0407
$\tilde{\beta}_u$	0.6236	0.7978	0.5419	0.5448	0.5207
$\tilde{\beta}_l$	0.4727	0.3669	0.3742	0.3684	0.3613
N	680	250	362	299	330
R^2	0.1988	0.1423	0.2252	0.2275	0.2201

The average of the OLS estimates, E($\hat{\beta}$), was 0.4893 with a maximum estimate of 0.7032 for P2, and a minimum of 0.3895 for P5. The median OLS estimate, M($\hat{\beta}$), was 0.4358. The evidence from Table 16 suggests that the majority of the OLS point estimates lie in the range 0.38 to 0.71.

The mean LAD estimate, E($\tilde{\beta}$) was 0.4972 with a maximum estimate of 0.5823 for P2 and a minimum of 0.4410 for P5. The median LAD estimate, M($\tilde{\beta}$), was 0.4580. The evidence from Table 16 suggests that the majority of the LAD point estimates lie in the range 0.44 to 0.59.

All the LAD estimates lie within the 95% OLS confidence interval for the relevant portfolio. This suggests that the information provided by the LAD estimates is consistent with the evidence provided by the OLS estimates.

Taken together, the evidence from Table 16 suggests that the point estimates of equity lie in the range 0.38 to 0.71. The evidence from Table 16 is, broadly speaking, consistent with the evidence presented in Table 14. The evidence in Table 14 is also, broadly speaking, consistent with the outcomes reported in Table 5.4 of Henry (2009).

Table 17 presents results for the longest available sample period – but excluding the tech boom and the GFC – for each fixed portfolio, value weighting each constituent stock by its market capitalisation. Note that there is one portfolio (P1) which encompasses both the tech boom (1 July 1998 to 31 December 2001) and the GFC (1 September 2008 to 31 October 2009). The other four portfolios are affected by only one of the exclusion periods – either the tech boom (P2, though only minimally) or the GFC (P3, P4 and P5). The regressions use a weekly sampling period, both OLS and LAD regression calculations, delever/relever each portfolio to the benchmark gearing (60 per cent), and included 95% confidence intervals around the estimates.

Table 17: Delevered/relevered estimates of β					
Fixed portfolio – Value weighting					
Longest sample available but excluding tech boom and GFC – Weekly frequency					
	P1	P2	P3	P4	P5
Firms	APA ENV	AAN AGL APA ENV GAS	APA DUE ENV HDF SPN	APA DUE ENV HDF SKI SPN	APA DUE ENV SKI SPN
Start	04/01/2002	04/01/2002	16/12/2005	02/03/2007	02/03/2007
End	28/06/2013	06/10/2006	23/11/2012	23/11/2012	28/06/2013
$\bar{G}$	0.6005	0.3755	0.6221	0.6305	0.6402
ω	0.9986	1.5613	0.9447	0.9237	0.8995
$\hat{\beta}$	0.5357	0.7022	0.5172	0.5028	0.4757
s.e	0.0472	0.1103	0.0457	0.0493	0.0458
$\hat{\beta}_u$	0.6283	0.9183	0.6067	0.5995	0.5653
$\hat{\beta}_l$	0.4431	0.4860	0.4277	0.4060	0.3860
$\tilde{\beta}$	0.5179	0.5675	0.4953	0.5424	0.5381
s.e	0.0473	0.1107	0.0458	0.0495	0.0458
$\tilde{\beta}_u$	0.6106	0.7844	0.5850	0.6394	0.6279
$\tilde{\beta}_l$	0.4252	0.3506	0.4056	0.4454	0.4483
N	537.0000	248.0000	300	237.0000	268.0000
R^2	0.1938	0.1415	0.3009	0.3064	0.2900

The average of the OLS estimates, E($\hat{\beta}$), was 0.5467 with a maximum estimate of 0.7022 for P2, and a minimum of 0.4757 for P5. The median OLS estimate, M($\hat{\beta}$), was 0.5172. The evidence from Table 17 suggests that the majority of the OLS point estimates lie in the range 0.47 to 0.71.

The mean LAD estimate, E($\tilde{\beta}$) was 0.5322, with a maximum estimate of 0.5675 for P2 and a minimum of 0.4953 for P3. The median LAD estimate, M($\tilde{\beta}$), was 0.5381. The evidence from Table 17 suggests that the majority of the LAD point estimates lie in the range 0.49 to 0.57.

All the LAD estimates lie within the 95% OLS confidence interval for the relevant portfolio. This suggests that the information provided by the LAD estimates is consistent with the evidence provided by the OLS estimates.

Taken together, the evidence from Table 17 suggests that the point estimates of equity lie in the range 0.47 to 0.71. The evidence from Table 17 is, broadly speaking, consistent with the evidence presented in Tables 14–16. The evidence in Table 17 is, broadly speaking, consistent with the outcomes reported in Table 5.3 of Henry (2009).

Table 18 presents the average and median equity beta estimates from Tables 14–17. This includes analysis across three different sample periods, and three different permutations of the sample frequency and calculation types.

Table 18: Delevered/relevered estimates of β

Summary of fixed portfolio equity β point estimates from tables 14–17

Fixed Portfolio	Regression	Full sample period		Ex tech boom /GFC	
		Ave	Med	Ave	Med
Equal weighted	OLS	0.4687	0.4759	0.5098	0.5209
Equal weighted	LAD	0.4628	0.4584	0.4900	0.5054
Value weighted	OLS	0.4893	0.4358	0.5467	0.5172
Value weighted	LAD	0.4972	0.4580	0.5322	0.5381

Taken together, the evidence presented in Table 18 (which summarises the evidence from Tables 14-17) suggests that the average and median point estimates of equity β lie in the range 0.43 to 0.55. The evidence from Table 18 is, broadly speaking, consistent with the evidence presented in Tables 2 and 3 above. Furthermore, the evidence in Table 18 is, in the main, consistent, with the outcomes reported in Table 5.3 of Henry (2009).

3.3. Thin trading analysis: Fixed weight portfolios

As with the individual firm analysis, thin trading can create issues with the estimation of equity β in portfolios. In comparison with individual firms, the likelihood of observing, let alone detecting thin trading is very small. In order to produce runs of zero portfolio returns requires that the component assets held in the portfolio would either have to produce runs of offsetting profits and losses or runs of zero returns or a combination of the two.

Table 19 presents estimates of Dimson's β for the equal weighted portfolios, using the longest available sample and a weekly sampling frequency. The equity β estimates underlying this analysis were presented in Table 14.

Table 19: Dimson's β – Portfolios

Fixed portfolio construction – Equal weighting

Longest sample available – Weekly frequency

	P1	P2	P3	P4	P5
Firms	APA ENV	AAN AGL APA ENV GAS	APA DUE ENV HDF SPN	APA DUE ENV HDF SKI SPN	APA DUE ENV SKI SPN
Start	16/06/2000	21/12/2001	16/12/2005	02/03/2007	02/03/2007
End	28/06/2013	06/10/2006	23/11/2012	23/11/2012	28/06/2013
β_{i-1}	0.0337	-0.0664	0.0850	0.1010	0.0814
s.e	0.0394	0.0582	0.0506	0.0542	0.0456
β_i	0.5063	0.4366	0.5432	0.5279	0.4650
s.e	0.0394	0.0576	0.0506	0.0542	0.0456
β_{i+1}	-0.0174	-0.0289	-0.0477	-0.0440	-0.0364
s.e	0.0395	0.0575	0.0505	0.0541	0.0456
β_i^D	0.5227	0.3413	0.5805	0.5849	0.5100
s.e	0.0699	0.0958	0.0909	0.0977	0.0816
β_i^{OLS}	0.5059	0.4245	0.5424	0.5260	0.4626
s.e	0.0392	0.0576	0.0504	0.0540	0.0456
$\beta_i^{OLS} = \beta_i^D$	-0.4277	1.4442	-0.7557	-1.0894	-1.0384

None of the β_{i-1} and β_{i+1} coefficients in Table 19 are significant, which is consistent with the view that there is a paucity of evidence of thin trading in the sample of firms considered in this report. Moreover, none of the t-statistics used to test the hypothesis $H_0 : \hat{\beta}_i = \beta_i^D$ reported in Table 19 are statistically significant. This indicates that there is an absence of evidence of thin trading in this sample period.

Table 20 presents estimates of Dimson's β for the equal weighted portfolios, using the longest available sample but excluding the tech boom and the GFC. The equity β estimates underlying this analysis were presented in Table 15.

Table 20: Dimson's β – Portfolios

Fixed portfolio construction – Equal weighting

Longest sample available but excluding tech boom and GFC – Weekly frequency

	P1	P2	P3	P4	P5
Firms	APA ENV	AAN AGL APA ENV GAS	APA DUE ENV HDF SPN	APA DUE ENV HDF SKI SPN	APA DUE ENV SKI SPN
Start	04/01/2002	04/01/2002	16/12/2005	02/03/2007	02/03/2007
End	28/06/2013	06/10/2006	23/11/2012	23/11/2012	28/06/2013
β_{i-1}	-0.0367	-0.0661	-0.0240	-0.0157	-0.0061
s.e	0.0449	0.0584	0.0480	0.0538	0.0499
β_i	0.5217	0.4366	0.5639	0.5636	0.5235
s.e	0.0446	0.0578	0.0477	0.0533	0.0495
β_{i+1}	-0.0010	-0.0291	-0.0219	-0.0096	0.0057
s.e	0.0444	0.0577	0.0476	0.0529	0.0492
β_i^D	0.4841	0.3414	0.5181	0.5382	0.5231
s.e	0.0772	0.0960	0.0855	0.0956	0.0875
β_i^{OLS}	0.5228	0.4253	0.5751	0.5672	0.5235
s.e	0.0445	0.0579	0.0474	0.0528	0.0493
$\beta_i^{OLS} = \beta_i^D$	0.8686	1.4485	1.2005	0.5486	0.0076

None of the β_{i-1} and β_{i+1} coefficients in Table 20 are significant, which is consistent with the view that there is a paucity of evidence of thin trading in the sample of firms considered in this report. Moreover, none of the t-statistics used to test the hypothesis $H_0 : \hat{\beta}_i = \beta_i^D$ reported in Table 20 are statistically significant. This indicates that there is an absence of evidence of thin trading in this sample period.

Table 21 presents estimates of Dimson's β for the value weighted portfolios, using the longest available sample and a weekly sampling frequency. The equity β estimates underlying this analysis were presented in Table 16.

Table 21: Dimson's β – Portfolios

Fixed portfolio construction – Value weighting

Longest sample available – Weekly frequency

	P1	P2	P3	P4	P5
Firms	APA ENV	AAN AGL APA ENV GAS	APA DUE ENV HDF SPN	APA DUE ENV HDF SKI SPN	APA DUE ENV SKI SPN
Start	16/06/2000	21/12/2001	16/12/2005	02/03/2007	02/03/2007
End	28/06/2013	06/10/2006	23/11/2012	23/11/2012	28/06/2013
β_{i-1}	0.0219	-0.0503	0.0684	0.0868	0.0759
s.e	0.0400	0.0717	0.0468	0.0506	0.0467
β_i	0.5163	0.4568	0.4750	0.4724	0.4508
s.e	0.0400	0.0710	0.0468	0.0506	0.0467
β_{i+1}	-0.0366	-0.0454	-0.0726	-0.0652	-0.0582
s.e	0.0401	0.0709	0.0467	0.0505	0.0467
β_i^D	0.5017	0.3610	0.4709	0.4940	0.4685
s.e	0.0710	0.1181	0.0841	0.0913	0.0836
β_i^{OLS}	0.5171	0.4505	0.4778	0.4730	0.4497
s.e	0.0399	0.0702	0.0467	0.0506	0.0467
$\beta_i^{OLS} = \beta_i^D$	0.3862	1.2738	0.1470	-0.4153	-0.4029

None of the β_{i-1} and β_{i+1} coefficients in Table 21 are significant at the 5% level of confidence, which is consistent with the view that there is a paucity of evidence of thin trading in the sample of firms considered in this report. Moreover, none of the t-statistics used to test the hypothesis $H_0 : \hat{\beta}_i = \beta_i^D$ reported in Table 21 is statistically significant. This indicates that there is little of evidence of thin trading in this sample period.

Table 22 presents estimates of Dimson's β for the equal weighted portfolios, using the longest available sample but excluding the tech boom and the GFC. The equity β estimates underlying this analysis were presented in Table 17.

Table 22: Dimson's β – Portfolios

Fixed portfolio construction – Value weighting

Longest sample available but excluding tech boom and GFC – Weekly frequency

	P1	P2	P3	P4	P5
Firms	APA ENV	AAN AGL APA ENV GAS	APA DUE ENV HDF SPN	APA DUE ENV HDF SKI SPN	APA DUE ENV SKI SPN
Start	04/01/2002	04/01/2002	16/12/2005	02/03/2007	02/03/2007
End	28/06/2013	06/10/2006	23/11/2012	23/11/2012	28/06/2013
β_{i-1}	-0.0498	-0.0509	-0.0332	-0.0176	-0.0122
s.e	0.0477	0.0720	0.0490	0.0544	0.0513
β_i	0.5363	0.4569	0.5406	0.5395	0.5288
s.e	0.0474	0.0712	0.0487	0.0539	0.0509
β_{i+1}	0.0009	-0.0451	-0.0382	-0.0209	-0.0063
s.e	0.0472	0.0711	0.0483	0.0534	0.0506
β_i^D	0.4873	0.3609	0.4692	0.5009	0.5103
s.e	0.0819	0.1183	0.0867	0.0966	0.0899
β_i^{OLS}	0.5364	0.4497	0.5475	0.5443	0.5288
s.e	0.0473	0.0706	0.0483	0.0534	0.0509
$\beta_i^{OLS} = \beta_i^D$	1.0391	1.2585	1.6195	0.8123	0.3643

None of the β_{i-1} and β_{i+1} coefficients in Table 22 are significant, which is consistent with the view that there is a paucity of evidence of thin trading in the sample of firms considered in this report. Moreover, none of the t-statistics used to test the hypothesis $H_0 : \hat{\beta}_i = \beta_i^D$ reported in Table 22 are statistically significant. This indicates that there is little evidence of thin trading in this sample period.

3.4. Stability and sensitivity analysis: Fixed weight portfolios

The appendix presents recursive estimates of β_i for each of the fixed portfolios (equal weighted and value weighted) using the longest available sample period. The estimates are produced using either a moving window with a fixed width of 1 year or an expanding window with initial width of 1 year.

Assessment of these graphs indicates that, irrespective of the construction of the recursion, the evidence for each portfolio is consistent. There is only very weak visual evidence of time variation in the estimates of β_i across the plots in the appendix. That is, there are relatively few occasions when the recursive estimates display sudden substantial jumps across all the cases considered. Moreover, there is no systematic evidence of regression to unity in the estimates of β. For example, the majority of figures 2.1 - 2.10 in the appendix suggest that the equity β lies somewhere between 0.2 and 0.8. This is consistent with the range of OLS point estimates reported in Table 14 of 0.3865 to 0.5198. In short, the recursive estimation provides no systematic evidence of parameter instability in the OLS estimates of β for the fixed weight portfolios considered in this report.

As with the individual firm analysis, we also report Hansen's (1992) test for parameter stability. The joint test tests the null hypothesis that the model does not display instability. Tests for the stability of the individual parameters of the model, α, β and σ^2, are also presented in Table 23.

Table 23 presents the Hansen (1992) Structural Stability Tests for the regressions underlying Table 14 – that is, the fixed weight portfolios using equal weighting, for the longest sample available.

Table 23: Hansen (1992) Structural Stability Tests – Portfolios

Fixed portfolio construction – Equal weighting

Longest sample available – Weekly frequency

	P1	P2	P3	P4	P5
Firms	APA ENV	AAN AGL APA ENV GAS	APA DUE ENV HDF SPN	APA DUE ENV HDF SKI SPN	APA DUE ENV SKI SPN
Start	16/06/2000	21/12/2001	16/12/2005	02/03/2007	02/03/2007
End	28/06/2013	06/10/2006	23/11/2012	23/11/2012	28/06/2013
Joint	0.00	0.00	0.05	0.05	0.00
σ^2	0.00	0.00	0.04	0.02	0.00
α	0.28	0.83	0.09	0.06	0.07
β	0.26	0.00	0.93	0.85	0.91

In three out of five cases, the joint test rejects the null of parametric stability. In the case of P3 and P4, the p-value of the joint test is 0.05, which indicates that the failure to reject the null hypothesis of model stability is reasonably marginal. With the exception of P2, this rejection of the null hypothesis of model stability is as a result of instability of the residual variance, σ_i^2, reflecting changes in the estimated level of portfolio-specific risk. However, the rejection of the null of parameter stability in β for P2 suggests that the estimates for this portfolio be interpreted with great caution.

Table 24 presents the Hansen (1992) Structural Stability Tests for the regressions underlying Table 16 – that is, the fixed weight portfolios using value weighting, for the longest sample available.

Table 24: Hansen (1992) Structural Stability Tests – Portfolios *Fixed portfolio construction – Value weighting* *Longest sample available – Weekly frequency*					
	P1	P2	P3	P4	P5
Firms	APA ENV	AAN AGL APA ENV GAS	APA DUE ENV HDF SPN	APA DUE ENV HDF SKI SPN	APA DUE ENV SKI SPN
Start	16/06/2000	21/12/2001	16/12/2005	02/03/2007	02/03/2007
End	28/06/2013	06/10/2006	23/11/2012	23/11/2012	28/06/2013
Joint	0.01	0.00	0.04	0.03	0.00
σ^2	0.00	0.02	0.01	0.00	0.00
α	0.36	0.91	0.14	0.10	0.13
β	0.26	0.00	0.92	0.89	0.88

The joint tests uniformly reject the null of parametric stability. With the exception of P2, this rejection is as a result of instability of the residual variance, σ_i^2, reflecting changes in the estimated level of portfolio-specific risk. However, the rejection of the null of parameter stability in β for P2 suggests that the estimates for this portfolio be interpreted with great caution.

3.5. Portfolio construction: Time-varying portfolios

Technically, a portfolio is defined using a fixed vector of weights. If the vector of weights changes a new portfolio is defined. Moreover, when a new business “drops in” and/ or “drops out” of the portfolio, both the investment opportunity set and/or the market portfolio may change as a result of takeovers and IPO activity. In short, great caution should be exercised when interpreting the β estimates from the resulting ‘time-varying portfolios’ as they are not grounded in financial theory.

In the case of the time varying portfolios, the consultant was advised that the composition of the portfolio should change across time to accommodate firms entering the data set (e.g. listing on the ASX) and leaving the dataset (e.g. delisting). At any given return interval, the weighting on each firm should be dependent on the number of firms currently in the portfolio, *n*. The consultant was instructed to construct average and median measures of returns for these portfolios with time varying weights. For the average portfolio, AVE, the return is the equally-weighted average return across the *n* constituent firms. For the median portfolio, the return on the portfolio will be the median return across the *n* firms.

It is important to note that equation (11) is written assuming that the weight *a* is constant. This assumption is clearly violated for the results presented below. Time variation in the portfolio weights means that the result underlying (11), $E(aX) = aE(X)$ does not apply as *a* is variable in the case of time varying portfolio weights. There is very likely to be substantial measurement error in the returns data for the average and median portfolios if we assume (11) obtains. This measurement error may occur because the return to the portfolio may vary because (i) the asset values in the portfolio vary, (ii) the weights in the portfolio vary, or (iii) both. Moreover, it is very likely that equation (12) will provide a very poor guide as to the variance of this second set of ‘portfolios’ as terms such as Var(a_t) and Cov(r_{it}, a_t) will be omitted from the measurement of the variance of returns. The resulting estimates and any associated inferences are extremely difficult to interpret. In particular, it is not clear whether Cov(r_{mt}, r_{pt}) will be affected by this measurement error, and what the impact of this measurement error might be. Any issues with bias in the β estimates obtained using this data are as a result of the particular approach used to construct the ‘portfolio’ returns and not due to problems with the OLS or LAD estimator. It is the strong view of the consultation that the analysis of these ‘time varying portfolios’ is unlikely to yield reliable evidence. At the minimum, extreme caution should be placed on any inference based on the estimates and confidence intervals report for these 'time-varying portfolios'.

The consultant was instructed to construct these time varying portfolio for three different time periods. Note that for each of the tables, the dates shown are all Fridays, and refer to the week ending on that date (in line with the Datastream calculation of weekly observations as set out at the beginning of this report). Table 25 shows the construction of the portfolio across the longest possible time period, from 29 August 1997 to 28 June 2013.[8]

Table 25: Construction of the time varying portfolio

Longest possible sample

Period Start	Change	Set	*n*	Period End
29/08/97		AGL ENV	2	09/06/00
16/06/00	+APA	AGL APA ENV	3	13/10/00
20/10/00	+AAN	AAN AGL APA ENV	4	14/12/01
21/12/01	+GAS	AAN AGL APA ENV GAS	5	06/08/04
13/08/04	+DUE	AAN AGL APA DUE ENV GAS	6	10/12/04
17/12/04	+HDF	AAN AGL APA DUE ENV GAS HDF	7	09/12/05
16/12/05	+SPN	AAN AGL APA DUE ENV GAS HDF SPN	8	06/10/06
13/10/06	-AGL	AAN APA DUE ENV GAS HDF SPN	7	10/11/06
17/11/06	-GAS	AAN APA DUE ENV HDF SPN	6	23/02/07
02/03/07	+SKI	AAN APA DUE ENV HDF SKI SPN	7	17/08/07
24/08/07	-AAN	APA DUE ENV HDF SKI SPN	6	23/11/12
30/11/12	-HDF	APA DUE ENV SKI SPN	5	28/06/13

[8] "The dates shown reflect the availability of price observations (consistent with dates presented elsewhere in the document). Where there is an addition to the available set of firms, the first portfolio return observation will reflect the previous firm composition, since there is a one week delay between the first price observation and the first return observation for the new firm. Where there is a deduction from the available set of firms, there is no delay and the first portfolio return observation will reflect the new portfolio construction."

Table 26 shows the construction of the portfolio for the period that starts after the end of the tech boom (1 January 2002), excludes the GFC (1 September 2008 to 31 October 2009) then concludes at the 30 June 2013.

Table 26: Construction of the time varying portfolio

Sample post tech boom, ex GFC

Period Start	Change	Set	*n*	Period End
04/01/02		AAN AGL APA ENV GAS	5	06/08/04
13/08/04	+DUE	AAN AGL APA DUE ENV GAS	6	10/12/04
17/12/04	+HDF	AAN AGL APA DUE ENV GAS HDF	7	09/12/05
16/12/05	+SPN	AAN AGL APA DUE ENV GAS HDF SPN	8	06/10/06
13/10/06	-AGL	AAN APA DUE ENV GAS HDF SPN	7	10/11/06
17/11/06	-GAS	AAN APA DUE ENV HDF SPN	6	23/02/07
02/03/07	+SKI	AAN APA DUE ENV HDF SKI SPN	7	17/08/07
24/08/07	-AAN	APA DUE ENV HDF SKI SPN	6	29/08/08
05/09/08		*(GFC period excluded)*		*30/10/09*
06/11/09		APA DUE ENV HDF SKI SPN	6	23/11/12
30/11/12	-HDF	APA DUE ENV SKI SPN	5	28/06/13

Table 27 shows the construction of the portfolio for the last five years, commencing on 1 July 2008 and ending on 30 June 2013.

Table 27: Construction of the time varying portfolio

Last five years (July 2008 to June 2013)

Period Start	Change	Set	*n*	Period End
04/07/08		APA DUE ENV HDF SKI SPN	6	23/11/12
30/11/12	-HDF	APA DUE ENV SKI SPN	5	28/06/13

3.6. Estimation results: Time-varying portfolios

Table 28 presents estimates of equity β for the time varying portfolios using a weekly sampling frequency. This analysis uses both OLS and LAD regression calculations. Three different sampling periods are presented in the one table (longest possible sample, sample post tech boom and excluding GFC, last five years).

Table 29 presents estimates of equity β for the time varying portfolios using a monthly sampling frequency as a robustness check on the weekly results. This analysis is restricted to the OLS regression calculation.

Table 30 is a summary table of the time varying portfolio analysis presented in Tables 28-29.

Table 28 presents estimates of equity β for the time varying portfolios using a weekly sampling frequency. The portfolio returns are calculated using both the average and the median of the returns for (time-varying) constituent firms, as instructed by the AER. Three different sampling periods are presented. This analysis uses both OLS and LAD regression calculations. Each portfolio is delevered/relevered to the benchmark gearing (60 per cent) and 95% confidence intervals are calculated. Great caution should be placed on any inference based on the estimates and confidence intervals report for these ‘time-varying portfolios’.

Table 28: Delevered/relevered estimates of β *Time-varying portfolio – Average and median returns* *Three different sample periods – Weekly frequency*						
Sample period	Longest possible 29/08/97 to 28/06/13		Post tech, ex GFC 04/01/02 to 29/08/08 06/11/09 to 28/06/13		Last five years 04/07/08 to 28/06/13	
	Ave	Med	Ave	Med	Ave	Med
$\bar{G}$	0.5666	0.5666	0.5835	0.5835	0.6490	0.6490
ω	1.0835	1.0835	1.0413	1.0413	0.8774	0.8774
$\hat{\beta}$	0.4954	0.4582	0.5304	0.5118	0.4336	0.3904
s.e	0.0365	0.0340	0.0365	0.0347	0.0503	0.0442
$\hat{\beta}_u$	0.5670	0.5248	0.6019	0.5798	0.5323	0.4770
$\hat{\beta}_l$	0.4239	0.3916	0.4589	0.4439	0.3349	0.3038
$\tilde{\beta}$	0.5080	0.4456	0.5318	0.4830	0.4647	0.3856
s.e	0.0365	0.0340	0.0365	0.0347	0.0504	0.0442
$\tilde{\beta}_u$	0.5796	0.5122	0.6033	0.5510	0.5635	0.4723
$\tilde{\beta}_l$	0.4365	0.3790	0.4603	0.4150	0.3659	0.2989
N	826	826	537	537	260	260
R^2	0.1827	0.1809	0.2833	0.3077	0.2233	0.2323

Consider first the OLS estimates. For portfolios using the average return of the constituent firms, the average estimate of equity β, E($\hat{\beta}$), was 0.4865 with a maximum estimate of 0.5304, and a minimum of 0.4336. For portfolios using the median return of the constituent firms, the average estimate of equity β was 0.4535 with a maximum estimate of 0.5118, and a minimum of 0.3904. The evidence from Table 28 suggests that the majority of the OLS point estimates lie in the range 0.39 to 0.54.

Turning to the LAD estimates, for portfolios using the average return of the constituent firms, the average estimate of equity β, E($\tilde{\beta}$), was 0.5015 with a maximum estimate of 0.5318, and a minimum of 0.4647. For portfolios using the median return of the constituent firms, the average estimate of equity β was 0.4381 with a maximum estimate of 0.4830, and a minimum of 0.3856. The estimates are broadly equivalent across the three different samples. The evidence from Table 28 suggests that the majority of the LAD point estimates lie in the range 0.38 to 0.54. All the LAD estimates lie within the 95% OLS confidence interval for the relevant portfolio. This suggests that the information provided by the LAD estimates is consistent with the evidence provided by the OLS estimates.

Given the concerns about measurement error and bias due to structural instability, extreme caution should be placed on any interpretation of the estimates in Table 28. Taken together, the evidence from Table 28 suggests that the point estimates of equity β lie in the range 0.38 to 0.54. The evidence in Table 28 is, broadly speaking, consistent with the outcomes reported in Table 5.6 of Henry (2009). However, given the concerns about measurement error and/or structural instability in the estimates obtained for the time-varying portfolios, it is the strong view of the consultant that there is no reliable evidence to be gained about the value of β from this exercise.

Table 29 presents estimates of equity β for the time varying portfolios. It differs from Table 28 in that it uses monthly sampling and does not present LAD regression calculations as instructed by the AER.

Table 29: Delevered/relevered estimates of β *Time-varying portfolio – Average and median returns* *Three different sample periods – Monthly frequency*						
Sample period	Longest possible 29/08/97 to 28/06/13		Post tech, ex GFC 04/01/02 to 29/08/08 06/11/09 to 28/06/13		Last five years 04/07/08 to 28/06/13	
	Ave	Med	Ave	Med	Ave	Med
$\bar{G}$	0.5666	0.5666	0.5835	0.5835	0.6490	0.6490
ω	1.0835	1.0835	1.0413	1.0413	0.8774	0.8774
$\hat{\beta}$	0.4471	0.4673	0.4506	0.4514	0.3949	0.4482
s.e	0.0790	0.0719	0.0850	0.0837	0.1074	0.0826
$\hat{\beta}_u$	0.6019	0.6083	0.6172	0.6154	0.6054	0.6101
$\hat{\beta}_l$	0.2924	0.3263	0.2839	0.2873	0.1844	0.2862
N	190	190	122	122	59	59
R^2	0.1457	0.1833	0.1897	0.1951	0.1917	0.3405

Consider first the OLS estimates. For portfolios using the average return of the constituent firms, the average estimate of equity β, E($\hat{\beta}$), was 0.4309 with a maximum estimate of 0.4506, and a minimum of 0.3949. For portfolios using the median return of the constituent firms, the average estimate of equity β was 0.4556 with a maximum estimate of 0.4673, and a minimum of 0.4482. The evidence from Table 28 suggests that the majority of the OLS point estimates lie in the range 0.39 to 0.47.

Given the concerns about measurement error and bias due to structural instability, great caution should be placed on the interpretation of the estimates in Table 29. Taken together, the evidence from Table 29 suggests that the point estimates of equity β lie in the range 0.39 to 0.47. The evidence in Table 29 is, broadly speaking, consistent with the outcomes reported in Table 5.6 of Henry (2009). However, given the concerns about measurement error and/or structural instability in the estimates obtained for the time-varying portfolios, it is the strong view of the consultant that there is no reliable evidence to be gained about the value of β from this exercise.

Table 30 presents a summary of the equity β estimates from Tables 28–29. Note that when comparing this table to Table 18 (summary of fixed portfolios) the headings ‘Ave’ and ‘Med’ have different interpretations. Here, these labels refer to the measure of central tendency for individual firm returns within the portfolio. In Table 18, they refer to the method of aggregating results across multiple different portfolios.

Table 30: Delevered/relevered estimates of β

Summary of time-varying portfolio equity β point estimates from tables 28–29

Sample period		Longest possible		Post tech, ex GFC		Last five years	
		Ave	Med	Ave	Med	Ave	Med
Weekly	OLS	0.4954	0.4582	0.5304	0.5118	0.4336	0.3904
Weekly	LAD	0.5080	0.4456	0.5318	0.4830	0.4647	0.3856
Monthly	OLS	0.4471	0.4673	0.4506	0.4514	0.3949	0.4482

Taken together, it is the view of the consultant that there is no reliable evidence about the value of β presented in Tables 28-30.

3.7. Thin trading analysis: Time-varying portfolios

Table 31 presents estimates of Dimson's β for the time varying portfolios, using a weekly sampling frequency. The equity β estimates underlying this analysis were presented in Table 28.

Table 31: Dimson's β – Portfolios

Time-varying portfolio – Average and median returns

Three different sample periods – Weekly frequency

Sample period	Longest possible		Post tech, ex GFC		Last five years	
	29/08/97 to 28/06/13		04/01/02 to 29/08/08 06/11/09 to 28/06/13		04/07/08 to 28/06/13	
	Ave	Med	Ave	Med	Ave	Med
β_{i-1}	0.055	0.033	-0.038	-0.025	0.114	0.084
s.e	0.034	0.031	0.035	0.034	0.057	0.050
β_i	0.458	0.423	0.510	0.492	0.500	0.449
s.e	0.034	-0.025	0.035	0.033	0.058	0.051
β_{i+1}	-0.032	-0.024	0.003	0.002	-0.058	-0.047
s.e	0.034	0.031	0.035	0.033	0.057	0.051
β_i^D	0.482	0.432	0.475	0.468	0.556	0.486
s.e	0.059	0.055	0.061	0.058	0.105	0.092
β_i^{OLS}	0.457	0.423	0.509	0.492	0.494	0.445
s.e	0.034	0.031	0.035	0.033	0.057	0.050
$\beta_i^{OLS} = \beta_i^D$	-0.725	-0.288	0.968	0.707	-1.073	-0.809

Of the β_{i-1} and β_{i+1} coefficients in Table 31, only the β_{i-1} value for the average returns over the last five years is marginally significant. The results in Table 31 are consistent with the view that there is a paucity of evidence of thin trading in the sample of firms considered in this report. Moreover, none of the t-statistics used to test the hypothesis $H_0 : \hat{\beta}_i = \beta_i^D$ reported in Table 31 are statistically significant. This indicates that there is an absence of evidence of thin trading in this sample period. However, it is the strong view of the consultant that, in the light of the discussion regarding Tables 28-30, Table 31 is uninformative.

3.8. Stability and sensitivity analysis: Time-varying portfolios

The appendix presents recursive estimates of β_i for the time–varying portfolio using the longest available sample period. It is the view of the consultant that Figures 3.1 – 3.4 in the appendix are entirely uninformative.

Table 32 presents the Hansen (1992) Structural Stability Tests for the time-varying portfolios presented in Table 28. The Hansen test has not been calculated for the non-contiguous sample (post-tech boom and ex-GFC) for the reasons set out above.

Table 32: Hansen (1992) Structural Stability Tests

Time-varying portfolio – Weekly frequency

	Longest possible sample 29/08/97 to 28/06/13		Last five years sample 04/07/08 to 28/06/13	
	Ave	Med	Ave	Med
Joint	0.05	0.00	0.01	0.00
σ^2	0.04	0.00	0.01	0.00
α	0.42	0.28	0.21	0.25
β	0.10	0.80	0.11	0.36

With the exception of the Average portfolio in the longest possible sample, the joint tests reject the null of parametric stability for all portfolios. This rejection is as a result of instability of the residual variance, σ_i^2, which may reflect changes in the estimated level of portfolio-specific risk. However, the rejection of the null of stability of the residual variance, σ_i^2, may also be a function of the potential measurement error associated with the construction of the Average and Median portfolios. However, it is the view of the consultant that, in the light of the discussion regarding Tables 28-31, Table 32 is uninformative.

4. Summary of advice

The following is a brief set of conclusions that the consultant has drawn from working with the data described in this document and in the reports submitted in 2008 and 2009.

4.1. Sampling Frequency

Henry (2009) advises the use of data sample at a weekly frequency. There is a tradeoff between the noisy nature of the daily data and the lack of degrees of freedom in monthly data. The consultant has no reason to alter this advice.

4.2. Construction of Returns

Henry (2009) advised that it is usual to employ continuously compounded returns. There was no evidence that β estimates obtained from discretely compounded data are manifestly different. Henry (2009) advised the use of raw as opposed to excess returns. The consultant has no reason to alter this advice.

4.3. Parameter Instability

There is no overwhelming issue with instability. The OLS and LAD estimates of β differ. However this difference in the point estimates of β is almost universally statistically insignificant.

Neither of the recursive least squares estimators appears to demonstrate convincing evidence of parameter instability. It is important to note that these estimators are not sufficient in the sense that they do not employ all available information. The use of the Hansen (1992) test for parameter instability produces evidence of instability in the regression models. Where this instability is detected it is almost uniformly due to a change in the error variance in the regression model. There is no evidence of parameter instability associated with the coefficients of the regression models themselves. This evidence is largely consistent with the view that asset specific volatility may have been unstable during the periods examined by the consultant in this report and Henry (2008), and Henry (2009).

4.4. Summary of advice on estimation of β

In terms of the sample period, it is the view of the consultant that the most appropriate approach is to use all available data. The purpose of this exercise is to obtain accurate estimates of the level of risk which is not diversifiable in each of the equities of interest. To omit data because of concerns about instability is only correct where there is strong evidence of instability. In this report there is little evidence of instability in the intercept or slope of the Security Market Line estimated using the full sample. This means that there is little or no reason to omit data and/or partition the sample. The consultant is of the opinion that the most reliable evidence about the magnitude of β is provided in Tables 2, 14 and 16 using individual assets and fixed weight portfolios.

The 'time-varying portfolios' are not well founded in financial theory. A portfolio is a linear combination of assets. The 'time-varying portfolios' do not satisfy this definition. Moreover, there is likely to be measurement error in the returns to the 'time-varying portfolios' arising variation in the weights and from co-variation between the returns to the constituent assets in the portfolio and the weights. In the opinion of the consultant there is no reliable evidence to be gained from the analysis of such 'time-varying portfolios'.

4.5. Magnitude of β

In the opinion of the consultant, the majority of the evidence presented in this report, across all estimators, firms and portfolios, and all sample periods considered, suggests that the point estimate for β lies in the range 0.3 to 0.8. Given the differences in sample periods and sizes underlying the various individual estimates provided in Tables 2, 14 and 16 using individual assets and fixed weight portfolios it is difficult to pin down a value for the beta of a typical firm, however within the range 0.3 to 0.8 the average of the OLS estimates for the individual firms reported in Table 2 is 0.5223 while the median estimate is 0.3285

Appendix: Recursive Estimation

This appendix presents recursive estimates of β_i for each of the firms and portfolios discussed in above. Two estimation strategies are employed using a moving window with a fixed width of 1 year of data and an expanding window with initial width of 1 year of data. The results are, in general, remarkably similar. First, irrespective of the construction of the recursion, the evidence for each firm or portfolio is consistent. Second, there is only weak visual evidence of time variation in the estimates of β_i across the plots in the Appendix. That is, there are no occasions when the recursive estimates display sudden substantial jumps across all the cases considered. Moreover, there is no systematic evidence of regression to unity. In short, there is no strong evidence of instability in the estimate of β.

1. Recursive Estimates of β: Individual firms

These recursive estimates are generated using the longest available sample for each stock.

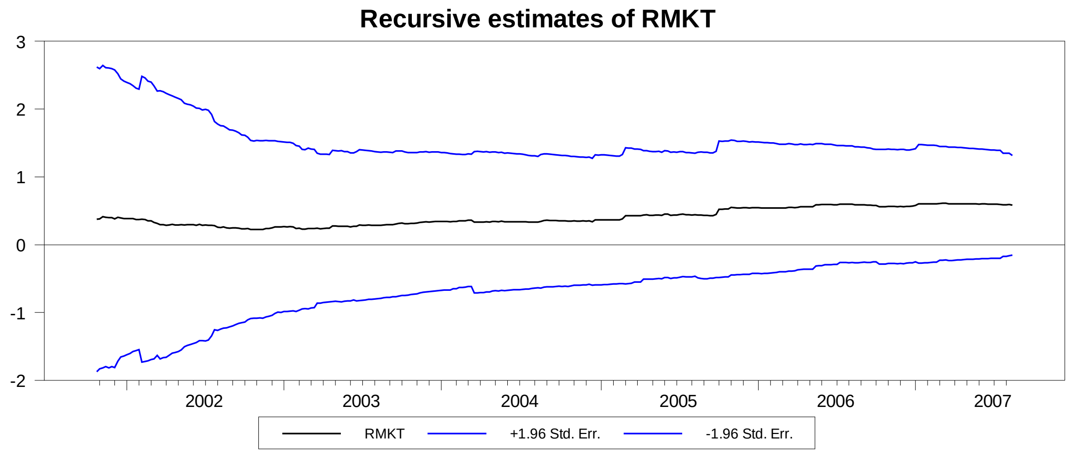

Figure 1.1: AAN 20/10/2000 – 17/08/2007

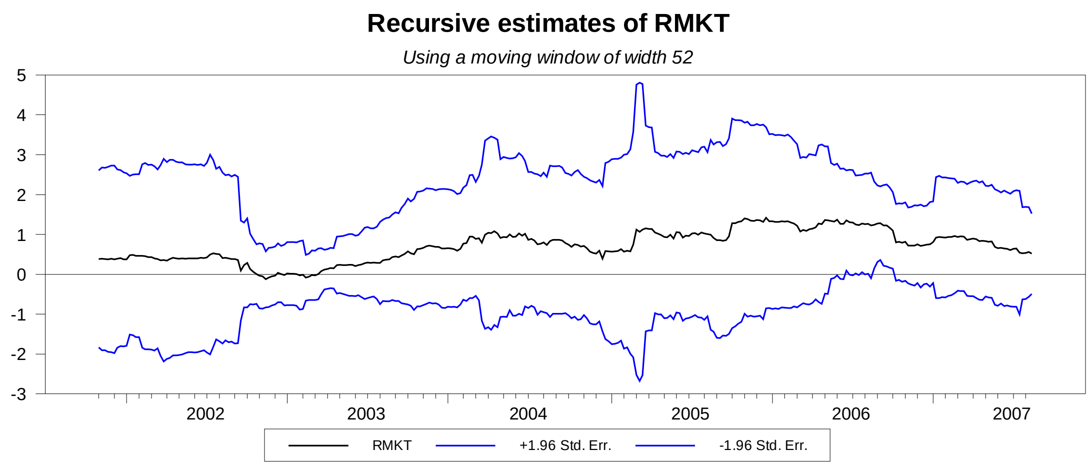

Figure 1.2: AAN 20/10/2000 – 17/08/2007

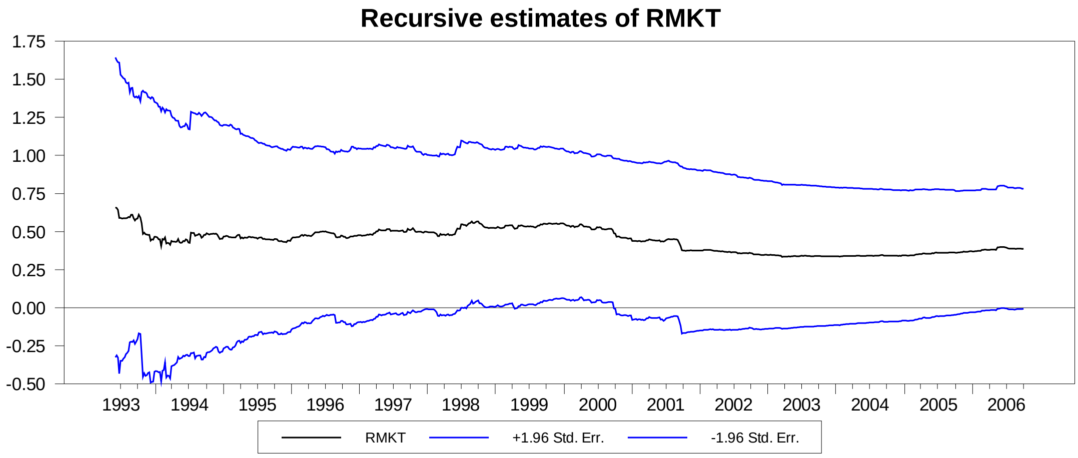

Figure 1.3: AGL 29/05/1992 to 06/10/2006

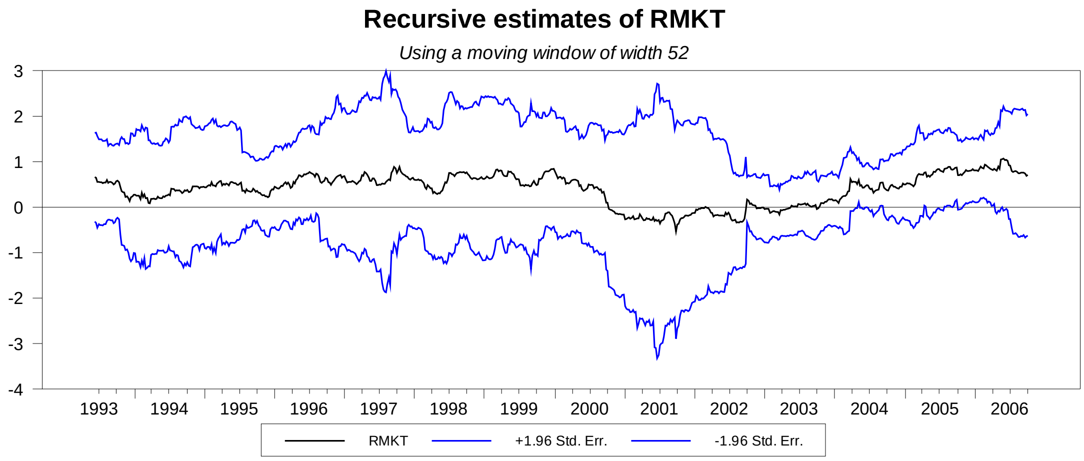

Figure 1.4: AGL 29/05/1992 to 06/10/2006

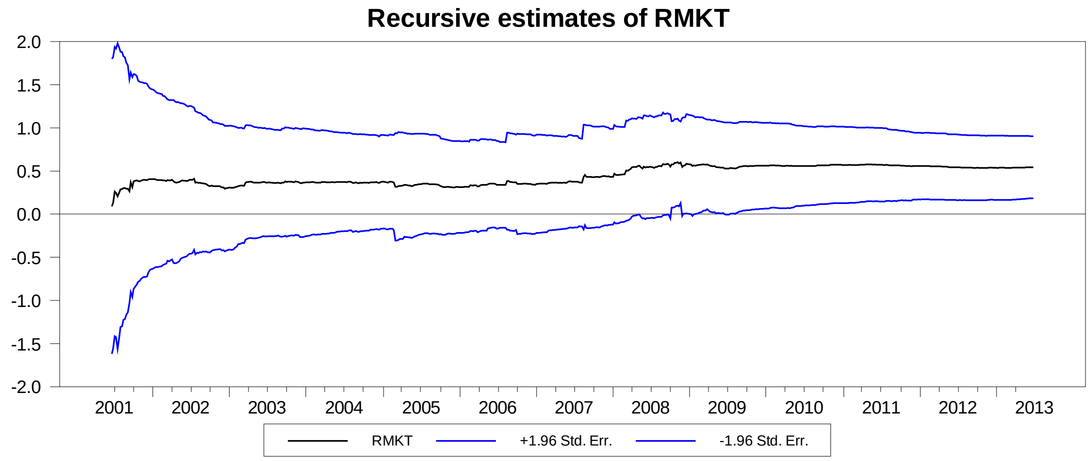

Figure 1.5 APA 16/6/2000 to 28/06/2013

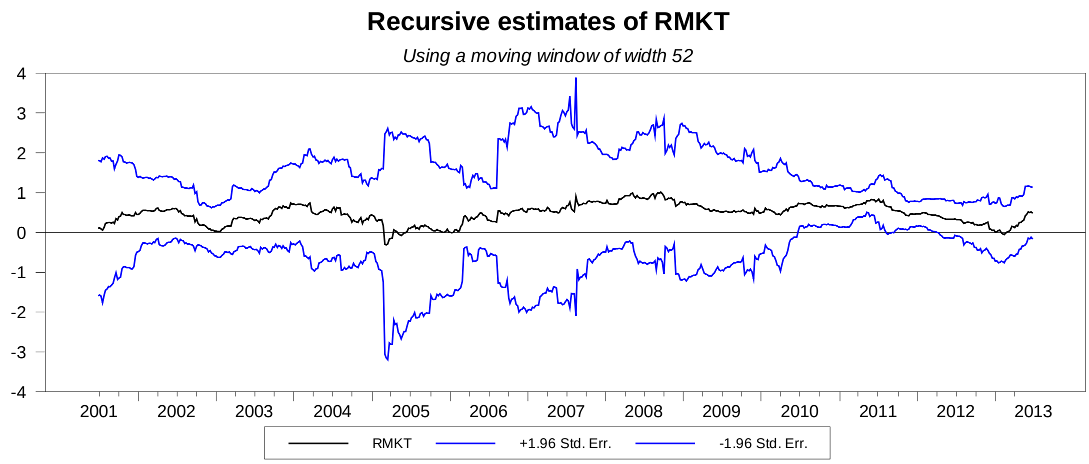

Figure 1.6 APA 16/6/2000 to 28/06/2013

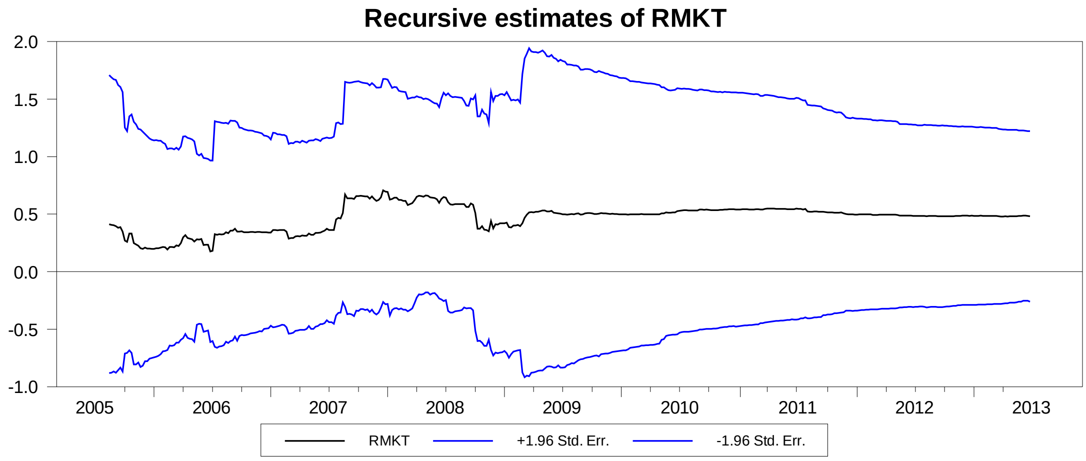

Figure 1.7 DUE 13/08/2004 to 28/06/2013

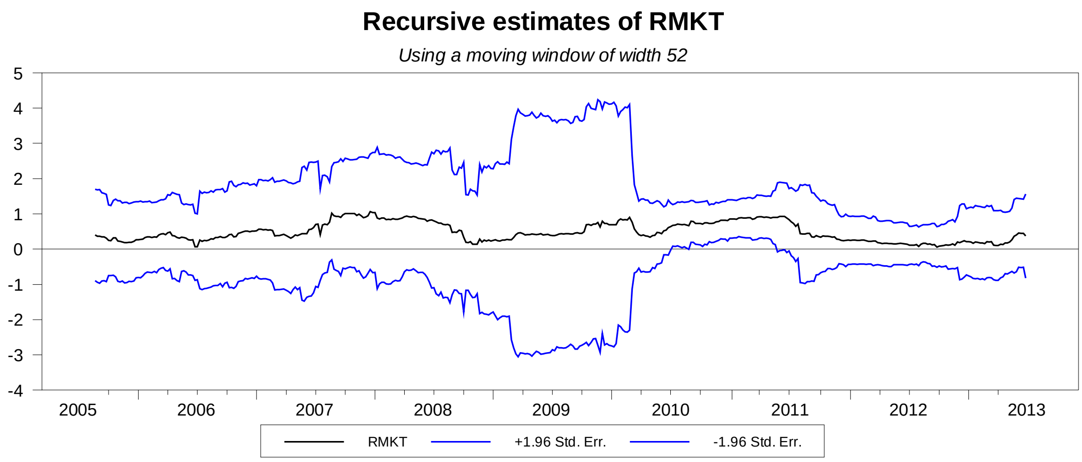

Figure 1.8 DUE 13/08/2004 to 28/06/2013

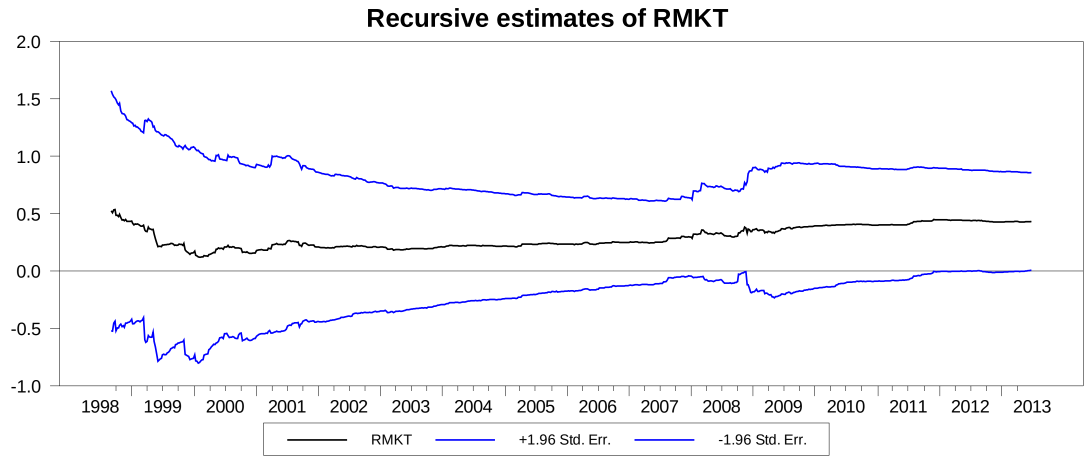

Figure 1.9 ENV 29/08/1997 to 28/06/2013

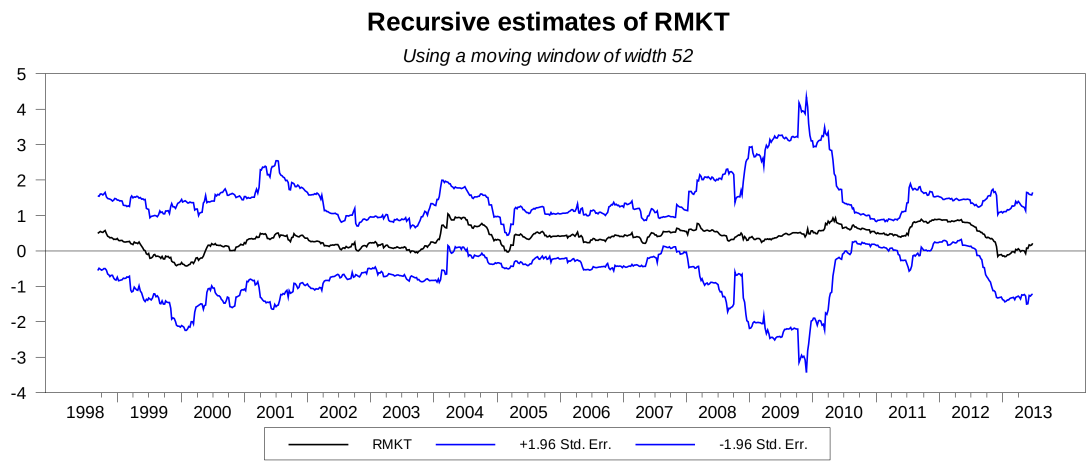

Figure 1.10 ENV 29/08/1997 to 28/06/2013

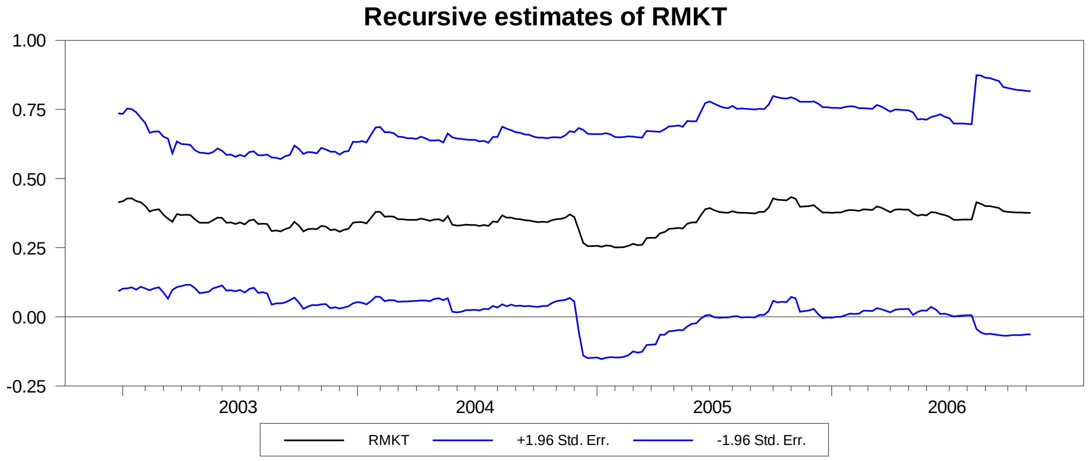

Figure 1.11: GAS 17/12/2001 to 10/11/2006

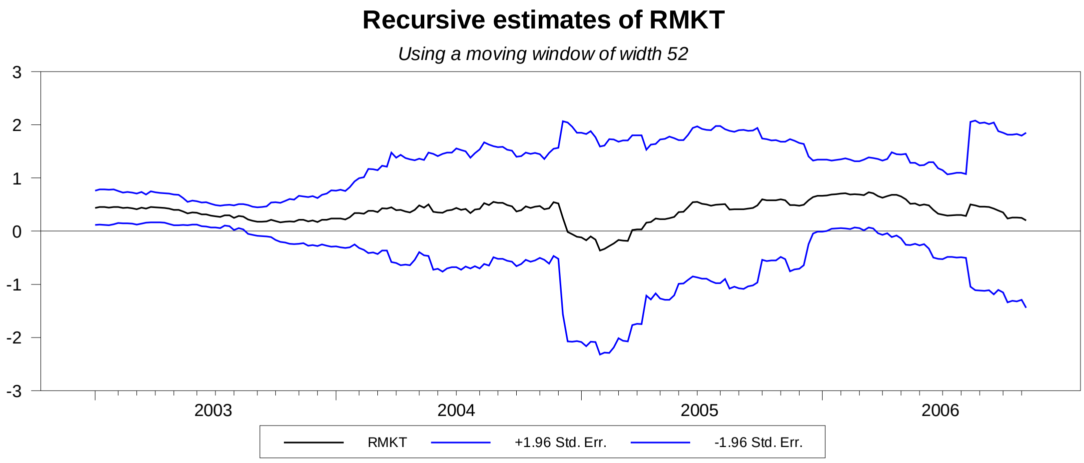

Figure 1.12: GAS 17/12/2001 to 10/11/2006

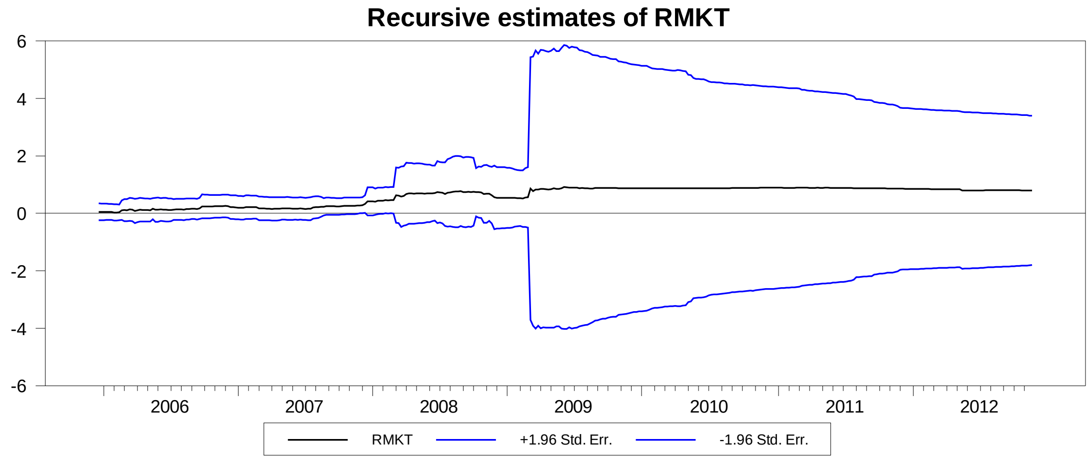

Figure 1.13 HDF 17/12/2004 to 23/11/2012

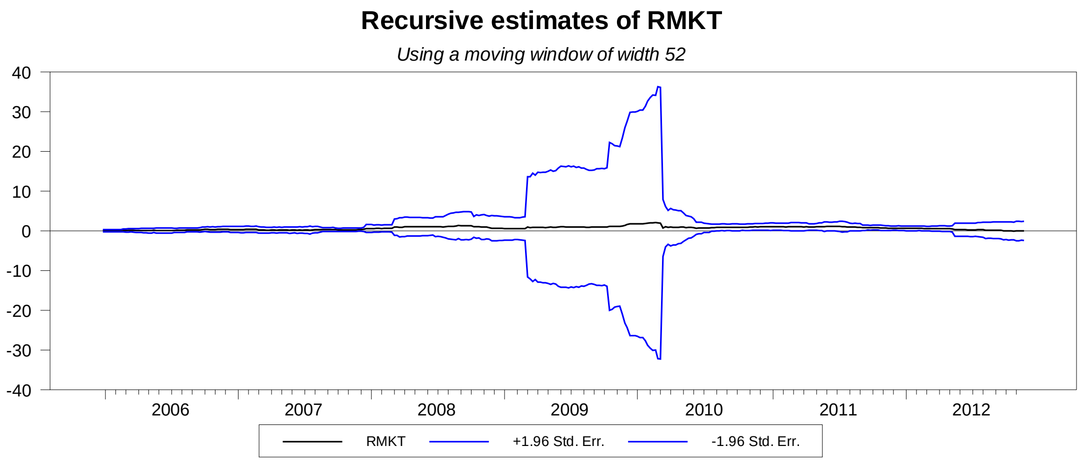

Figure 1.14 HDF 17/12/2004 to 23/11/2012

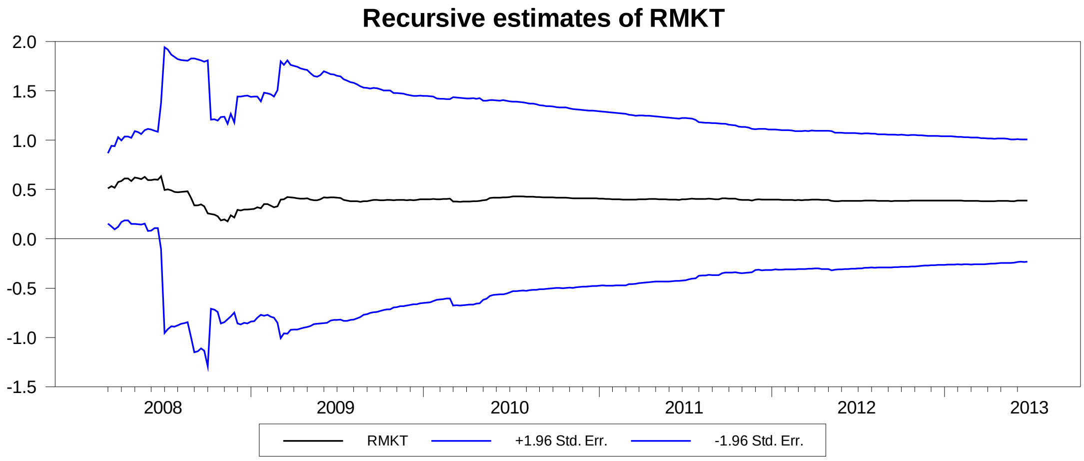

Figure 1.15: SKIX 02/03/2007 to 28/06/2013

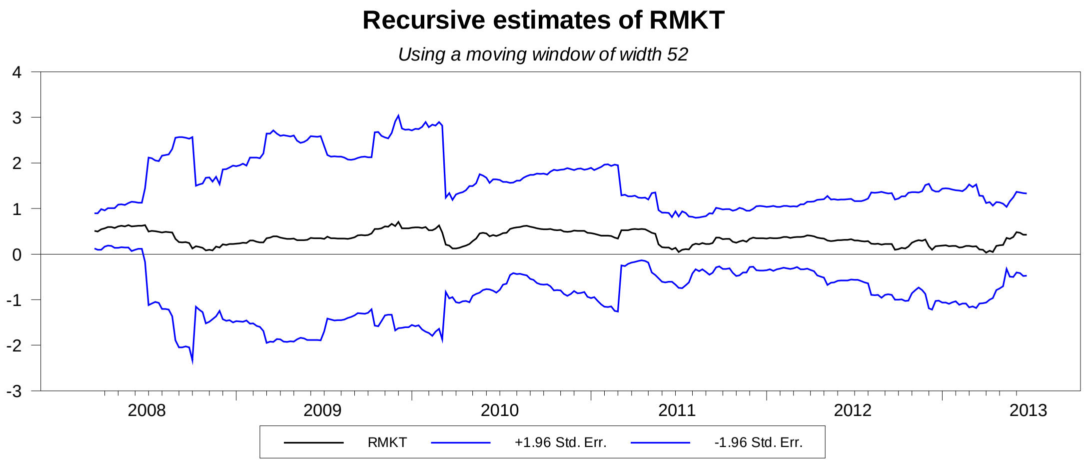

Figure 1.16: SKIX 02/03/2007 to 28/06/2013

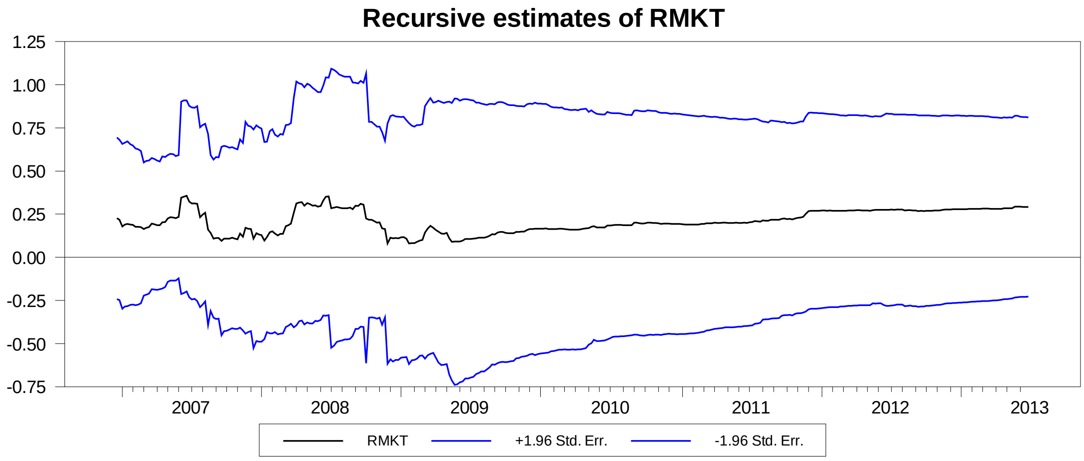

Figure 1.17: SPN 16/12/2005 to 28/06/2013

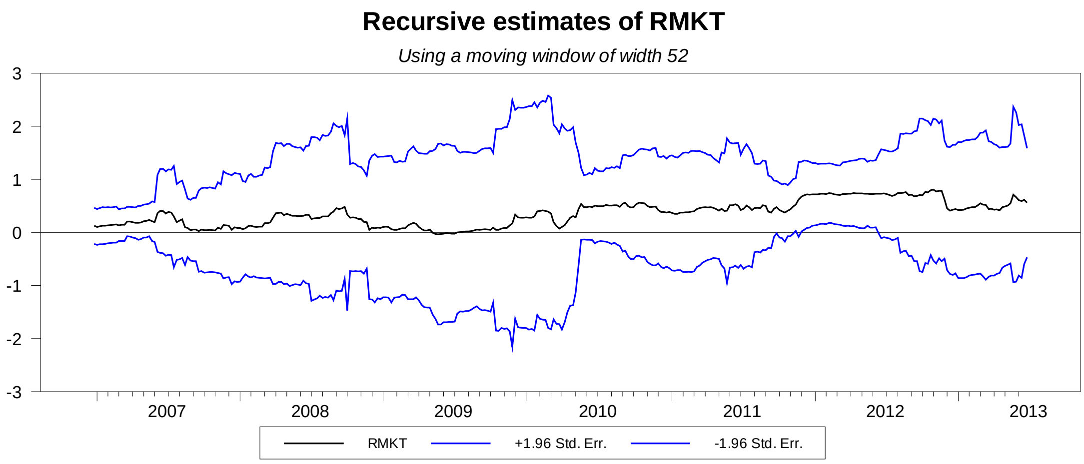

Figure 1.18: SPN 16/12/2005 to 28/06/2013

2. Recursive Estimates of β: Portfolios

These recursive estimates are generated using the longest available sample for each fixed weight portfolio.

2.1. Fixed weight portfolios: Equal weighted

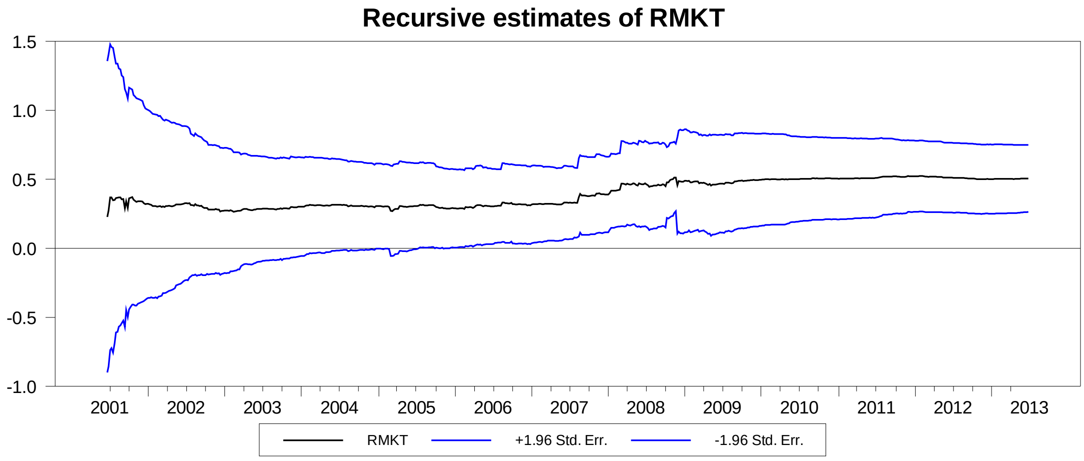

Figure 2.1: P1: 16/6/2000 to 28/6/2013

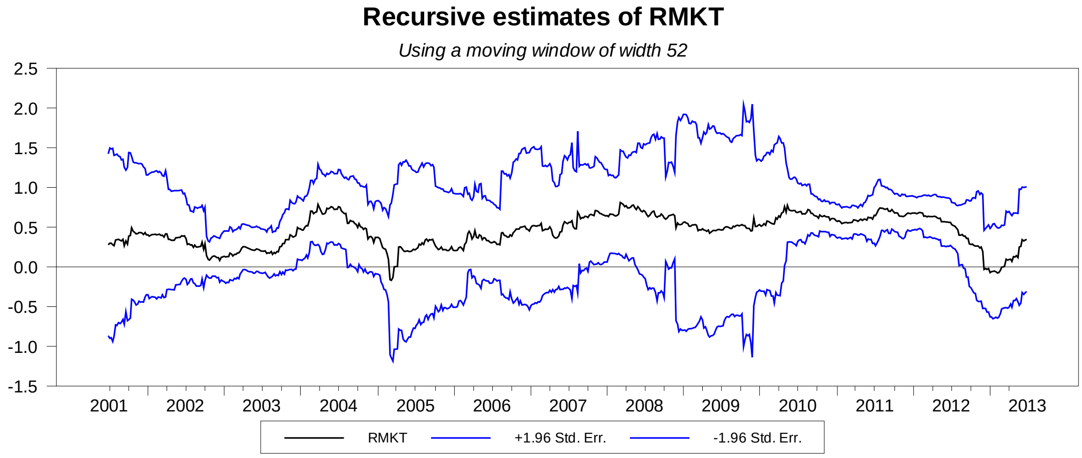

Figure 2.2: P1: 16/6/2000 to 28/6/2013

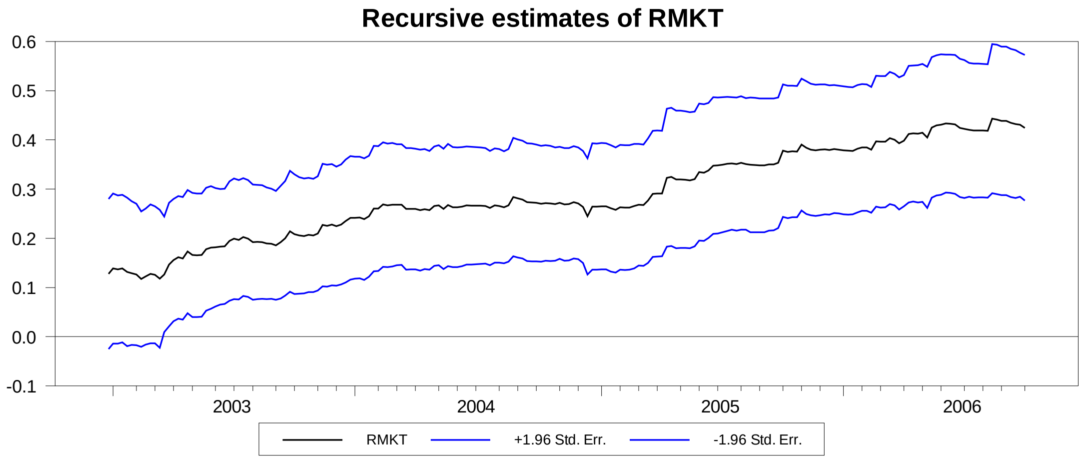

Figure 2.3: P2: 21/12/2001 to 06/10/2006

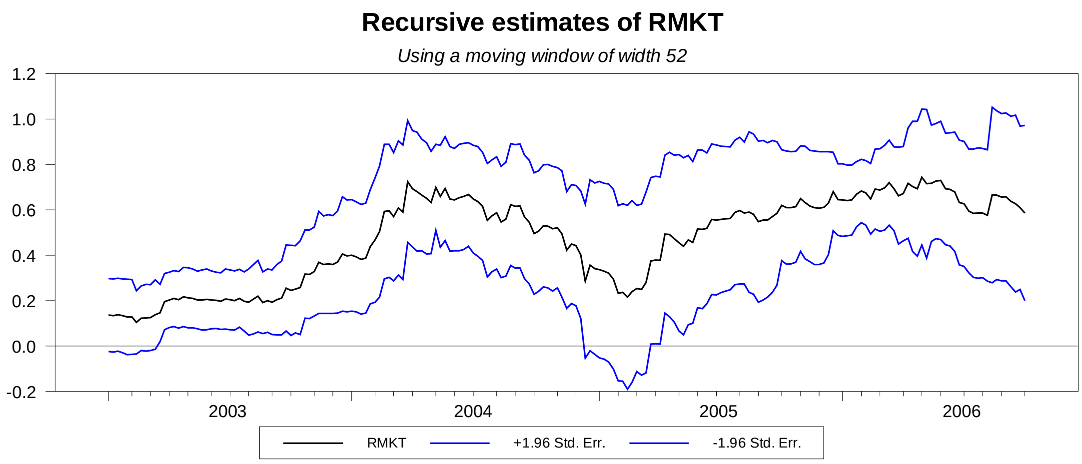

Figure 2.4: P2: 21/12/2001 to 06/10/2006

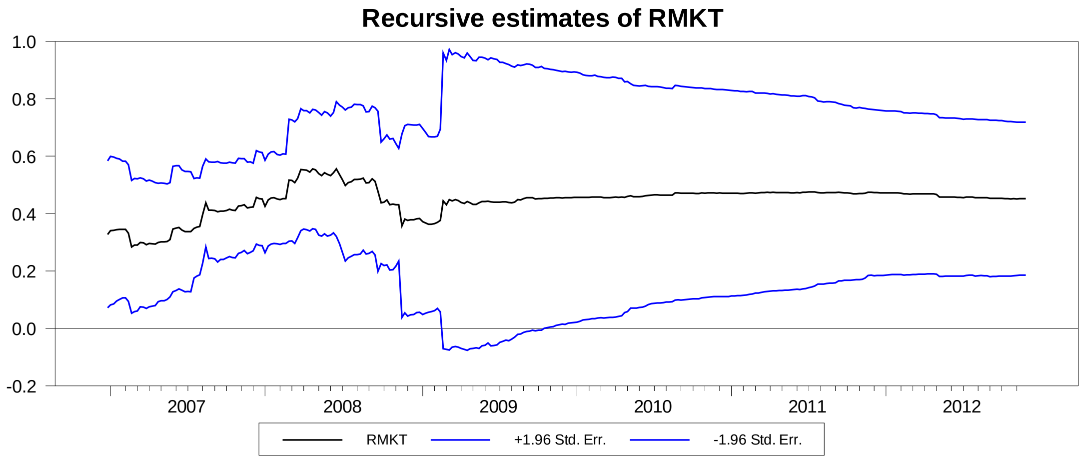

Figure 2.5: P3: 16/12/2005 – 23/11/2012

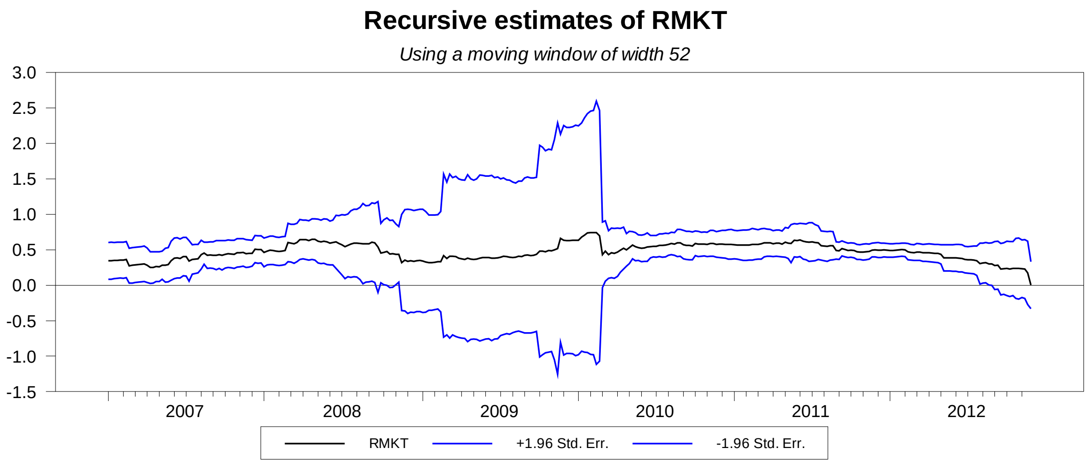

Figure 2.6: P3: 16/12/2005 – 23/11/2012

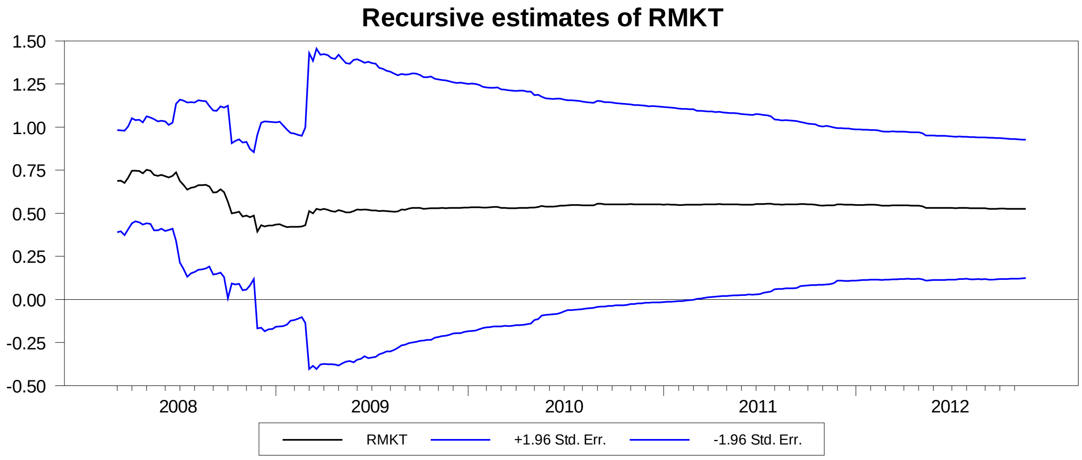

Figure 2.7: P4: 02/03/2007 to 23/11/2012

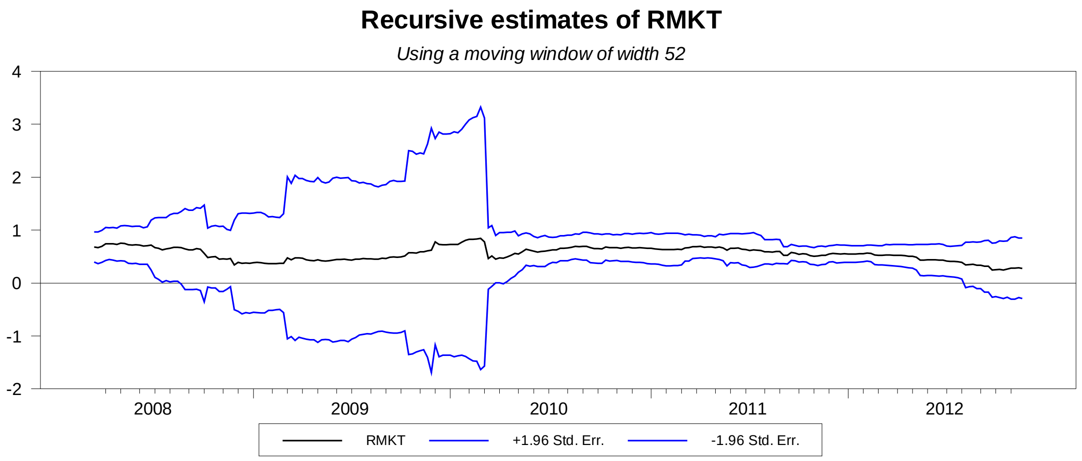

Figure 2.8: P4: 02/03/2007 to 23/11/2012

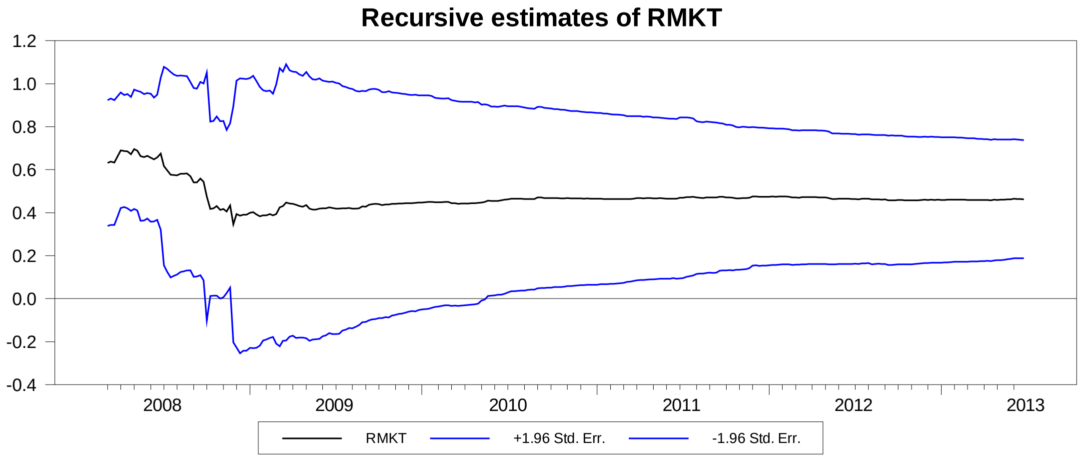

Figure 2.9: P5: 02/03/2007 to 28/06/2013

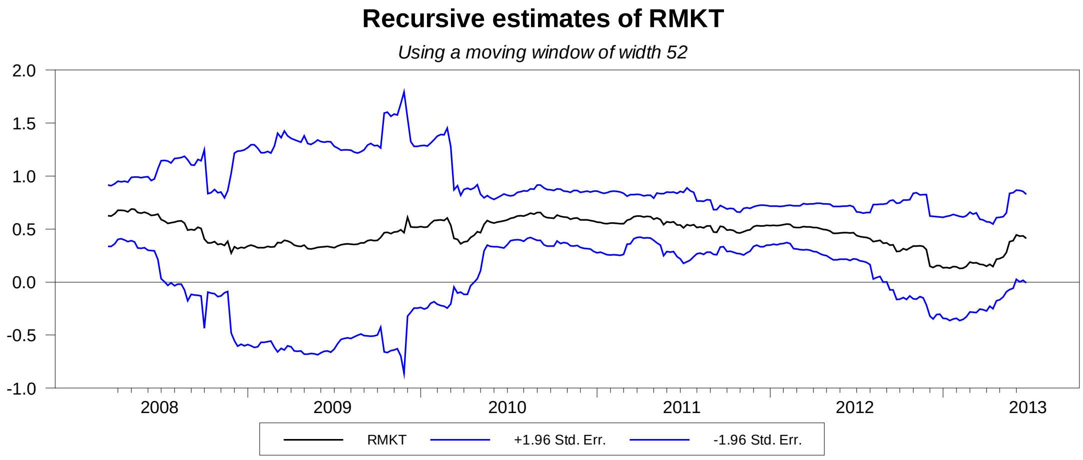

Figure 2.10: P5: 02/03/2007 to 28/06/2013

2.2. *Fixed weight portfolios: Value weighted*

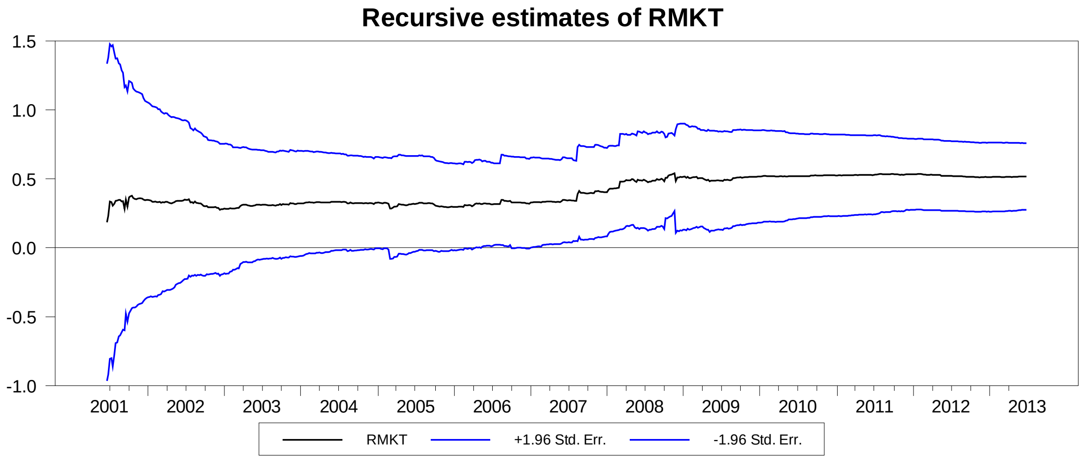

Figure 2.11: P1 – Value Weight 16/6/2000 – 28/6/2013

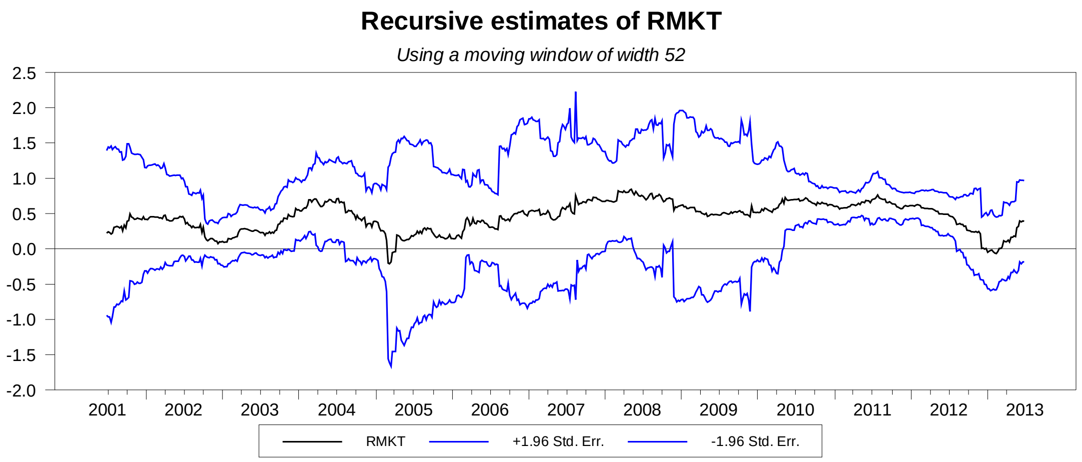

Figure 2.12 P1 – Value Weight 16/6/2000 – 28/6/2013

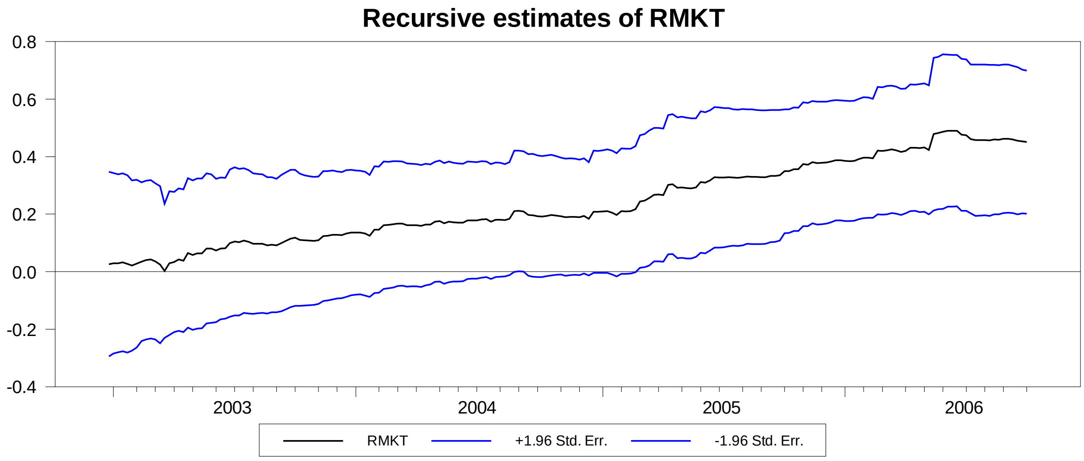

Figure 2.13 P2 – Value Weight 21/12/2001 – 06/10/2006

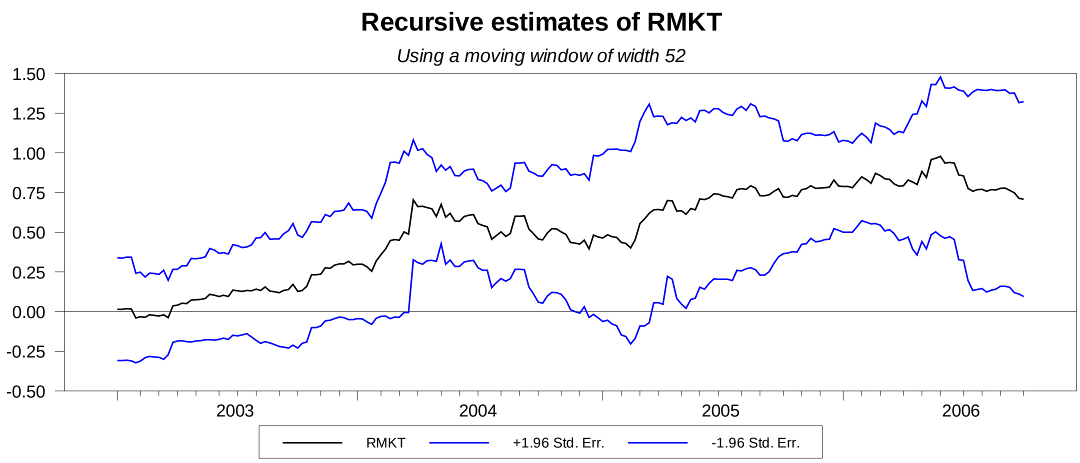

Figure 2.14 P2 – Value Weight 21/12/2001 – 06/10/2006

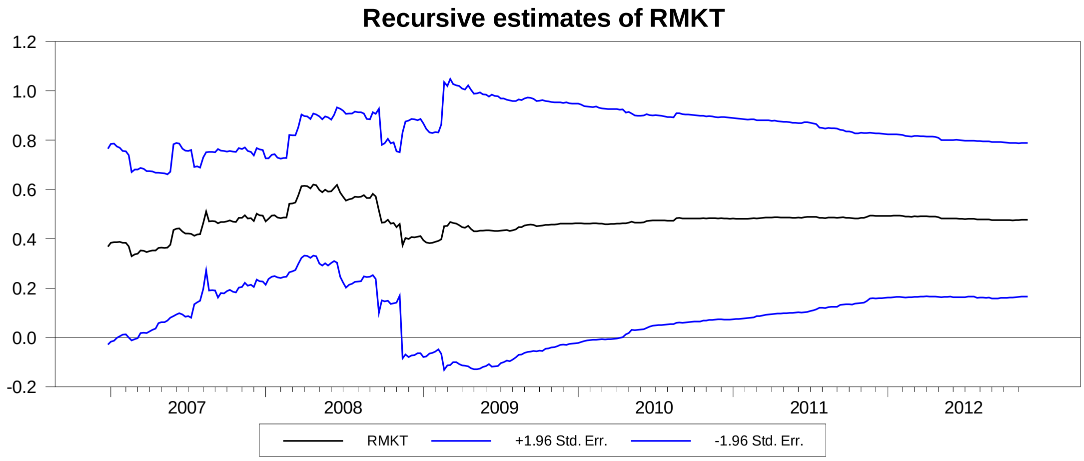

Figure 2.15 P3 – Value Weight 16/12/2005 – 23/11/2012

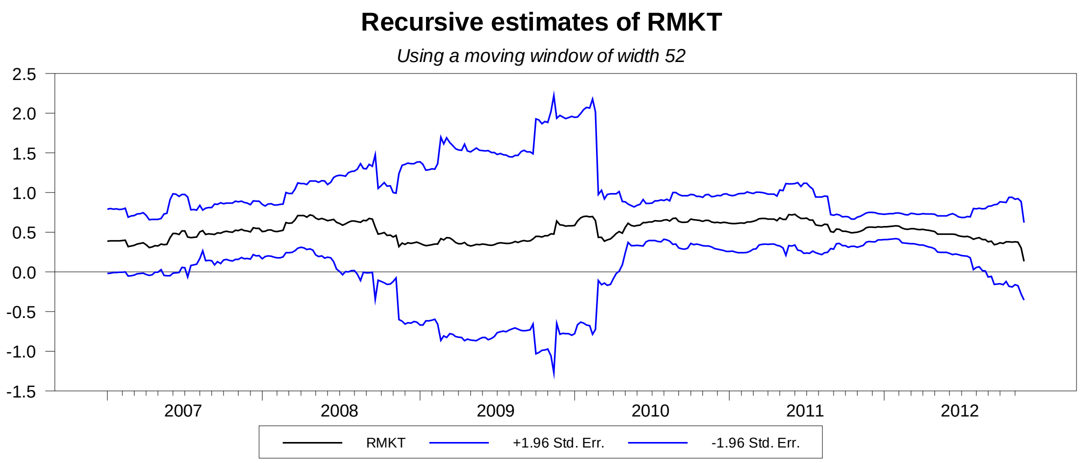

Figure 2.16 P3 – Value Weight 16/12/2005 – 23/11/2012

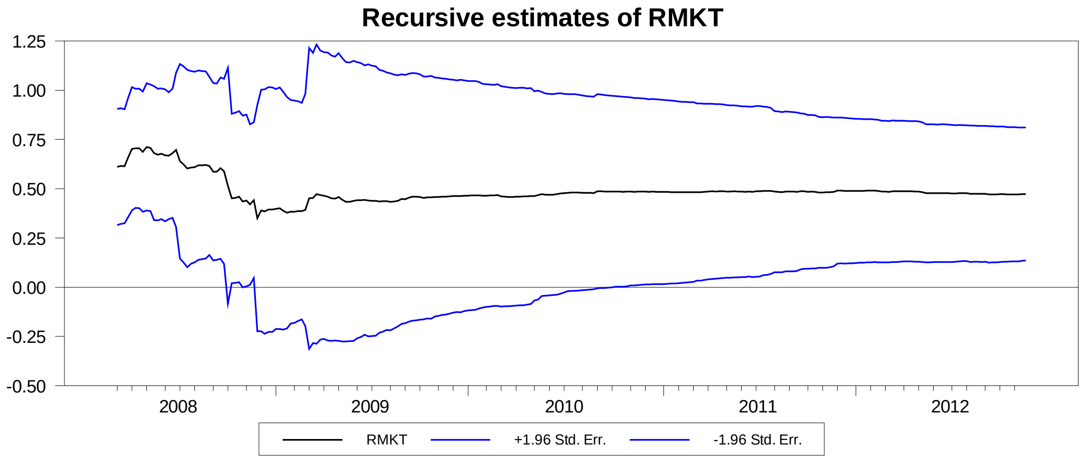

Figure 2.17 P4 – Value Weight 02/03/2007 – 23/11/2012

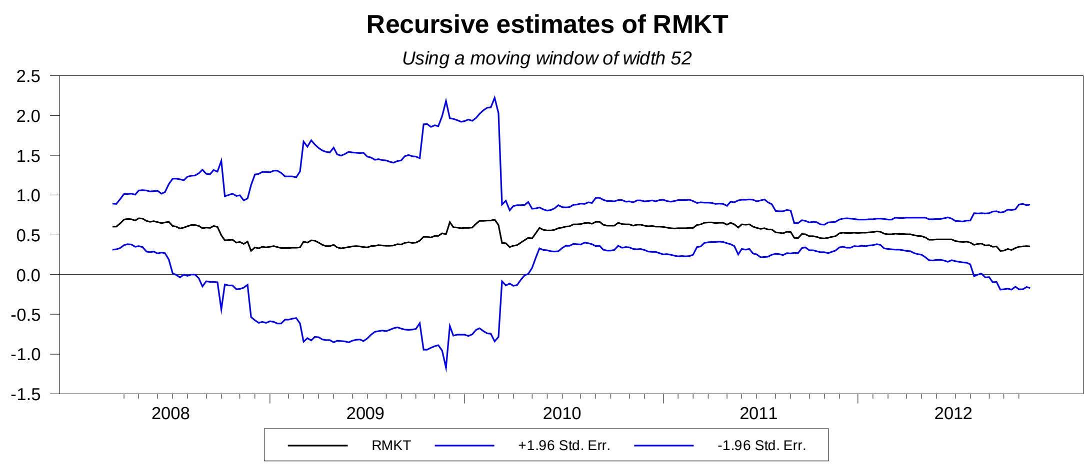

Figure 2.18 P4 – Value Weight 02/03/2007 – 23/11/2012

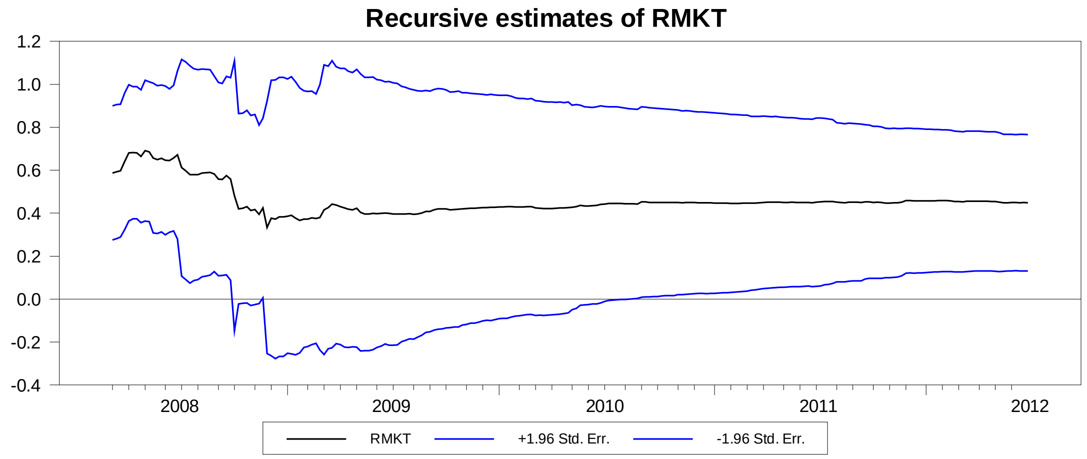

Figure 2.19 P5 – Value Weight 02/03/2007 – 28/06/2013

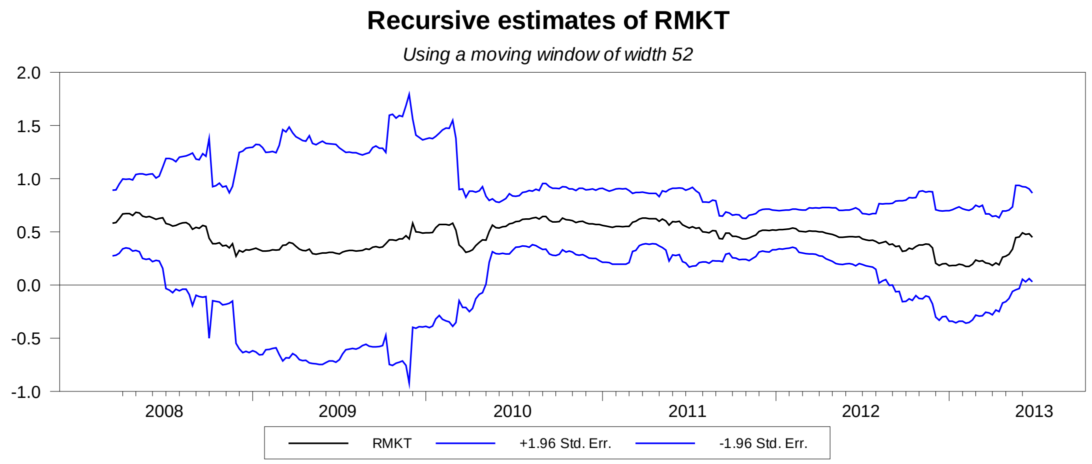

Figure 2.19 P5 – Value Weight 02/03/2007 – 28/06/2013

2.3. Time–varying portfolios

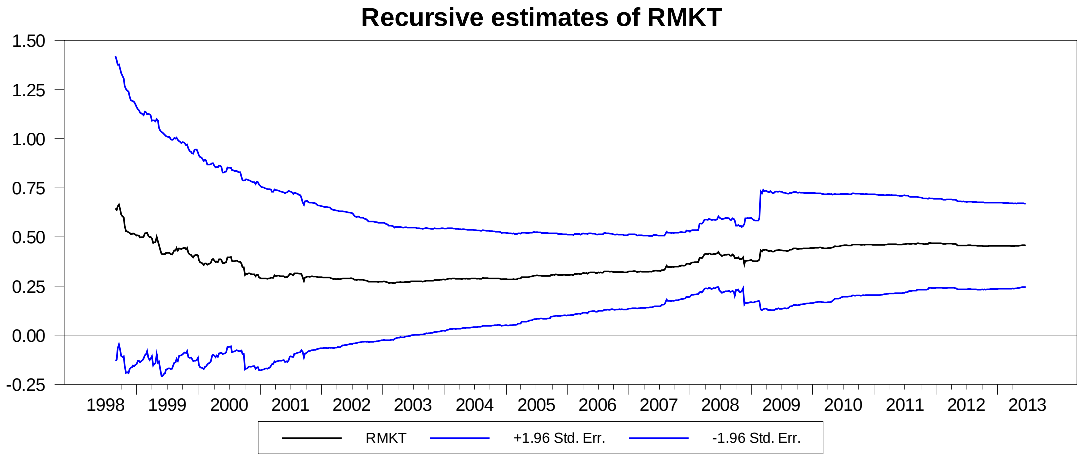

Figure 3.1: Average Portfolio 29/08/1997 – 28/06/2013

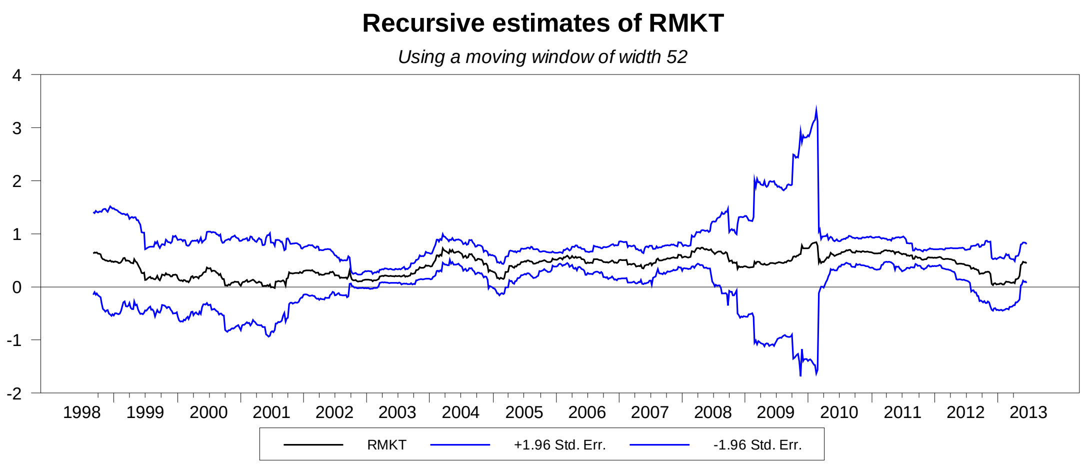

Figure 3.2: Average Portfolio 29/08/1997 – 28/6/2013

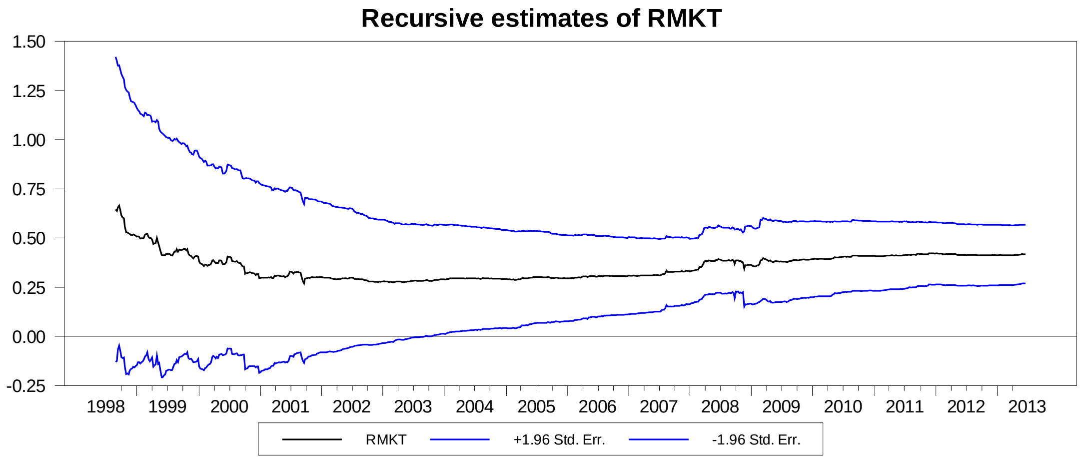

Figure 3.3: Median Portfolio 29/08/1997 – 28/6/2013

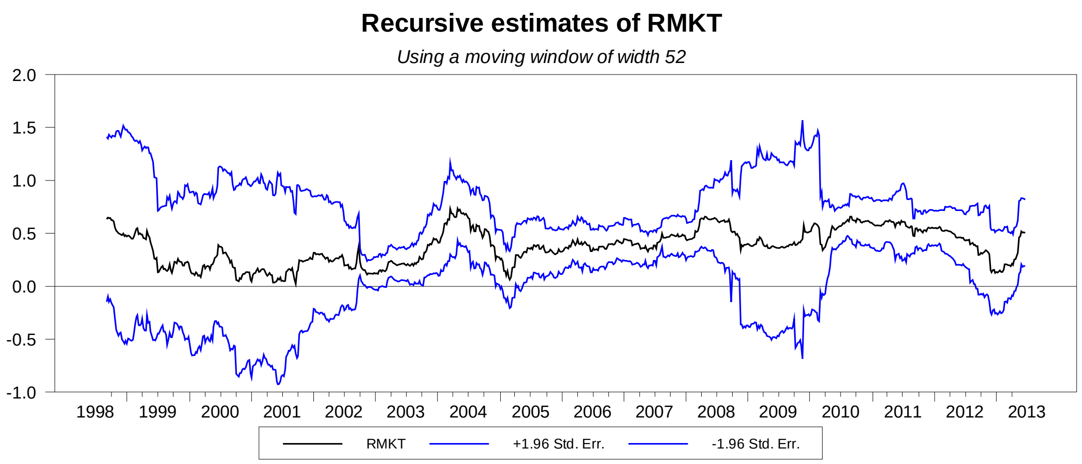

Figure 3.4: Median Portfolio 29/08/1997 – 28/6/2013

Appendix: Unadjusted betas

This appendix presents estimates of equity β that have not been delevered/relevered to the benchmark gearing (60 per cent) for the individual equity issues. The results are presented for Tables 2-4 (the individual equity) and 14-17 (the fixed weight portfolios) in the report. As no weight is placed on the 'time-varying portfolios' the raw results are not presented in this appendix.

Table A1: Raw estimates of β for Table 2

Longest sample available – Weekly frequency

	AAN	AGL	APA	DUE	ENV	GAS	HDF	SKI	SPN
Start	20/10/00	29/05/92	16/06/00	13/08/04	29/08/97	21/12/01	17/12/04	02/03/07	16/12/05
End	17/08/07	06/10/06	28/06/13	28/06/13	28/06/13	10/11/06	23/11/12	28/06/13	28/06/13
$\hat{\beta}$	0.5818	0.3874	0.5418	0.4813	0.4331	0.3761	0.7979	0.3869	0.2914
s.e	0.1057	0.0629	0.0512	0.0699	0.0517	0.1014	0.1203	0.0726	0.0632
$\hat{\beta}_u$	0.7889	0.5106	0.6423	0.6183	0.5344	0.5749	1.0336	0.5293	0.4152
$\hat{\beta}_l$	0.3747	0.2642	0.4414	0.3444	0.3318	0.1773	0.5622	0.2445	0.1676
$\tilde{\beta}$	0.3969	0.4091	0.5388	0.3641	0.4055	0.2699	0.5471	0.3763	0.2885
s.e	0.1064	0.0629	0.0513	0.0702	0.0517	0.1021	0.1209	0.0727	0.0632
$\tilde{\beta}_u$	0.6053	0.5324	0.6392	0.5016	0.5069	0.4699	0.7841	0.5187	0.4123
$\tilde{\beta}_l$	0.1884	0.2858	0.4383	0.2266	0.3042	0.0698	0.3102	0.2339	0.1647
N	356	749	680	463	826	255	414	330	393
R^2	0.0789	0.0484	0.1416	0.0933	0.0785	0.0516	0.0965	0.0796	0.0516

Table A2: Raw estimates of β for Table 3

Sample from 2002 to present, excluding GFC – Weekly frequency

	AAN	AGL	APA	DUE	ENV	GAS	HDF	SKI	SPN
Start	04/01/02	04/01/02	04/01/02	13/08/04	04/01/02	04/01/02	17/12/04	02/03/07	16/12/05
End	17/08/07	06/10/06	29/08/08	29/08/08	29/08/08	10/11/06	29/08/08	29/08/08	29/08/08
Start			06/11/09	06/11/09	06/11/09		06/11/09	06/11/09	06/11/09
End			28/06/13	28/06/13	28/06/13		12/11/12	28/06/13	28/06/13
$\hat{\beta}$	0.6587	0.4257	0.5664	0.4975	0.4791	0.3797	0.6893	0.3873	0.4592
s.e	0.1220	0.0958	0.0628	0.0702	0.0581	0.1019	0.0842	0.0880	0.0735
$\hat{\beta}_u$	0.8977	0.6135	0.6896	0.6352	0.5929	0.5795	0.8543	0.5597	0.6033
$\hat{\beta}_l$	0.4196	0.2380	0.4433	0.3598	0.3653	0.1800	0.5243	0.2149	0.3151
$\tilde{\beta}$	0.4059	0.3316	0.4761	0.4281	0.4395	0.2699	0.5514	0.4665	0.4572
s.e	0.1231	0.0961	0.0630	0.0704	0.0581	0.1026	0.0846	0.0881	0.0735
$\tilde{\beta}_u$	0.6471	0.5200	0.5995	0.5660	0.5534	0.4710	0.7172	0.6392	0.6014
$\tilde{\beta}_l$	0.1647	0.1432	0.3526	0.2902	0.3257	0.0687	0.3855	0.2938	0.3131
N	293	248	537	401	537	253	352	268	331
R^2	0.0911	0.0743	0.1319	0.1117	0.1128	0.0524	0.1607	0.0679	0.1060

Table A3: Raw estimates of β for Table 4

Last five years sample – Weekly frequency

	AAN	AGL	APA	DUE	ENV	GAS	HDF	SKI	SPN
Start	*NA*	*NA*	04/07/08	04/07/08	04/07/08	*NA*	04/07/08	04/07/08	04/07/08
End			28/06/13	28/06/13	28/06/13		12/11/12	28/06/13	28/06/13
$\hat{\beta}$			0.5427	0.4295	0.5218		0.8247	0.3647	0.2939
s.e			0.0670	0.0913	0.0898		0.1767	0.0757	0.0752
$\hat{\beta}_u$			0.6740	0.6084	0.6977		1.1710	0.5130	0.4414
$\hat{\beta}_l$			0.4114	0.2506	0.3459		0.4784	0.2163	0.1465
$\tilde{\beta}$			0.5415	0.3813	0.5453		0.6288	0.2627	0.4371
s.e			0.0670	0.0915	0.0900		0.1772	0.0761	0.0760
$\tilde{\beta}_u$			0.6728	0.5606	0.7217		0.9761	0.4118	0.5860
$\tilde{\beta}_l$			0.4102	0.2020	0.3689		0.2816	0.1136	0.2883
N			260	260	260		229	260	260
R^2			0.2028	0.0791	0.1158		0.0876	0.0825	0.0559

Table A4: Raw estimates of β for Table 14

Fixed portfolio– Equal weighting

Longest sample available – Weekly frequency

	P1	P2	P3	P4	P5
Firms	APA ENV	AAN AGL APA ENV GAS	APA DUE ENV HDF SPN	APA DUE ENV HDF SKI SPN	APA DUE ENV SKI SPN
Start	16/06/2000	21/12/2001	16/12/2005	02/03/2007	02/03/2007
End	28/06/2013	06/10/2006	23/11/2012	23/11/2012	28/06/2013
$\hat{\beta}$	0.5059	0.4245	0.5424	0.5260	0.4626
s.e	0.0392	0.0576	0.0504	0.0540	0.0456
$\hat{\beta}_{\text{u}}$	0.5828	0.5374	0.6412	0.6319	0.5520
$\hat{\beta}_{\text{l}}$	0.4290	0.3116	0.4437	0.4201	0.3733
$\tilde{\beta}$	0.5068	0.3345	0.5515	0.5514	0.5205
s.e	0.0393	0.0581	0.0505	0.0541	0.0458
$\tilde{\beta}_{\text{u}}$	0.5838	0.4484	0.6506	0.6575	0.6103
$\tilde{\beta}_{\text{l}}$	0.4298	0.2207	0.4525	0.4453	0.4308
N	680	250	362	299	330
R^2	0.1968	0.1796	0.2436	0.2419	0.2389

Table A5: Raw estimates of β for Table 15

Fixed portfolio– Equal weighting

Longest sample available but excluding tech boom and GFC – Weekly frequency

	P1	P2	P3	P4	P5
Firms	APA ENV	AAN AGL APA ENV GAS	APA DUE ENV HDF SPN	APA DUE ENV HDF SKI SPN	APA DUE ENV SKI SPN
Start	16/06/2000	21/12/2001	16/12/2005	02/03/2007	02/03/2007
End	28/06/2013	06/10/2006	23/11/2012	23/11/2012	28/06/2013
$\hat{\beta}$	0.5228	0.4253	0.5751	0.5672	0.5235
s.e	0.0445	0.0579	0.0474	0.0528	0.0493
$\hat{\beta}_u$	0.6100	0.5389	0.6680	0.6707	0.6201
$\hat{\beta}_l$	0.4355	0.3118	0.4822	0.4637	0.4268
$\tilde{\beta}$	0.4761	0.3119	0.5788	0.5978	0.5832
s.e	0.0446	0.0585	0.0475	0.0529	0.0495
$\tilde{\beta}_u$	0.5635	0.4265	0.6719	0.7015	0.6802
$\tilde{\beta}_l$	0.3887	0.1972	0.4856	0.4940	0.4862
N	537	248	300	237	268
R^2	0.2049	0.1797	0.3307	0.3292	0.2976

Table A6:Raw estimates of β for Table 16

Fixed portfolio– Value weighting

Longest sample available – Weekly frequency

	P1	P2	P3	P4	P5
Firms	APA ENV	AAN AGL APA ENV GAS	APA DUE ENV HDF SPN	APA DUE ENV HDF SKI SPN	APA DUE ENV SKI SPN
Start	16/06/2000	21/12/2001	16/12/2005	02/03/2007	02/03/2007
End	28/06/2013	06/10/2006	23/11/2012	23/11/2012	28/06/2013
$\hat{\beta}$	0.5171	0.4505	0.4778	0.4730	0.4497
s.e	0.0399	0.0702	0.0467	0.0506	0.0467
$\hat{\beta}_u$	0.5952	0.5881	0.5693	0.5721	0.5413
$\hat{\beta}_l$	0.4389	0.3128	0.3862	0.3739	0.3581
$\tilde{\beta}$	0.5688	0.3730	0.5022	0.5145	0.5091
s.e	0.0399	0.0704	0.0469	0.0507	0.0470
$\tilde{\beta}_u$	0.6471	0.5110	0.5941	0.6139	0.6011
$\tilde{\beta}_l$	0.4905	0.2351	0.4103	0.4151	0.4171
N	680	250	362	299	330
R^2	0.1988	0.1423	0.2252	0.2275	0.2201

Table A7: Raw estimates of β for Table 17

Fixed portfolio – Value weighting

Longest sample available but excluding tech boom and GFC – Weekly frequency

	P1	P2	P3	P4	P5
Firms	APA ENV	AAN AGL APA ENV GAS	APA DUE ENV HDF SPN	APA DUE ENV HDF SKI SPN	APA DUE ENV SKI SPN
Start	16/06/2000	21/12/2001	16/12/2005	02/03/2007	02/03/2007
End	28/06/2013	06/10/2006	23/11/2012	23/11/2012	28/06/2013
$\hat{\beta}$	0.5364	0.4497	0.5475	0.5443	0.5288
s.e	0.0473	0.0706	0.0483	0.0534	0.0507
$\hat{\beta}_u$	0.6291	0.5882	0.6422	0.6490	0.6282
$\hat{\beta}_l$	0.4437	0.3113	0.4527	0.4396	0.4294
$\tilde{\beta}$	0.5186	0.3635	0.5243	0.5872	0.5982
s.e	0.0474	0.0709	0.0484	0.0536	0.0509
$\tilde{\beta}_u$	0.6114	0.5024	0.6192	0.6922	0.6980
$\tilde{\beta}_l$	0.4257	0.2245	0.4294	0.4823	0.4984
N	537	248	300	237	268
R^2	0.1938	0.1415	0.3009	0.3064	0.2900

Annex A: Value Weights:

Weights used to construct value weighted portfolios in Tables 16, 21, and 24.

P1	APAX	ENVX
Market Cap	1561	821
Weight	**0.6553**	**0.3447**

P2	AAN	AGKX	APAX	ENVX	GASX
Market Cap	1642	5784	839	787	323
Weight	**0.1752**	**0.6169**	**0.0895**	**0.0839**	**0.0345**

P3	APAX	DUEX	ENVX	HDFX	SPAU
Market Cap	1941	1632	856	670	2611
Weight	**0.2518**	**0.2116**	**0.1110**	**0.0870**	**0.3386**

P4	APAX	DUEX	ENVX	HDFX	SKIX	SPAU
Market Cap	2059	1707	840	718	1604	2583
Weight	**0.2165**	**0.1795**	**0.0884**	**0.0754**	**0.1687**	**0.2715**

P5	APAX	DUEX	ENVX	SKIX	SPAU
Market Cap	2333	1790	917	1666	2703
Weight	**0.2480**	**0.1903**	**0.0975**	**0.1770**	**0.2873**

Weights used to construct value weighted portfolios in Tables 17, 22.

P1	APAX	ENVX
Market Cap	1721	900
Weight	**0.6567**	**0.3433**

P2	AAN	AGKX	APAX	ENVX	GASX
Market Cap	1651	5804	841	789	323
Weight	**0.1755**	**0.6169**	**0.0894**	**0.0838**	**0.0344**

P3	APAX	DUEX	ENVX	HDFX	SPAU
Market Cap	2043	1685	916	730	2706
Weight	**0.2529**	**0.2086**	**0.1134**	**0.0903**	**0.3349**

P4	APAX	DUEX	ENVX	HDFX	SKIX	SPAU
Market Cap	2218	1794	913	804	1700	2696
Weight	**0.2190**	**0.1772**	**0.0901**	**0.0794**	**0.1679**	**0.2662**

P5	APAX	DUEX	ENVX	SKIX	SPAU
Market Cap	2535	1886	998	1765	2831
Weight	**0.2531**	**0.1883**	**0.0997**	**0.1762**	**0.2827**